

LAKSHYA BATCH



Magnetism and Matter
Questions on Bar magnet

LECTURE - 4



GOALS OF THE DAY

- ❖ Questions on bar magnet





Units:

SI
 Magnetic field Intensity
 Magnetic flux density
 Magnetic induction
 \vec{B} T
 Tesla, (Wb/m²)

Cgs
 Gauss 10⁻⁴
 (G)
 (1 G = 10⁻⁴ T)

Magnet Pole strength (m) = $\frac{M}{l}$ Am
 $\vec{M} = m l$

⇒ Magnetic Dipole moment: Am²
 $\vec{M} = I \cdot A$

Stat A cm² (3.3 x 10⁻¹⁴)
 1 Stat A cm² = 3.3 x 10⁻¹⁴ Am²

⇒ Magnetic flux Weber

Maxwell or abweber 10⁻⁸

$$\Phi_B = B \cdot A$$

$$B = \frac{\Phi_B}{A} = \frac{Wb}{m^2}$$

$$\left(\begin{array}{l} 1 \text{ abwb} = 10^{-8} \text{ Wb} \\ 1 \text{ Mx} \end{array} \right)$$

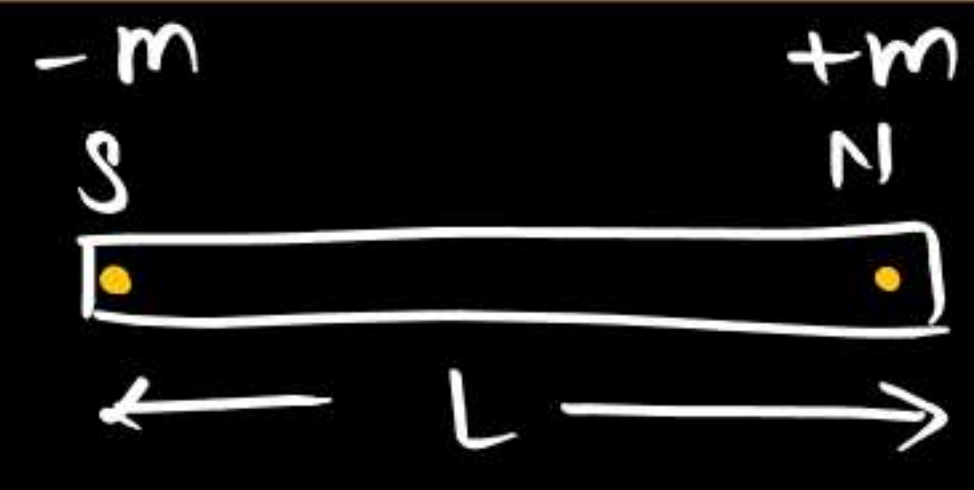




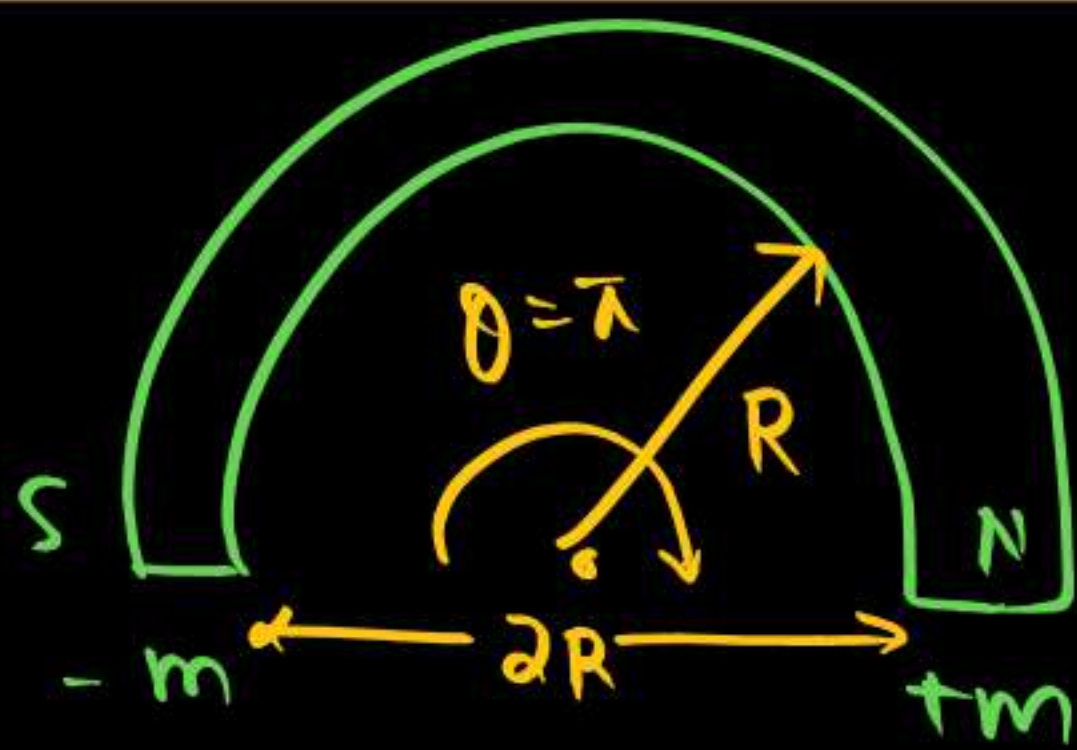
An iron rod of length L and magnetic moment M is bent in the form of a semicircle. Now its magnetic moment will be (March) 2021.

- (a) M
- (b) $\frac{2M}{\pi}$ Ans
- (c) $\frac{M}{\pi}$
- (d) $M\pi$

Gen for



$$M = mL$$



$$\pi R = L$$
$$R = \frac{L}{\pi}$$

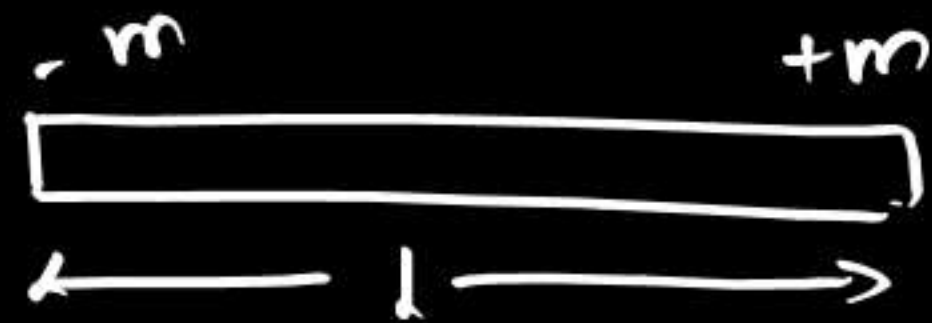
$$\bar{m} = \frac{2M \sin(\frac{\theta}{2})}{\theta}$$

$$\theta = \pi$$
$$= \frac{2M \sin 90}{\pi}$$
$$= \frac{2M}{\pi}$$

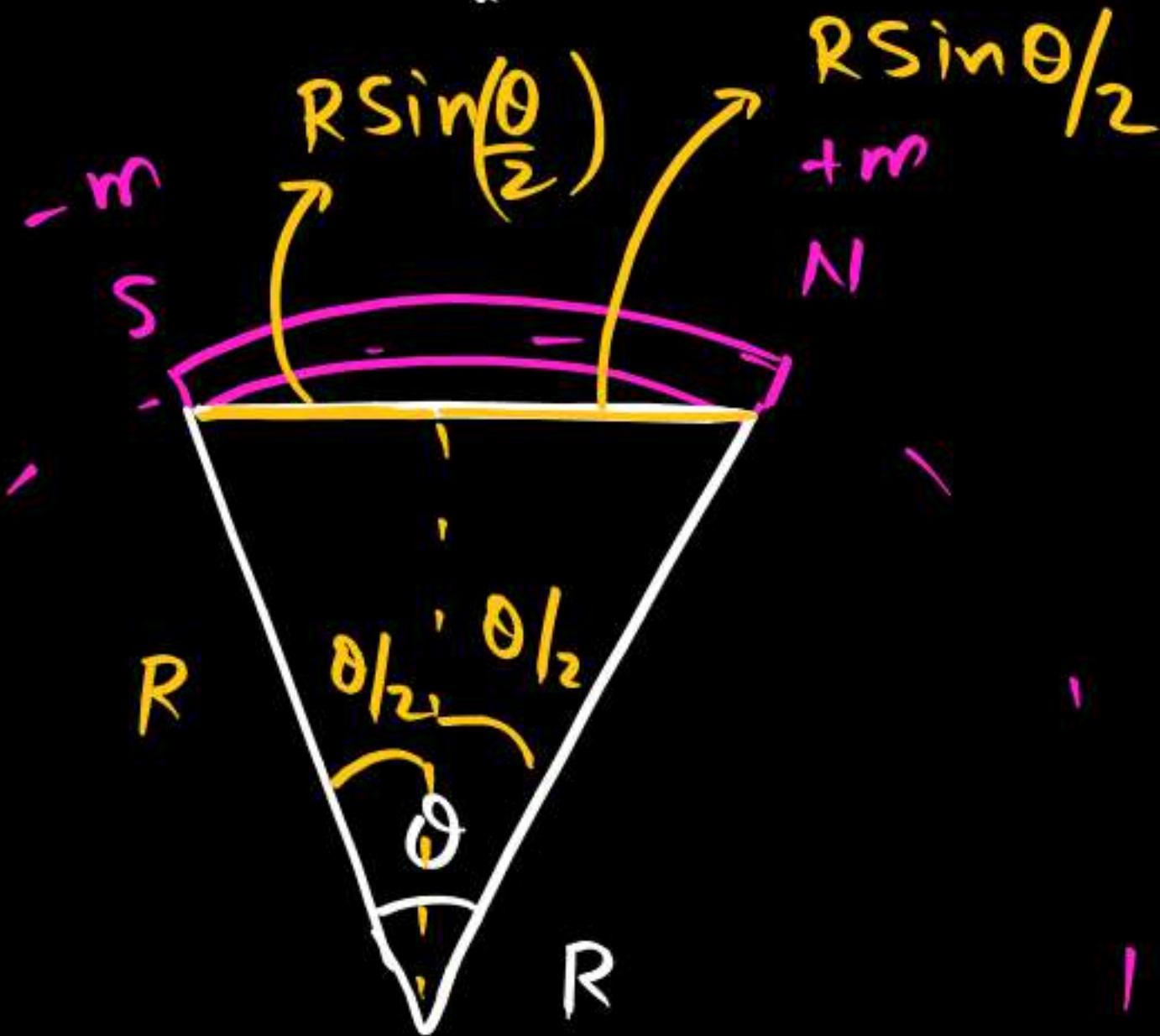
$\vec{M} = (\text{either Pole}) \left(\begin{array}{l} \text{Sep between} \\ \text{them} \end{array} \right)$
Strength (Magnetic length.)

$$\vec{M}' = m \cdot 2R$$
$$= 2m \frac{L}{\pi} = \frac{2M}{\pi}$$





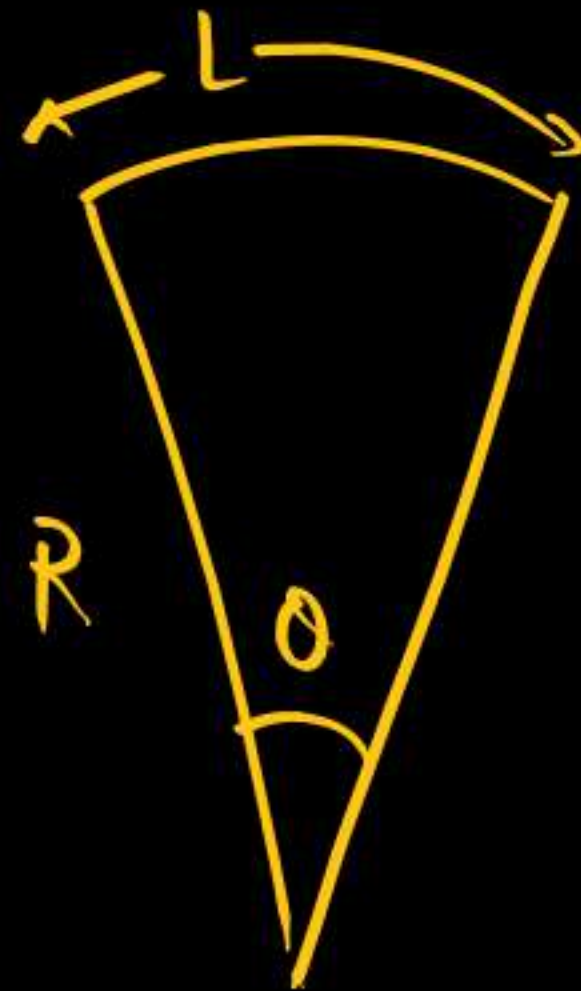
$$\vec{M} = mL$$



$$\vec{M}_{\text{new}} = m \cdot 2R \sin\left(\frac{\theta}{2}\right)$$

$$= \frac{2mL}{\theta} \sin\left(\frac{\theta}{2}\right)$$

$$= \frac{2M}{\theta} \sin\left(\frac{\theta}{2}\right)$$



$$\theta = \frac{L}{R}$$

$$R = \frac{L}{\theta}$$

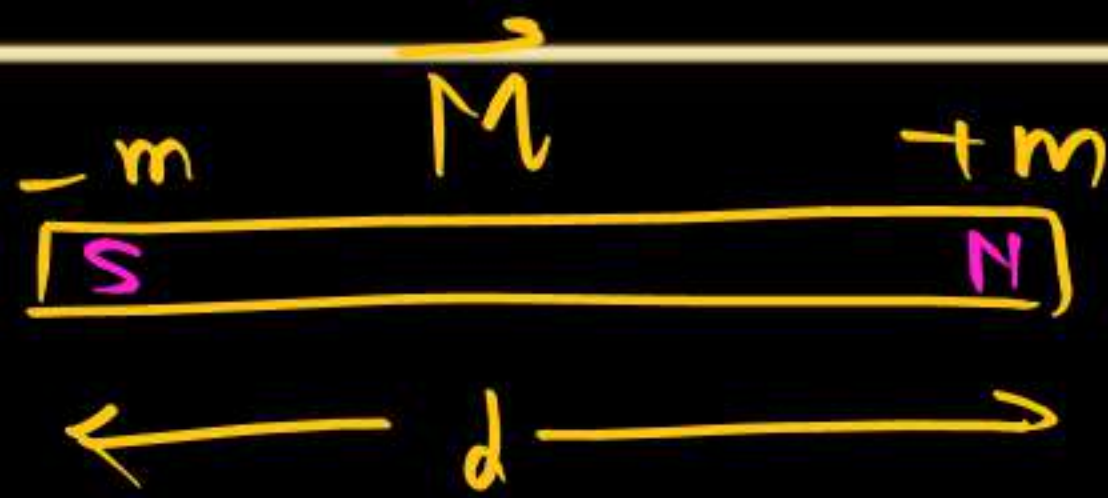
The length of a magnetized steel wire is l and its magnetic moment is M . It is bent into the shape of L with two sides equal. The magnetic moment now will be

(a) $\frac{M}{2}$

(b) $2M$

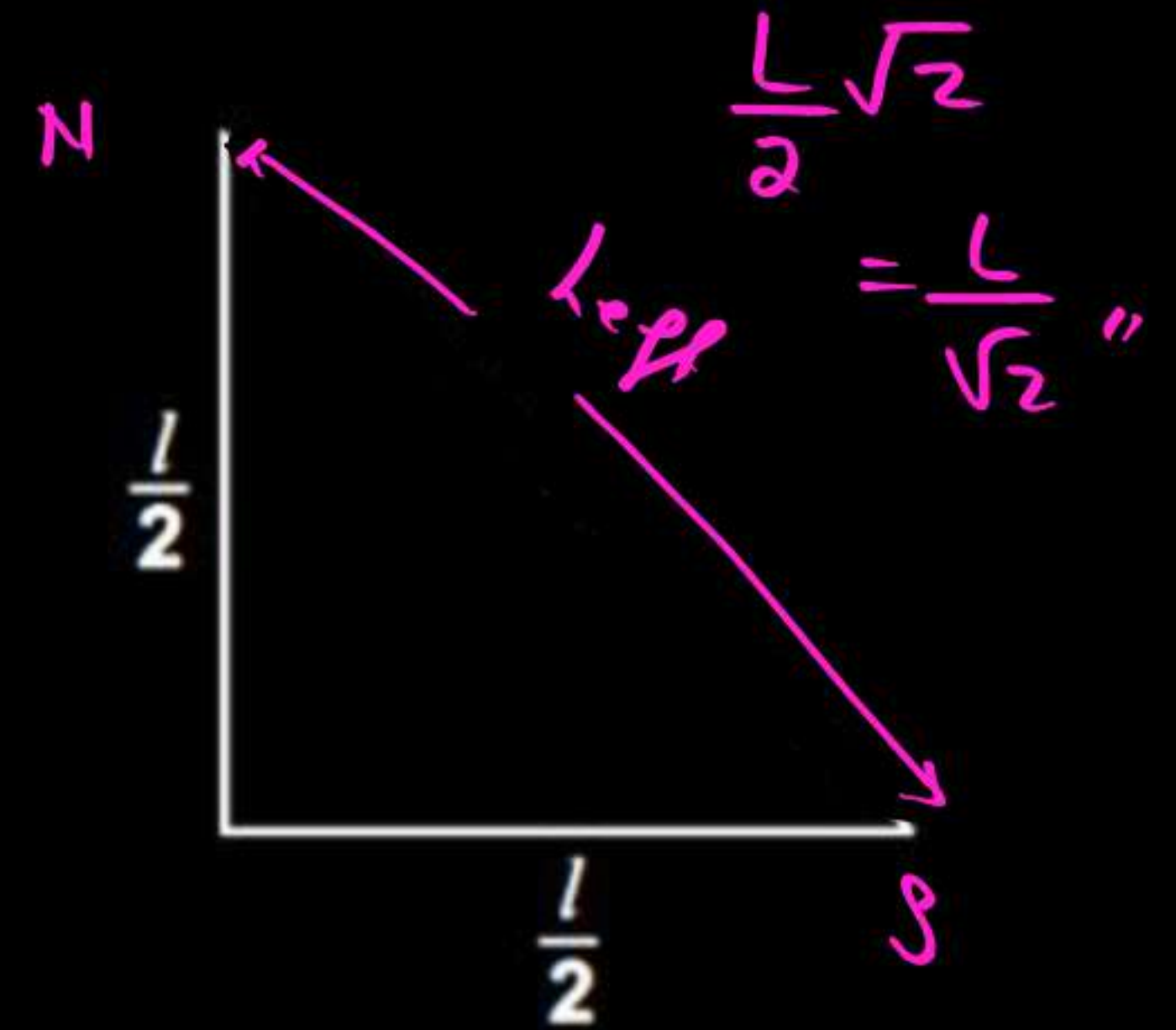
(c) $\sqrt{2} M$

(d) $M/\sqrt{2}$ Ans



$$|\vec{M}| = M = m l$$

$$M_{\text{new}} = \frac{m l}{\sqrt{2}} = \frac{M}{\sqrt{2}}$$



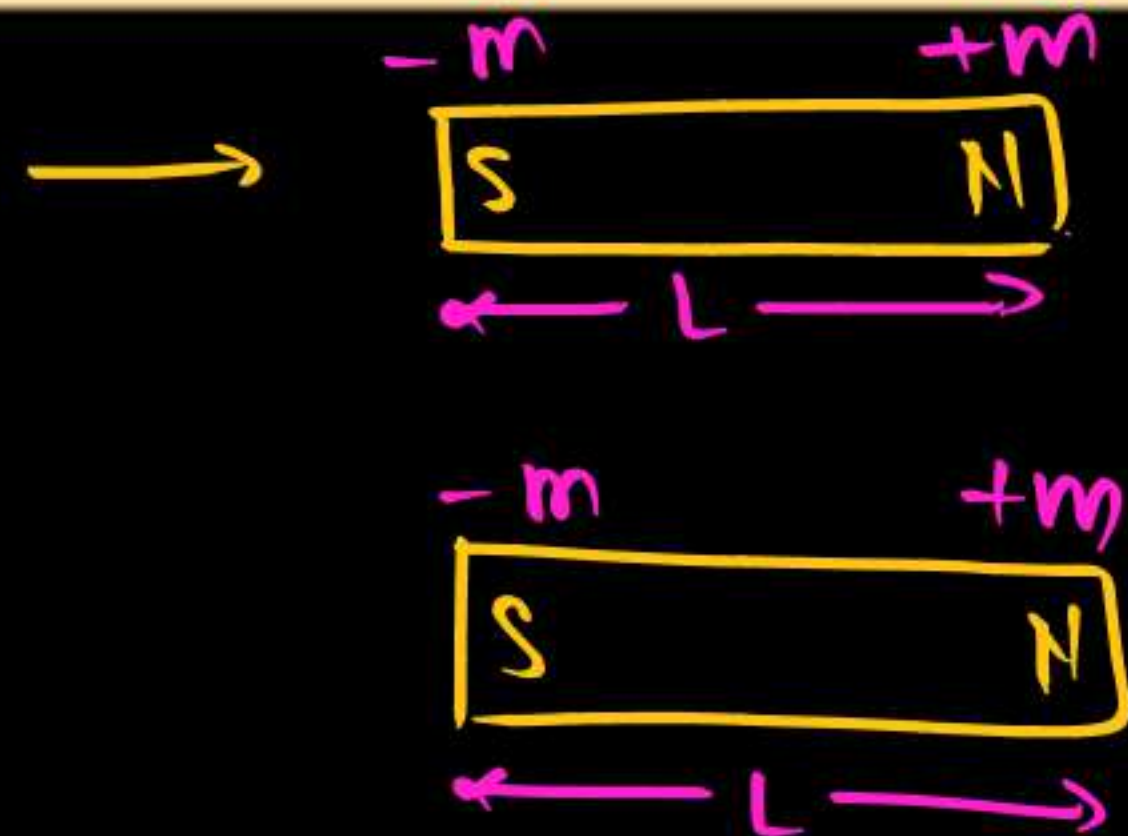
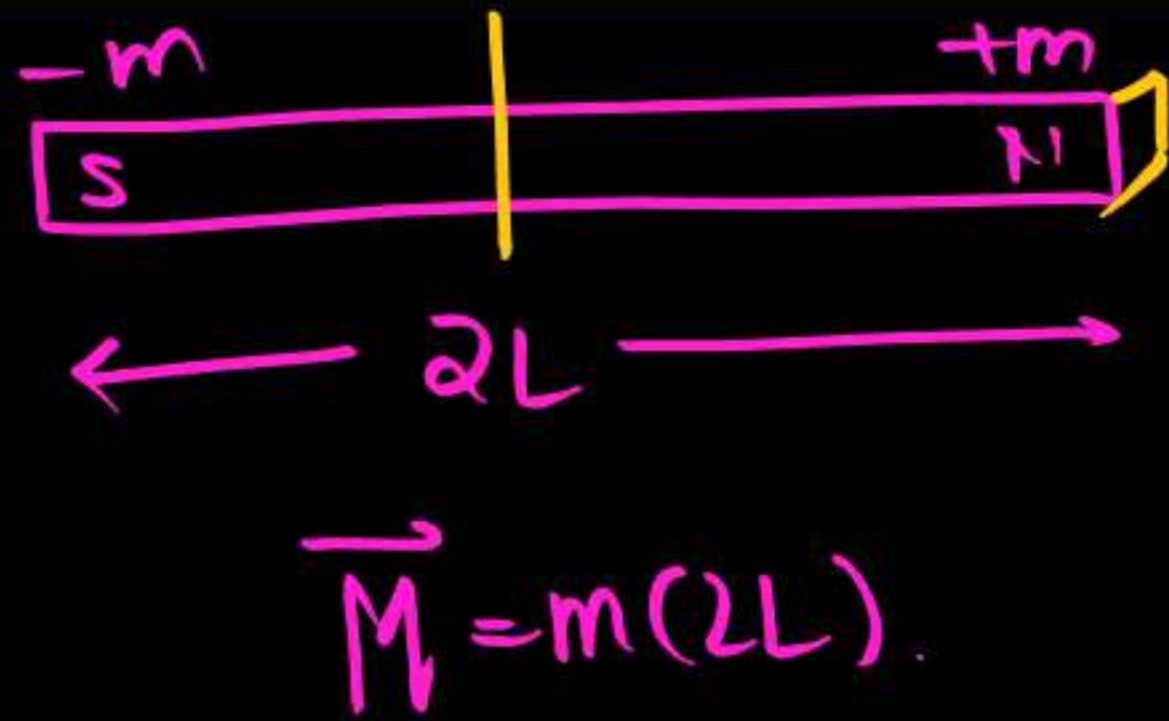
A long magnetic needle of length $2L$, magnetic moment M and pole strength m units is broken into two pieces at the middle. The magnetic moment and pole strength of each piece will be

(a) $\frac{M}{2}, \frac{m}{2}$

(b) $M, \frac{m}{2}$

(c) $\frac{M}{2}, m$ **Ans**
Cut

(d) M, m



Pole Strength = $\pm m$ Same

$\vec{M} = (\text{Pole strength}) (\text{Sep})$

$= mL$

$|\vec{M}| = \frac{M}{2}$



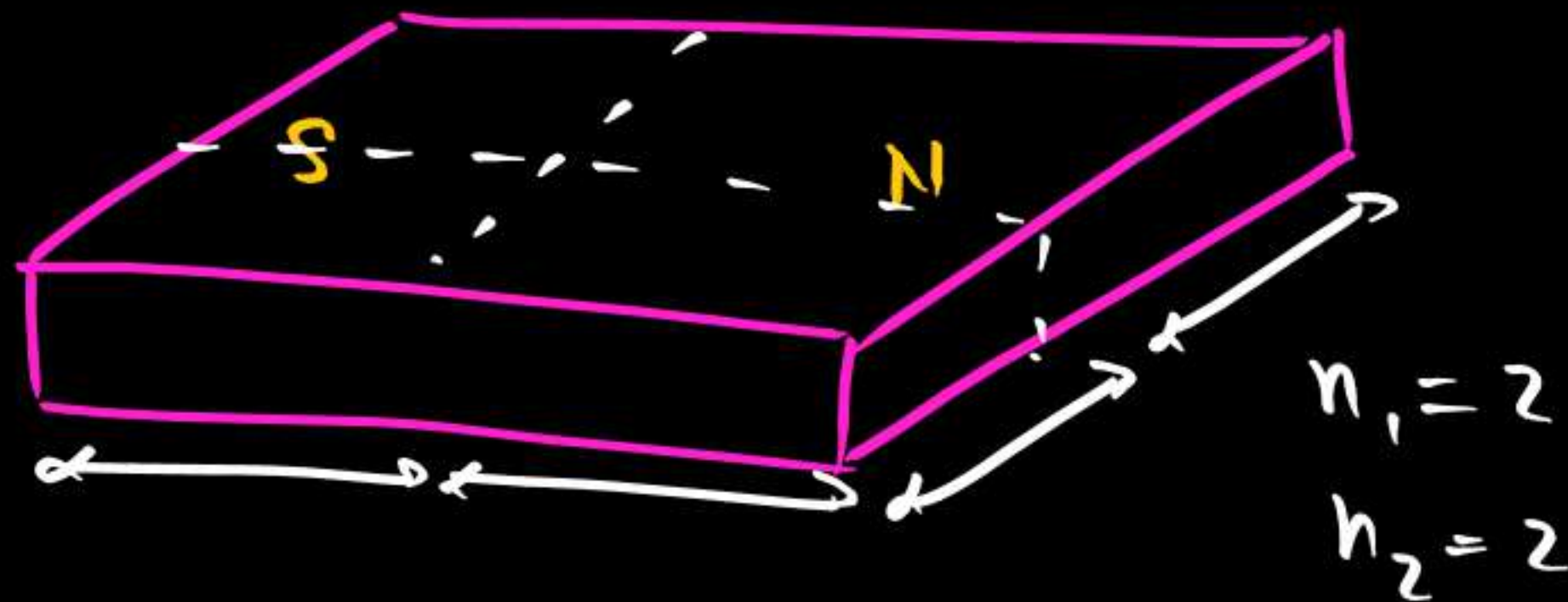
If a magnet of pole strength m is divided into four parts such that the length and width of each part is half that of initial one, then the pole strength of each part will be

(a) $m/4$ *Ans.*

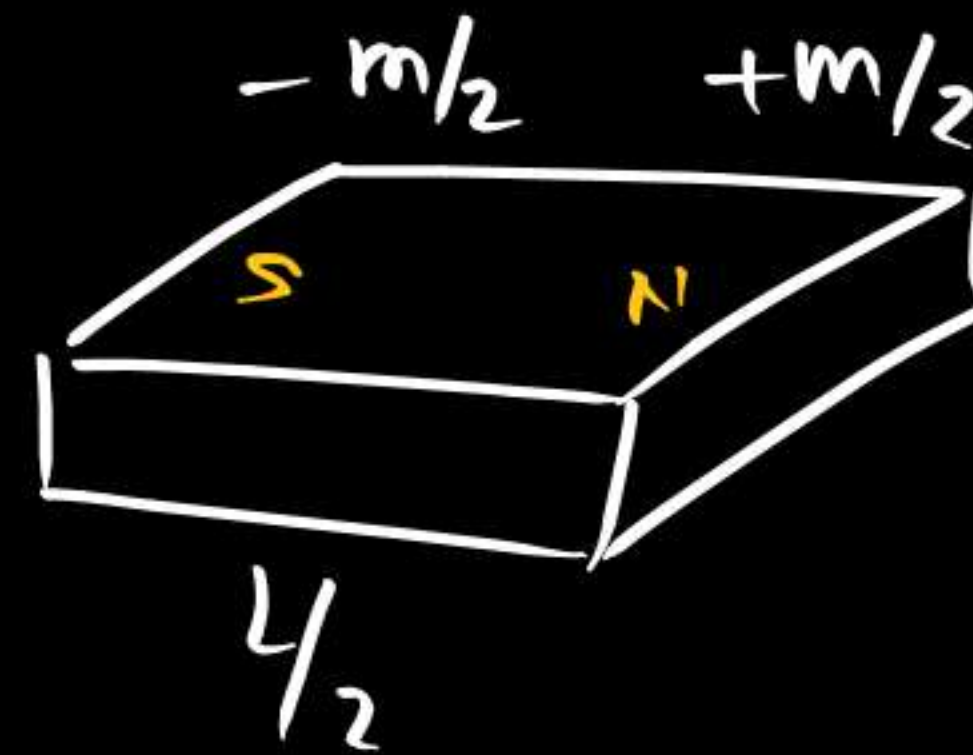
(b) $m/2$

(c) $m/8$

(d) $4m$



$$\vec{M} = \frac{M}{n_1 n_2} = \frac{M}{4}$$



$$\vec{M} = \left(\frac{m}{2}\right) \left(\frac{L}{2}\right) = \frac{mL}{4} = \frac{M}{4}$$



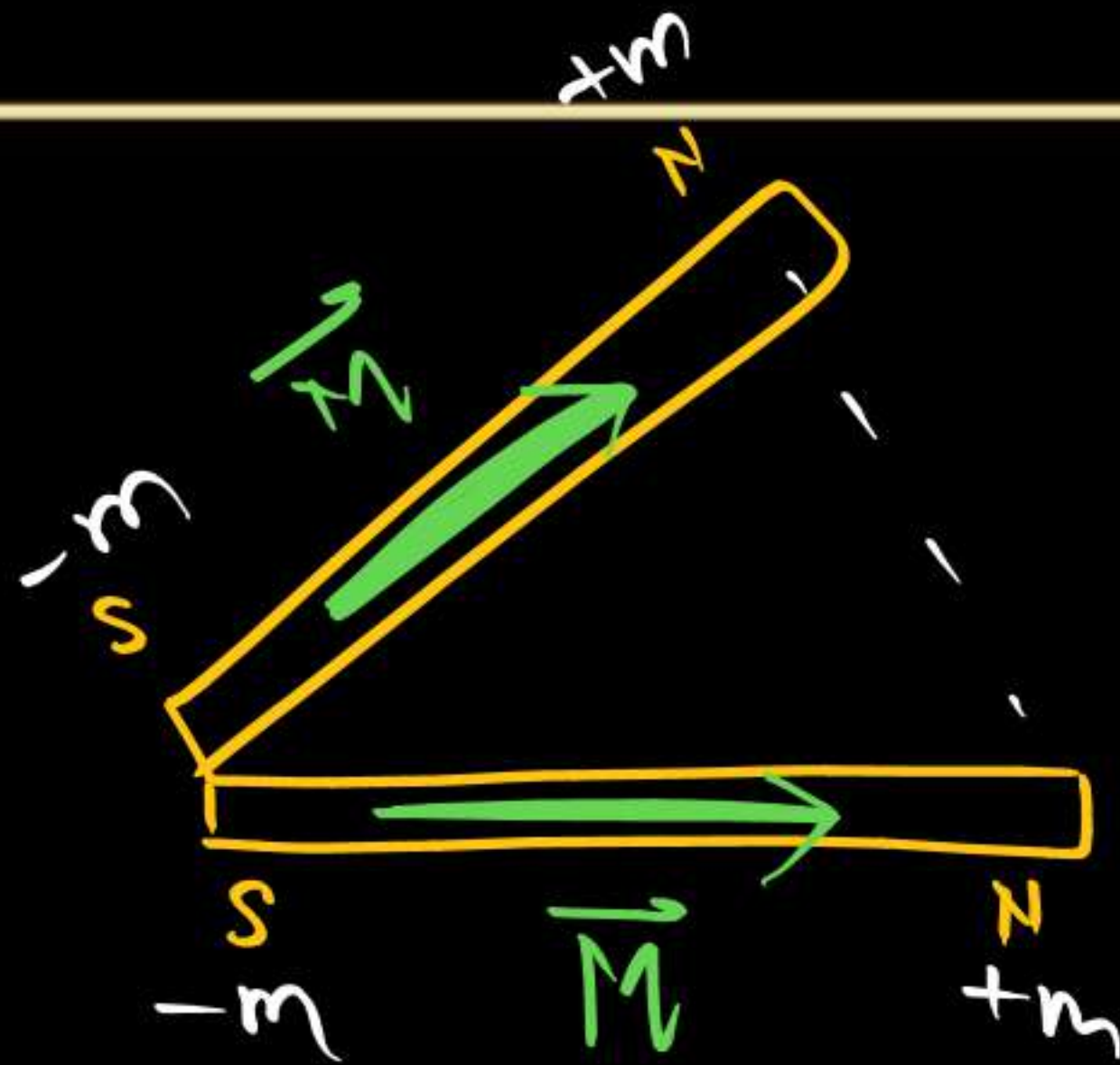
The magnetic moment of the system as shown in figure, will be :

(a) $\sqrt{3} ma$ Ans

(b) ma

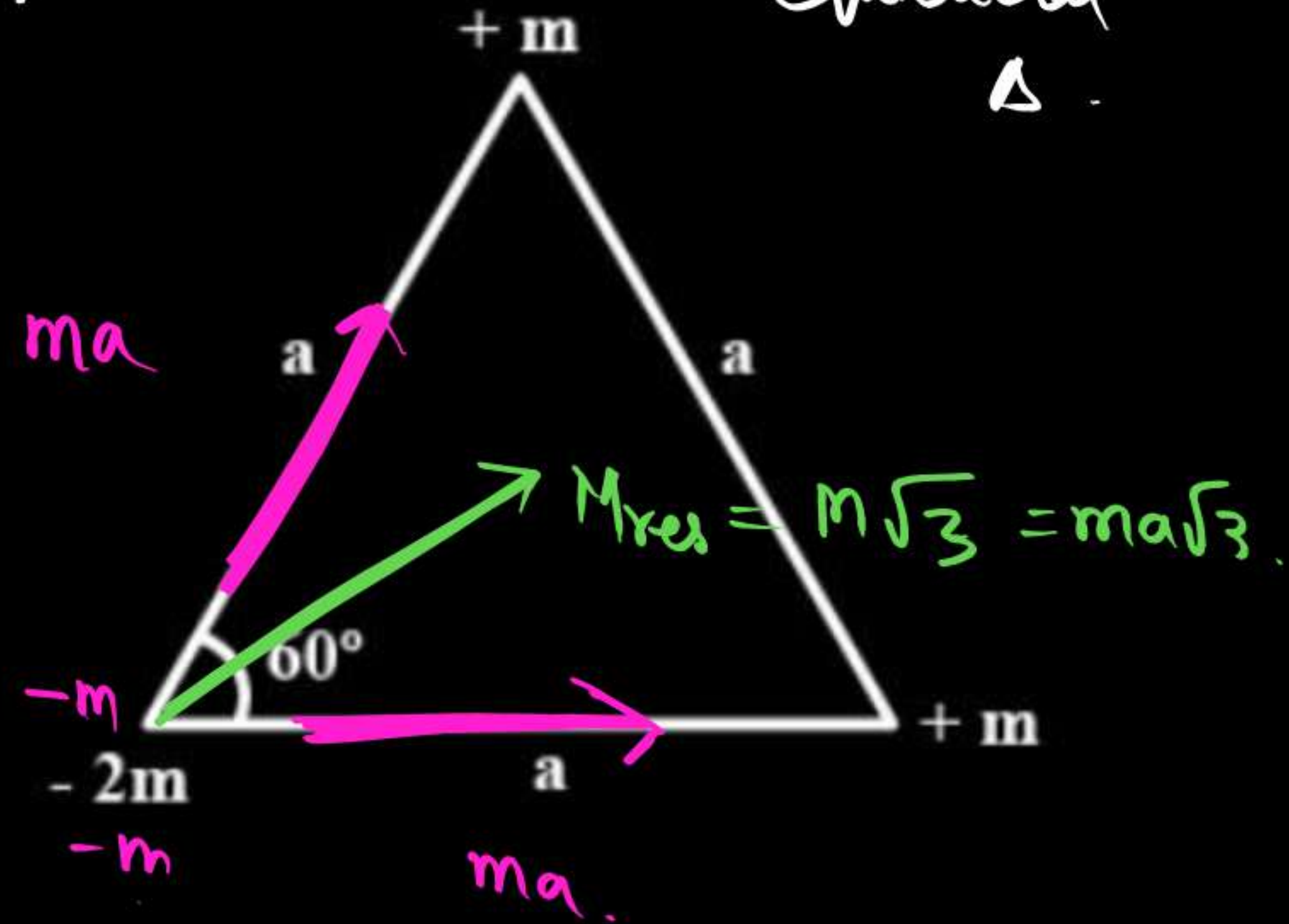
(c) $2ma$

(d) none of the above



Vector Qty hence follows Vector Addition.

⊗ $\vec{M} \Rightarrow$ (magnetic) (Position Vector) Equilateral Δ
Pole Strength with Sign

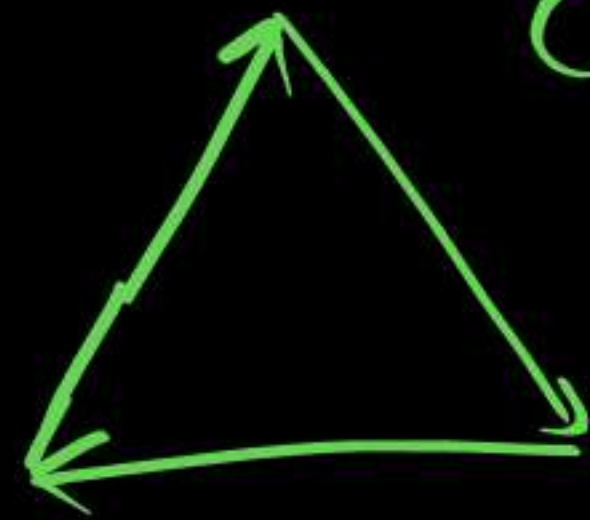


Three identical magnets are arranged as shown in the figure. The magnetic moment of each magnet is M . The effective magnetic moment of the given combination is:

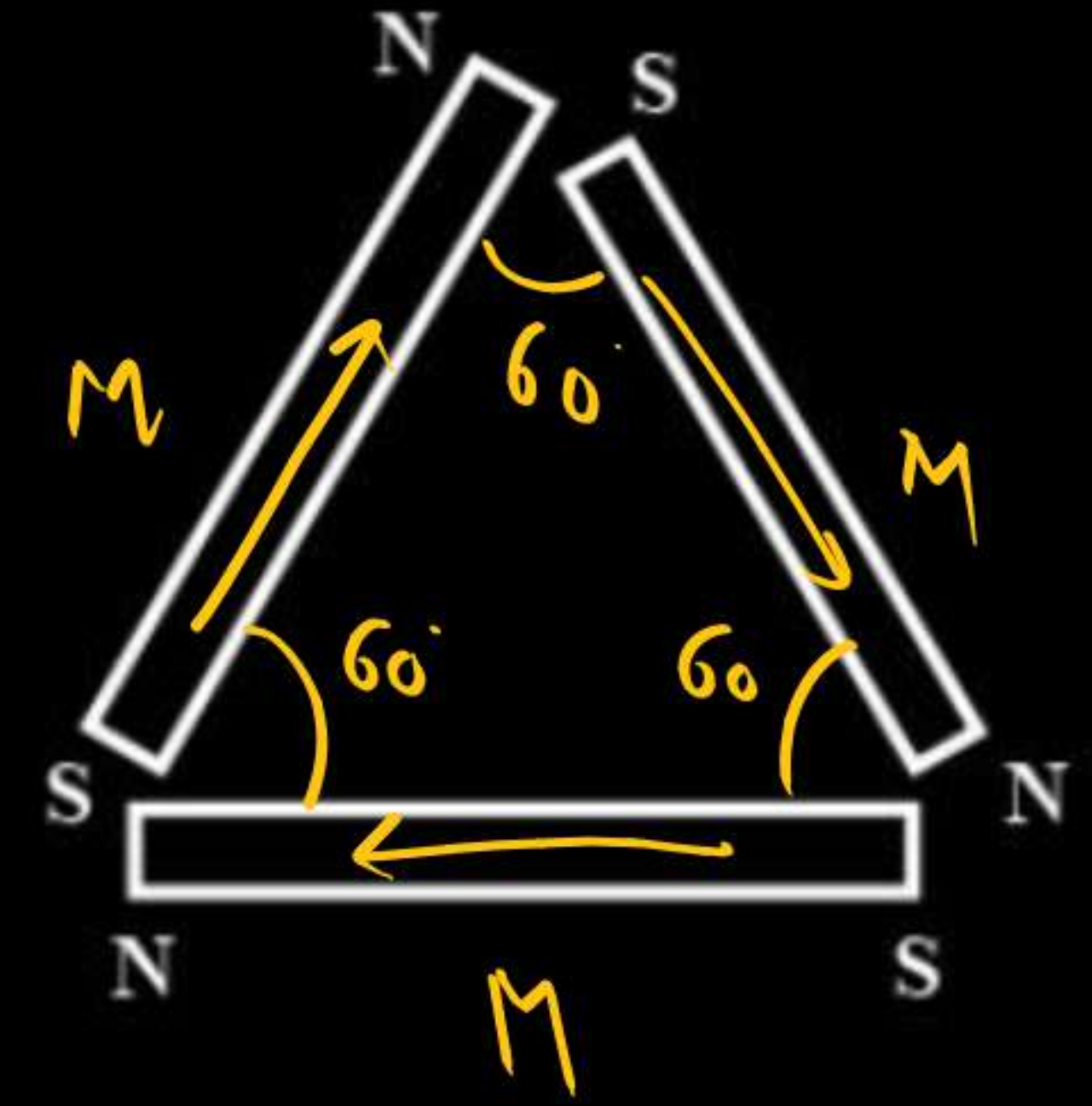
- (a) $6m$
- (b) $3m$
- (c) zero
- (d) $2m$

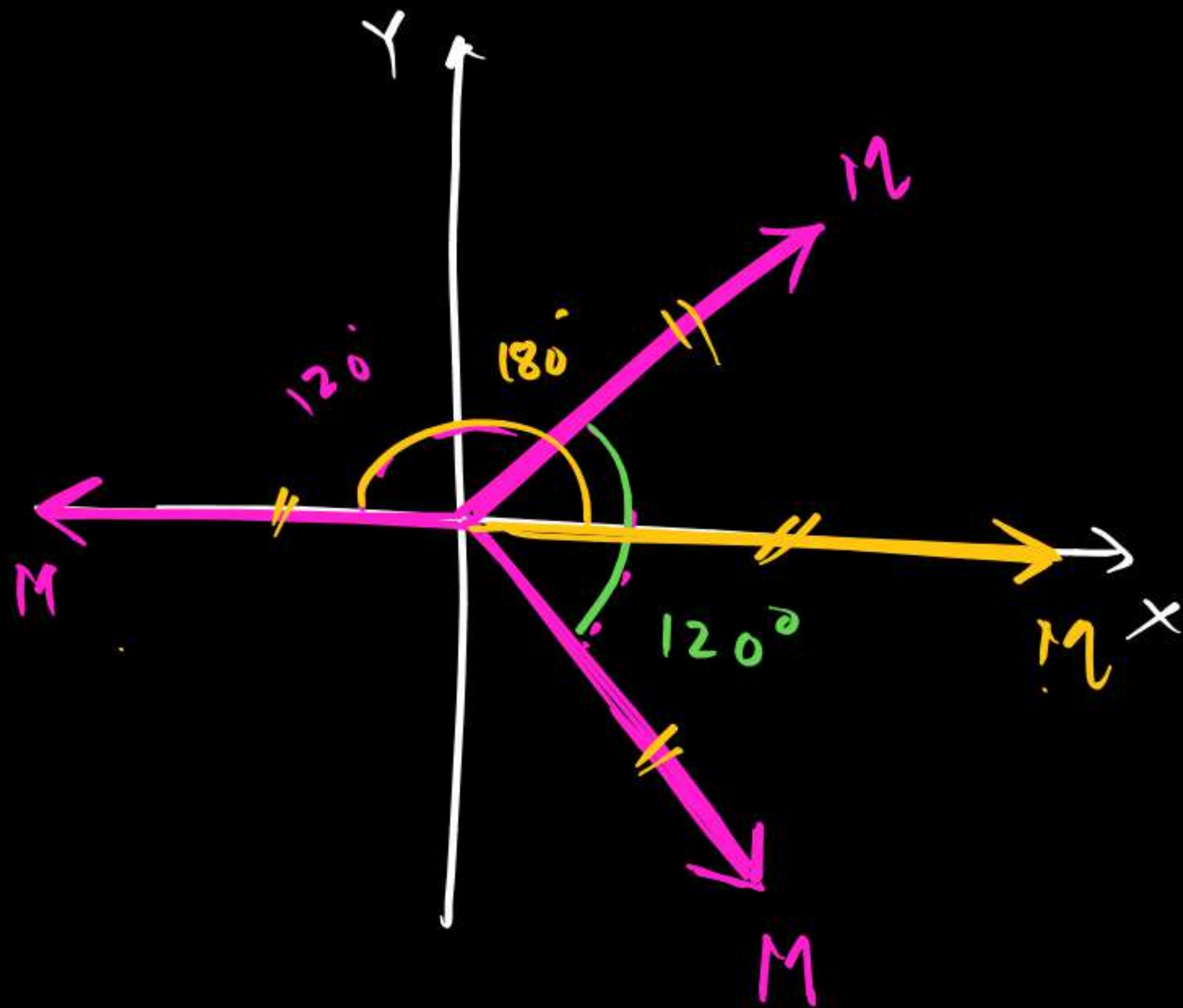
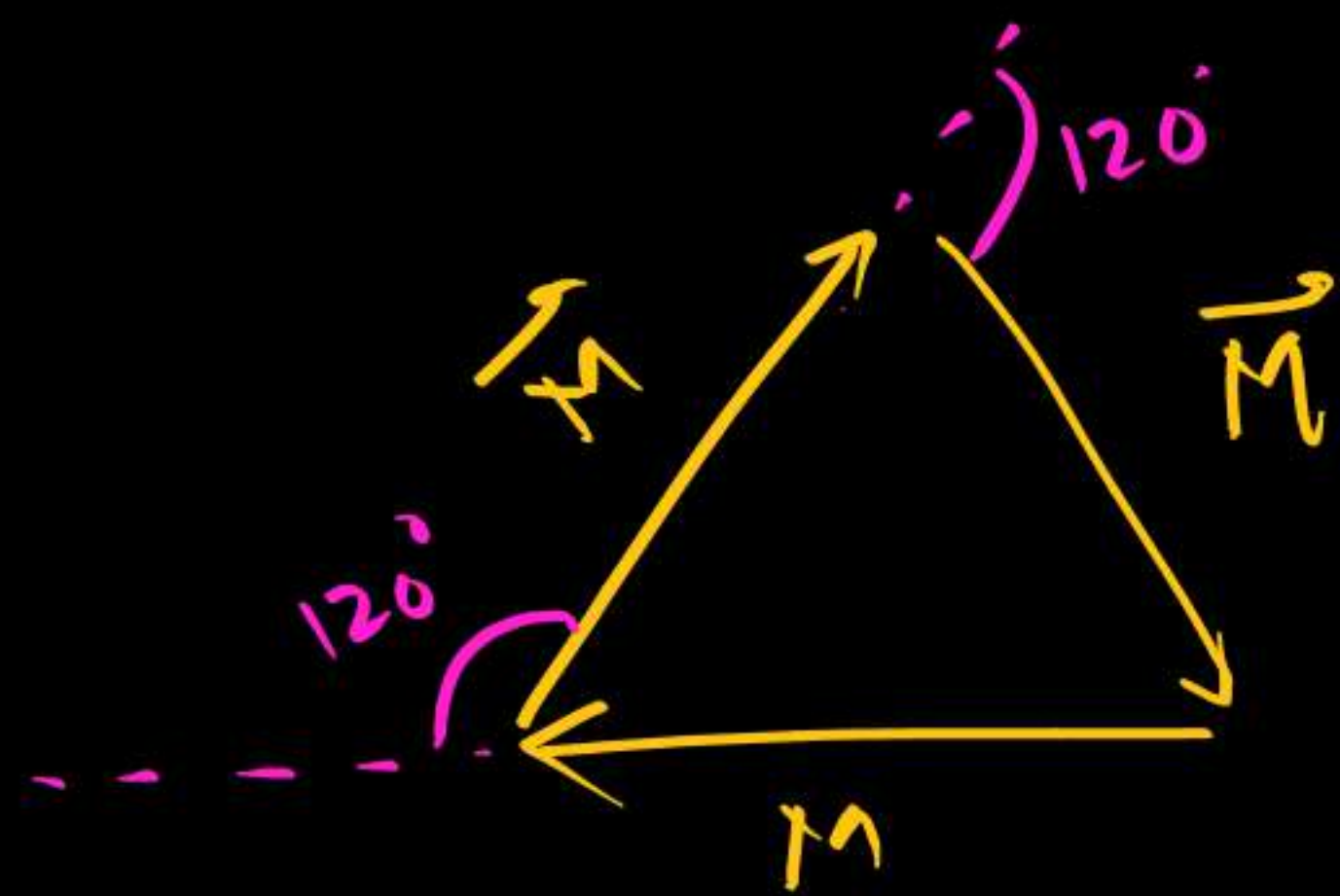
Equilateral Δ

Polygon Law



Closed Polygon
 $R=0$

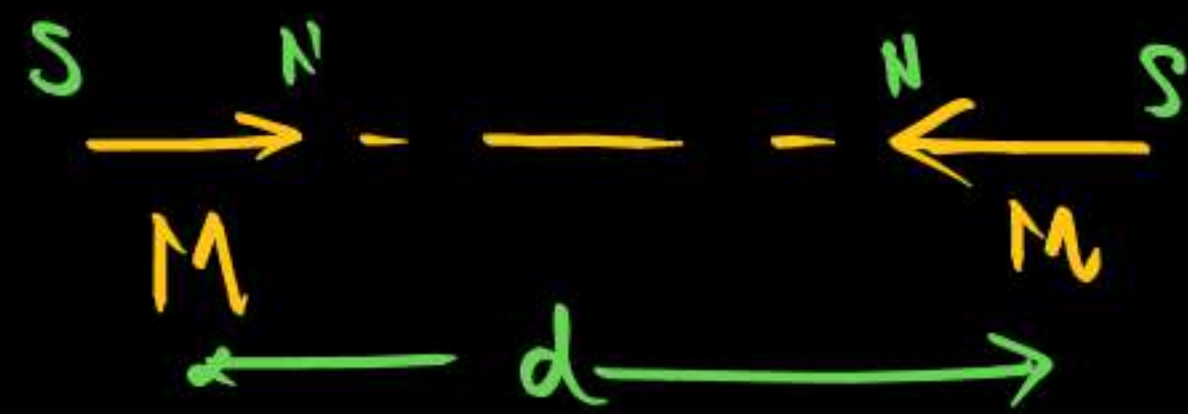




$R=0$

Two small bar magnets are placed in a line with like poles facing each other at a certain distance d apart if the length of each magnet is negligible as compared to d , the force between them will be inversely proportional to

- (a) d
- (b) d^2
- (c) $1/d^2$
- (d) ~~d^4~~ Ans



(In class discussed)

$$f = + \frac{6 \mu_0 m_1 m_2}{4\pi r^4} \text{ (Repulsion)}$$

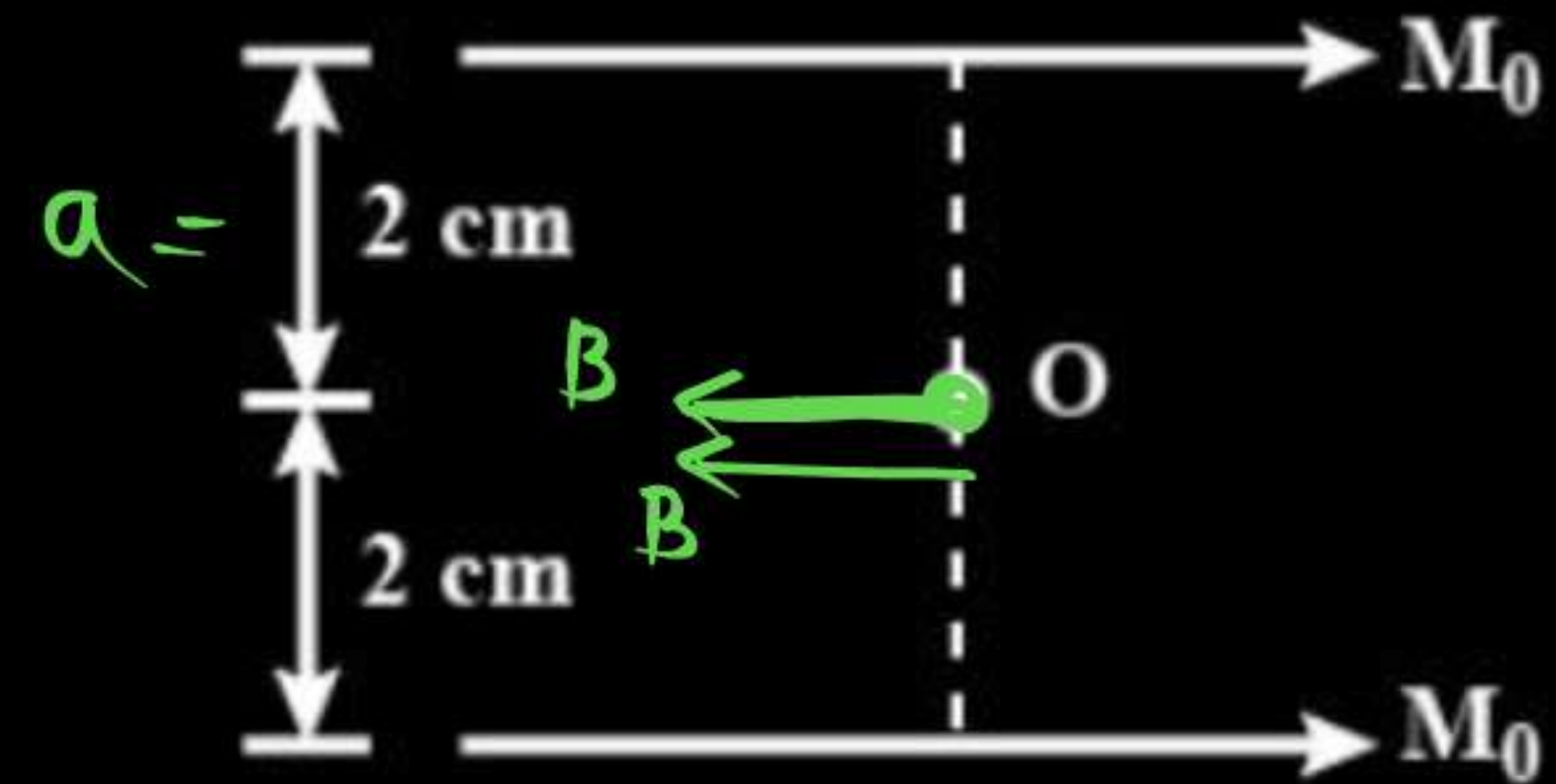
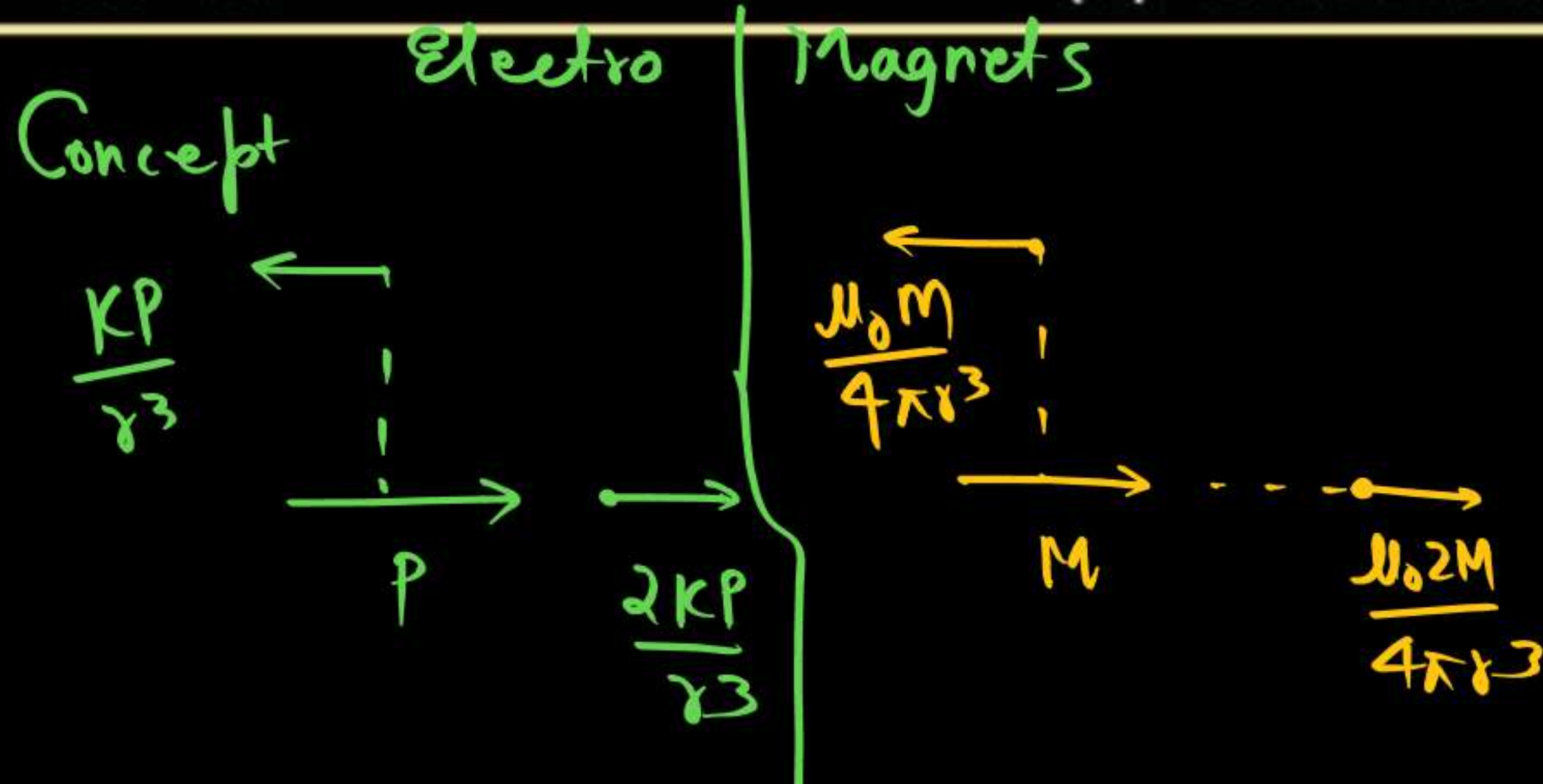
$$f \propto \frac{1}{r^4}$$



Two small magnets each of magnetic moment M_0 is placed parallel to each other (shown in figure). The magnetic field at point O is :

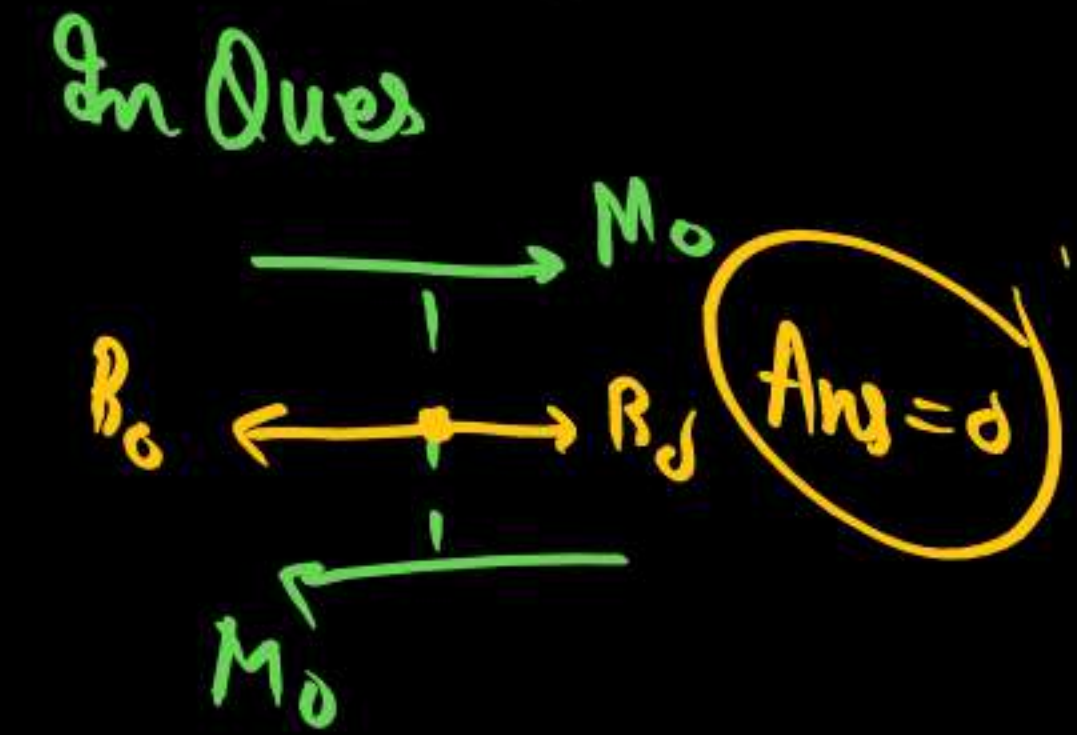
- (a) zero
- (b) $4.5 \times 10^{-4} \text{ N}$
- (c) $2 \times 10^{-4} \text{ N}$
- (d) none of these

option should be in terms of M_0 .



$$B_{\text{Total}} = 2 B_0 = 2 \times \frac{\mu_0 M_0}{4\pi a^3} = \frac{10^{-7} \times 2 M_0}{\left(\frac{2}{100}\right)^3}$$

$$B = \frac{\mu_0 M_0}{4\pi a^3}$$



The magnetic field at point C as shown in figure is :

(a) $\frac{\mu_0 M_0}{2\pi r_0^3}$

(b) $\frac{\sqrt{2}\mu_0 M_0}{2\pi r_0^3}$

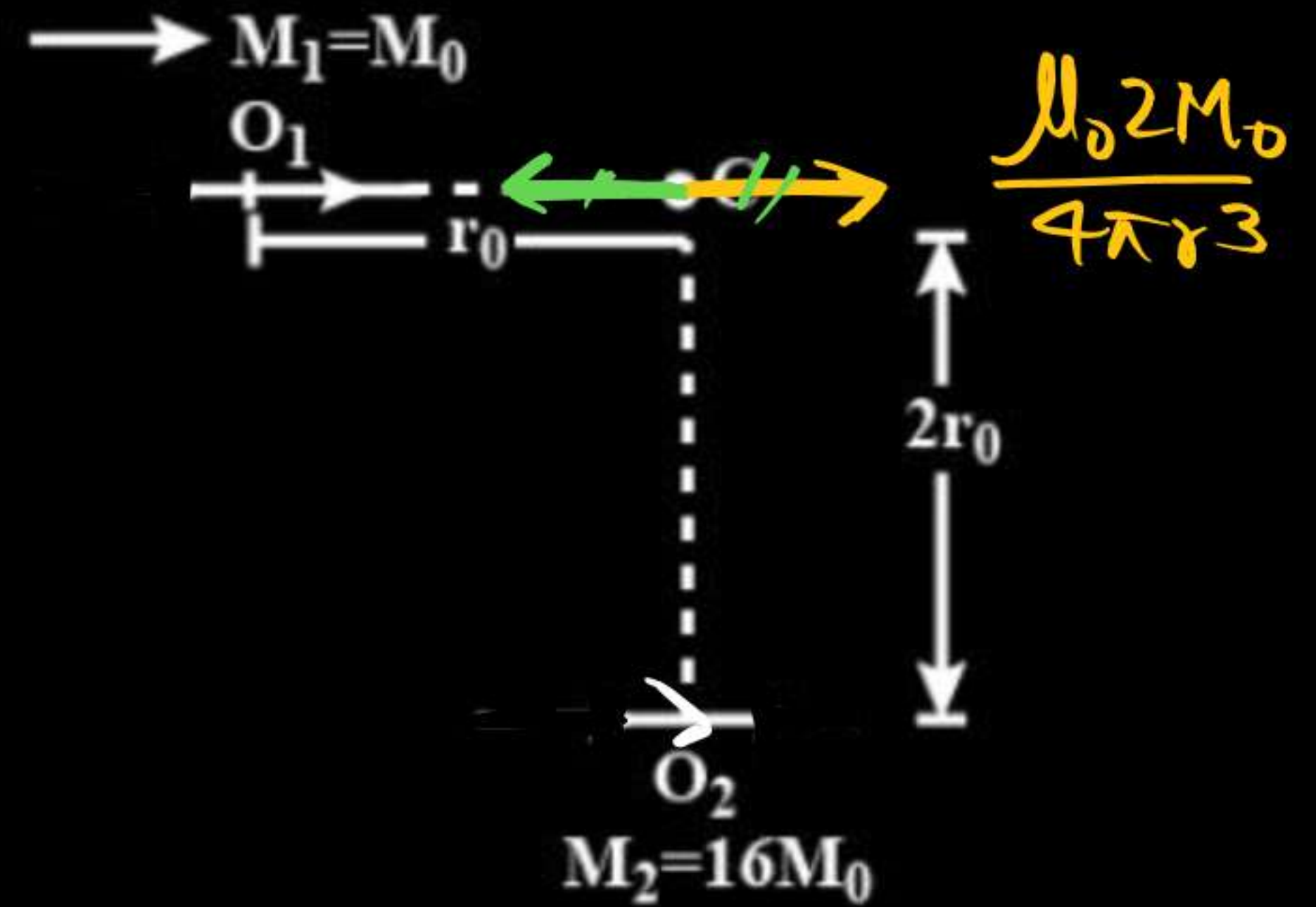
(c) ✓ zero **Ans.**

(d) $\frac{\sqrt{2}\mu_0 M_0}{4\pi r_0^3}$

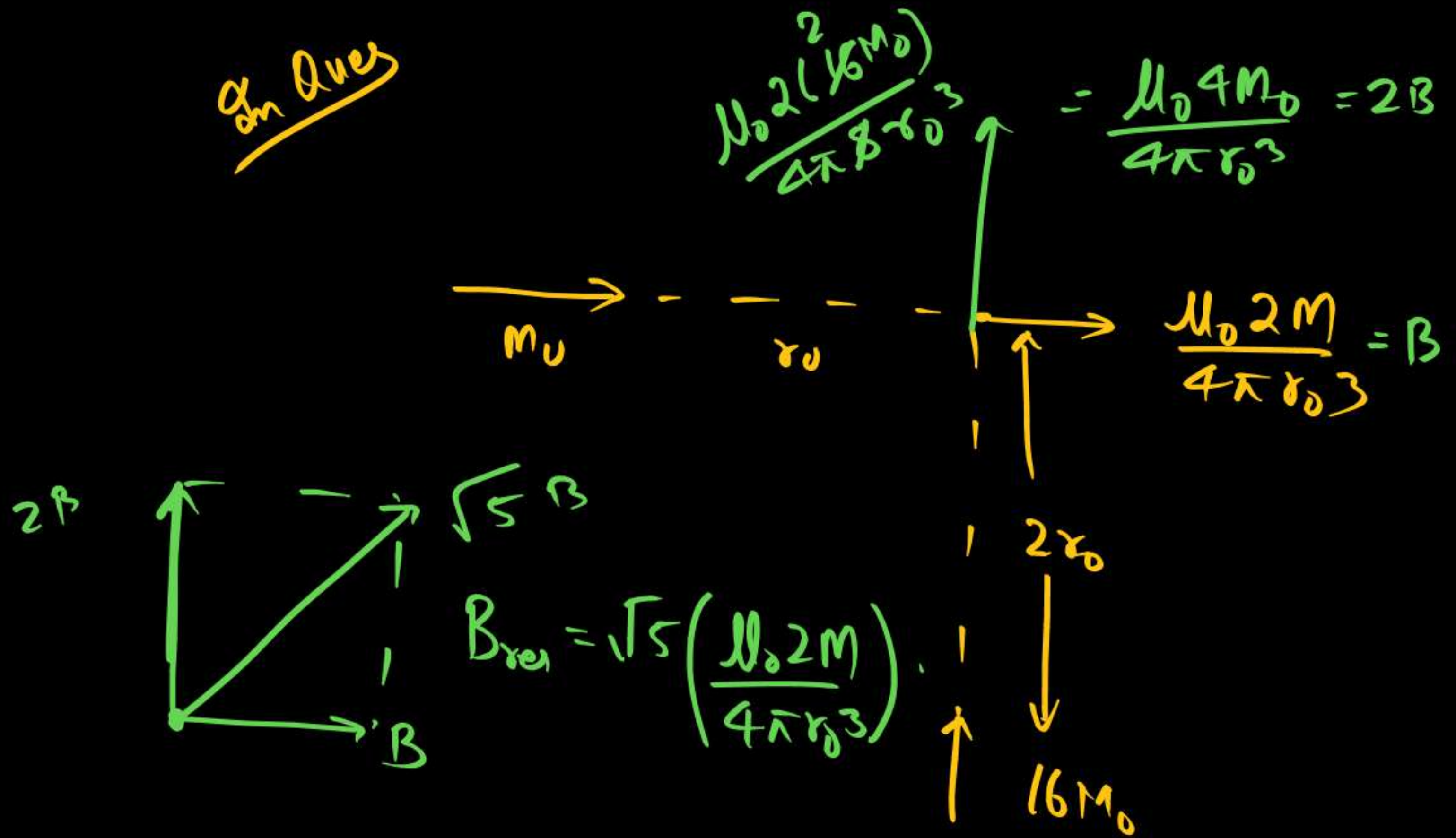
$\frac{\mu_0 16 M_0}{4\pi 8 r_0^3} = \frac{\mu_0 2 M_0}{4\pi r_0^3}$

Pt C for M_1 (axial)

Pt C for M_2 (Equatorial).



Ans Ques



A magnetic needle is kept in a non-uniform magnetic field. It experiences

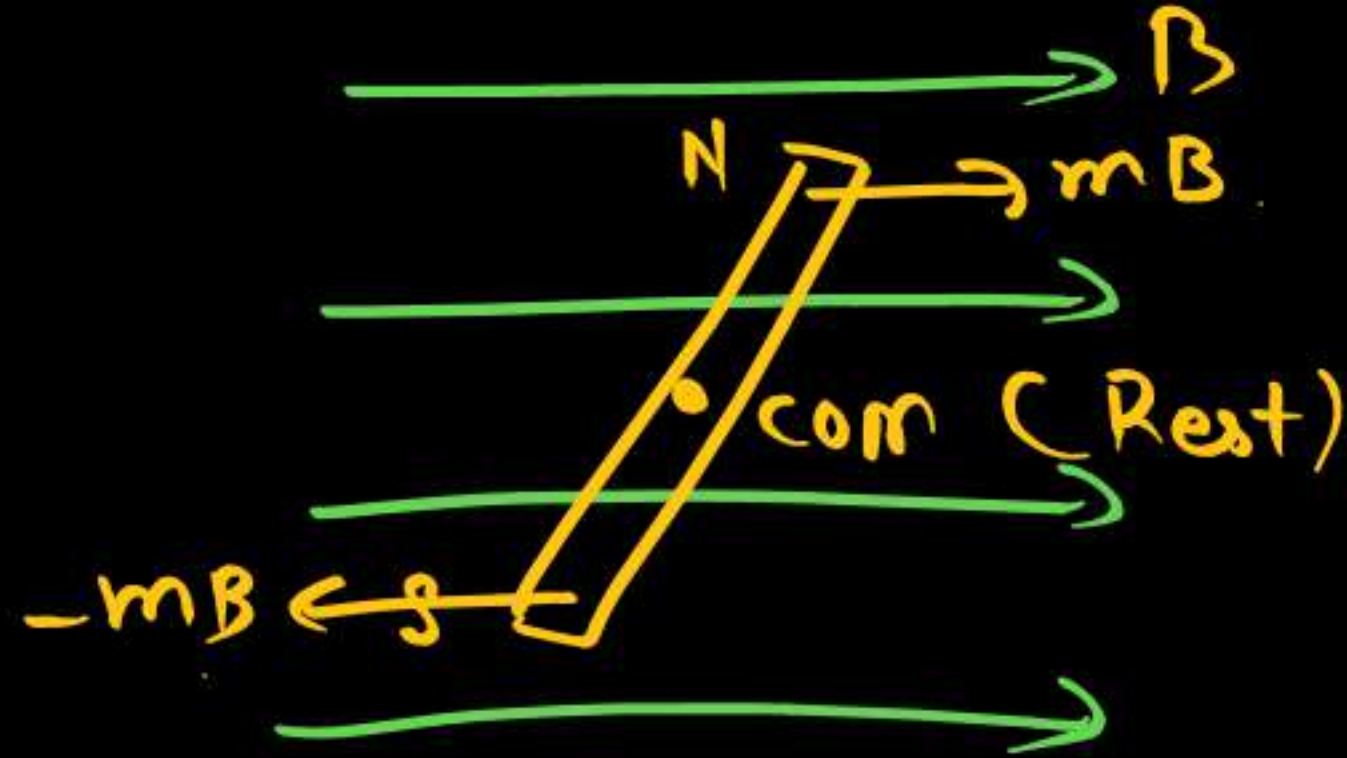
(a) A force and a torque Ans

(c) A torque but not a force

(b) A force but not a torque

(d) Neither a torque nor a force

In uniform \vec{B}



$$f_{net} = 0$$

$$\tau_{net} \neq 0$$

Non-uniform Magnetic field.

$$f_T \neq 0$$

$$\tau_T \neq 0$$



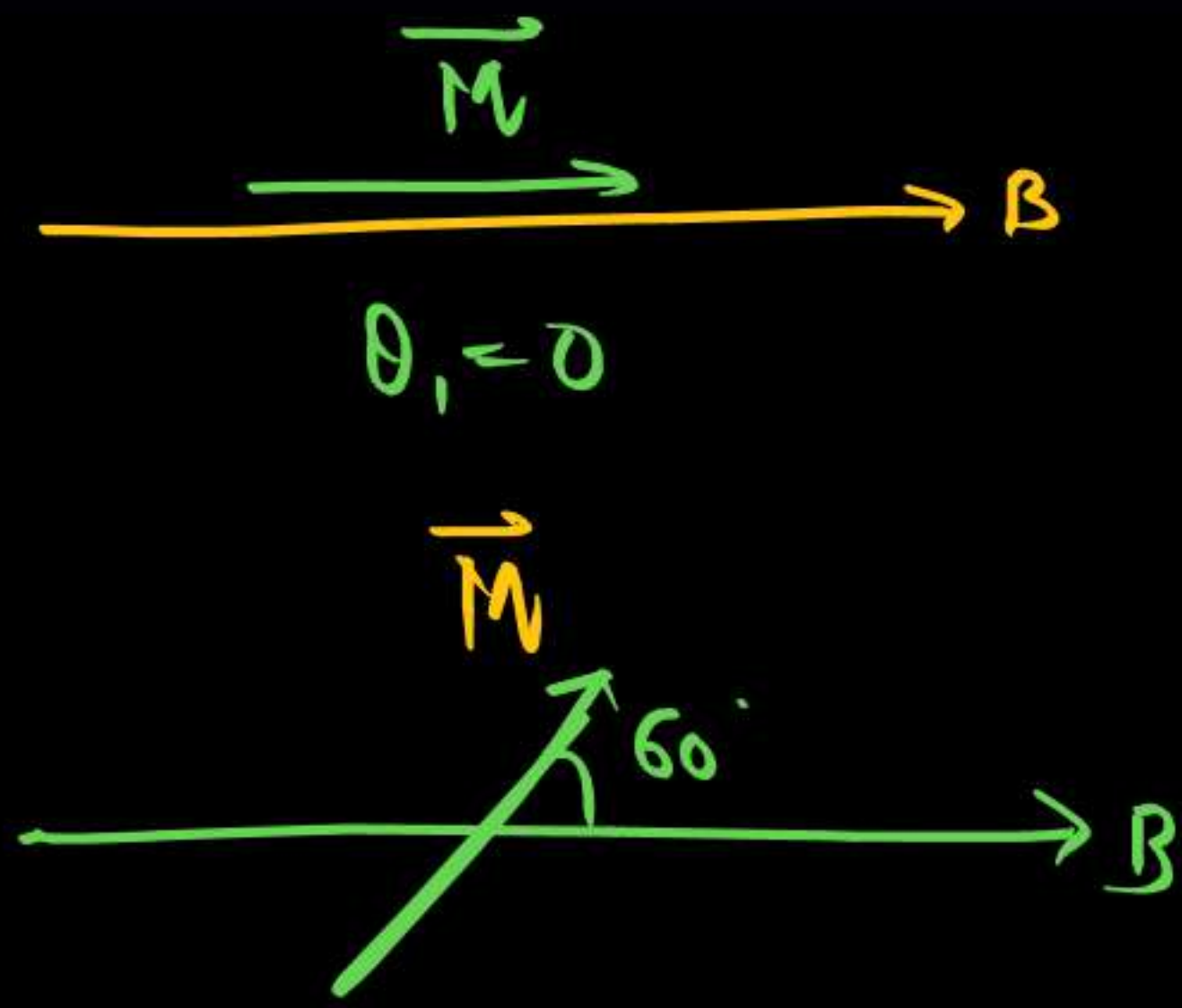
A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60° . The torque required to maintain the needle in this position will be

(a) $\sqrt{3} W$ *Ans.*

(b) W

(c) $\sqrt{3}/2 W$

(d) $2W$



$$\begin{aligned}
 W &= mB (\cos \theta_1 - \cos \theta_2) \\
 &= mB (\cos 0 - \cos 60) \\
 &= mB \left(1 - \frac{1}{2}\right)
 \end{aligned}$$

$$\boxed{W = \frac{mB}{2}}$$

In Second Scenario

$$\vec{\tau} = \vec{M} \times \vec{B} \quad (-\hat{k})$$

$$|\tau| = MB \sin 60$$

$$= \frac{MB \sqrt{3}}{2}$$

$$\boxed{|\tau| = W\sqrt{3}}$$

External Torque to Keep it at Rest $W\sqrt{3} (\hat{k})$



A small bar magnet placed with its axis at 30° ~~Will~~ⁱⁿ an external field of 0.06 T experiences a torque of 0.018 Nm . The minimum work required to rotate it from its stable to unstable equilibrium position is: [Main 2021]

- (a) $6.4 \times 10^{-2}\text{ J}$
- (b) $9.2 \times 10^{-3}\text{ J}$
- (c) $7.2 \times 10^{-2}\text{ J}$ Ans
- (d) $11.7 \times 10^{-3}\text{ J}$



$$\tau = MB \sin 30^\circ$$

$$0.018 = \frac{MB}{2}$$

$$MB = 2 \times 0.018$$



$$W = MB (\cos 0 - \cos \pi)$$

$$= MB (1 - (-1))$$

$$= 2MB = 2 \times 2 \times 0.018$$

$$= 4 \times 0.018 = 7.2 \times 10^{-2}\text{ J}$$



The magnetic induction at P , for the arrangement shown in the figure, when two similar short magnets of magnetic moment M are joined at the middle so that they are mutually perpendicular, will be :

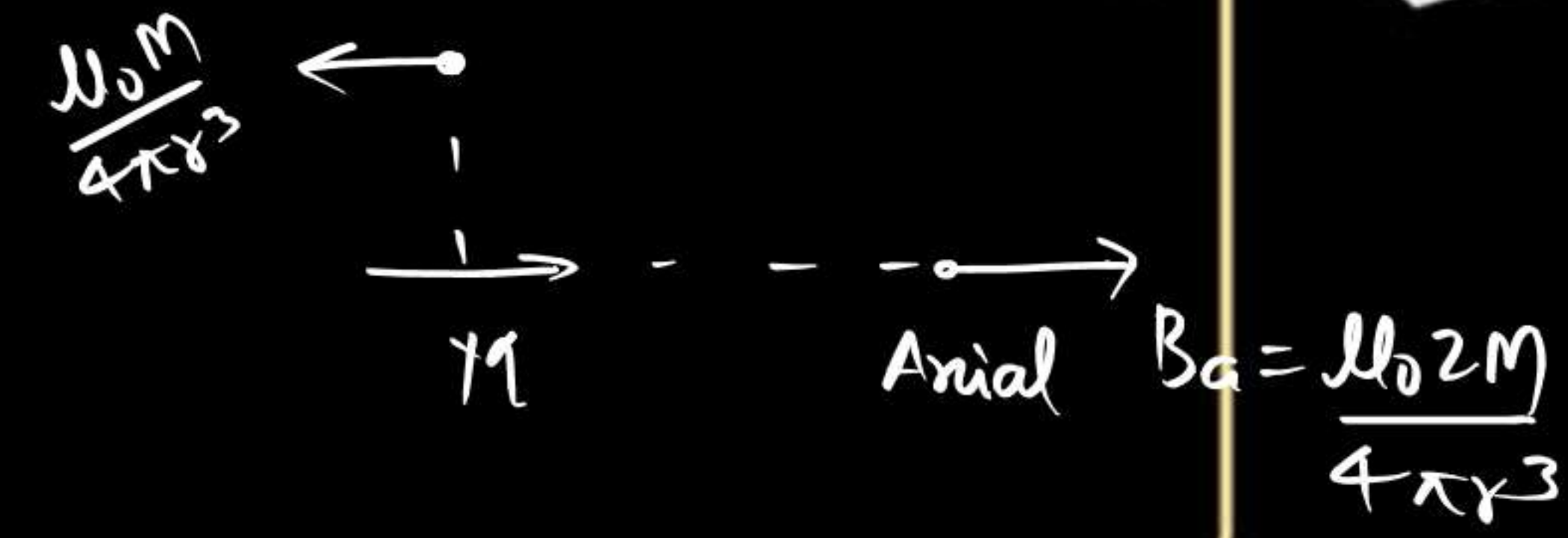
(a) $\frac{\mu_0 M \sqrt{3}}{4\pi d^3}$

(c) $\frac{\mu_0 M \sqrt{5}}{4\pi d^3}$

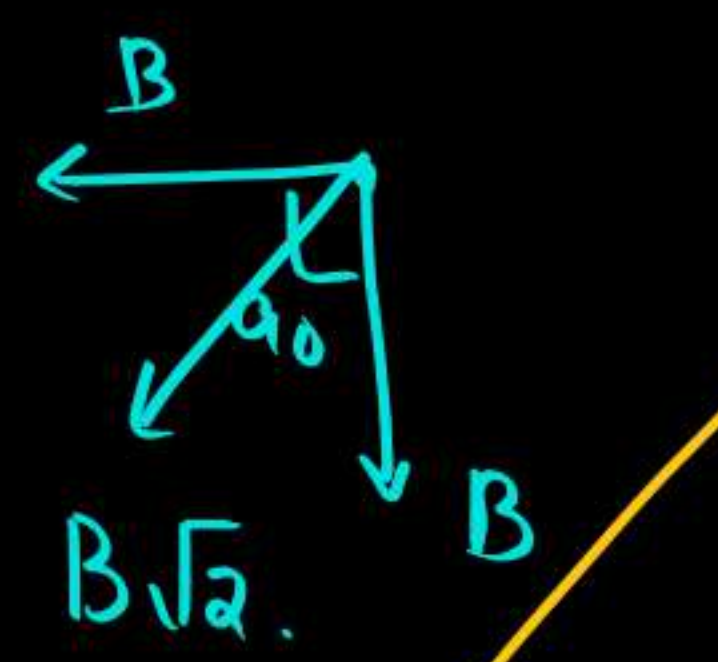
(b) $\frac{\mu_0 2M}{4\pi d^3}$

(d) $\frac{\mu_0 \sqrt{2}M}{4\pi d^3}$

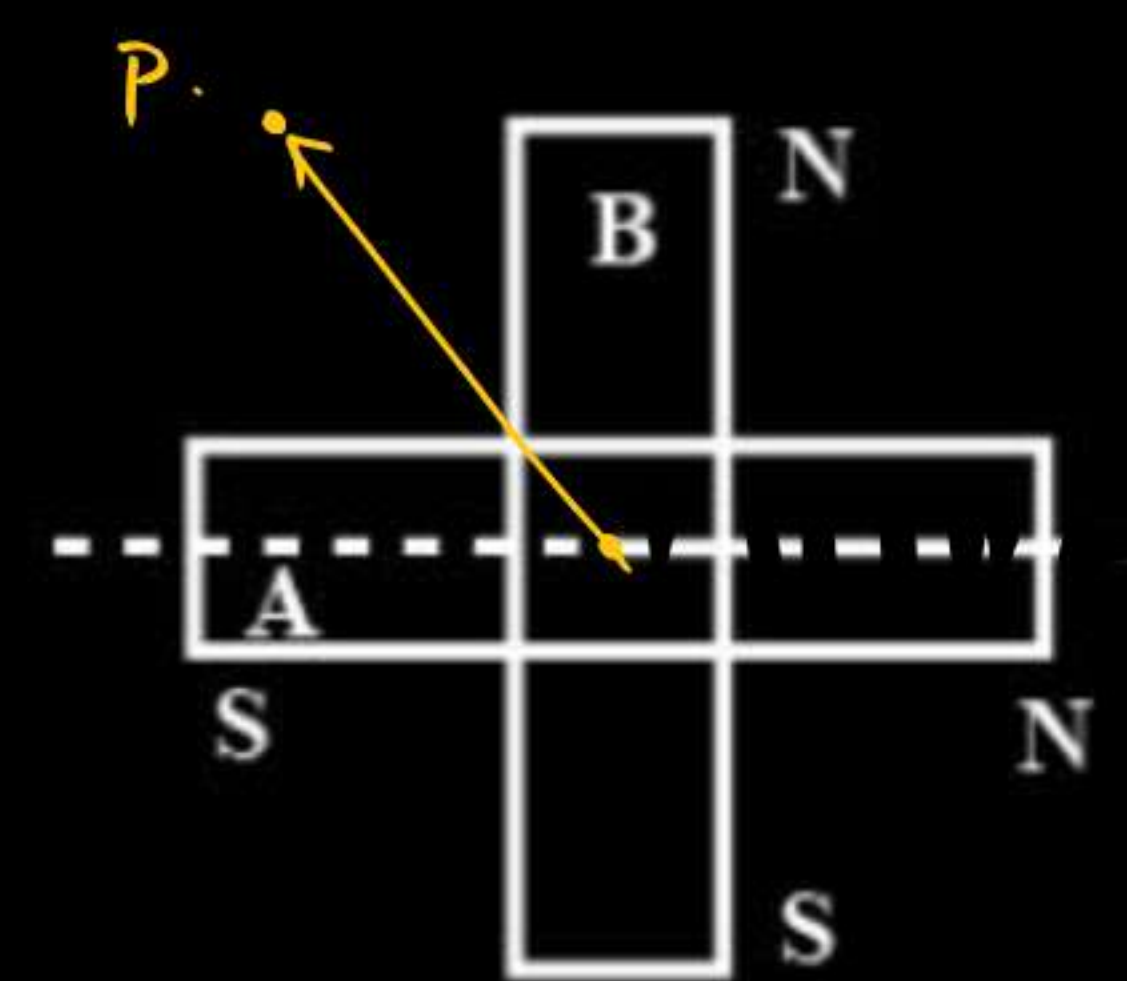
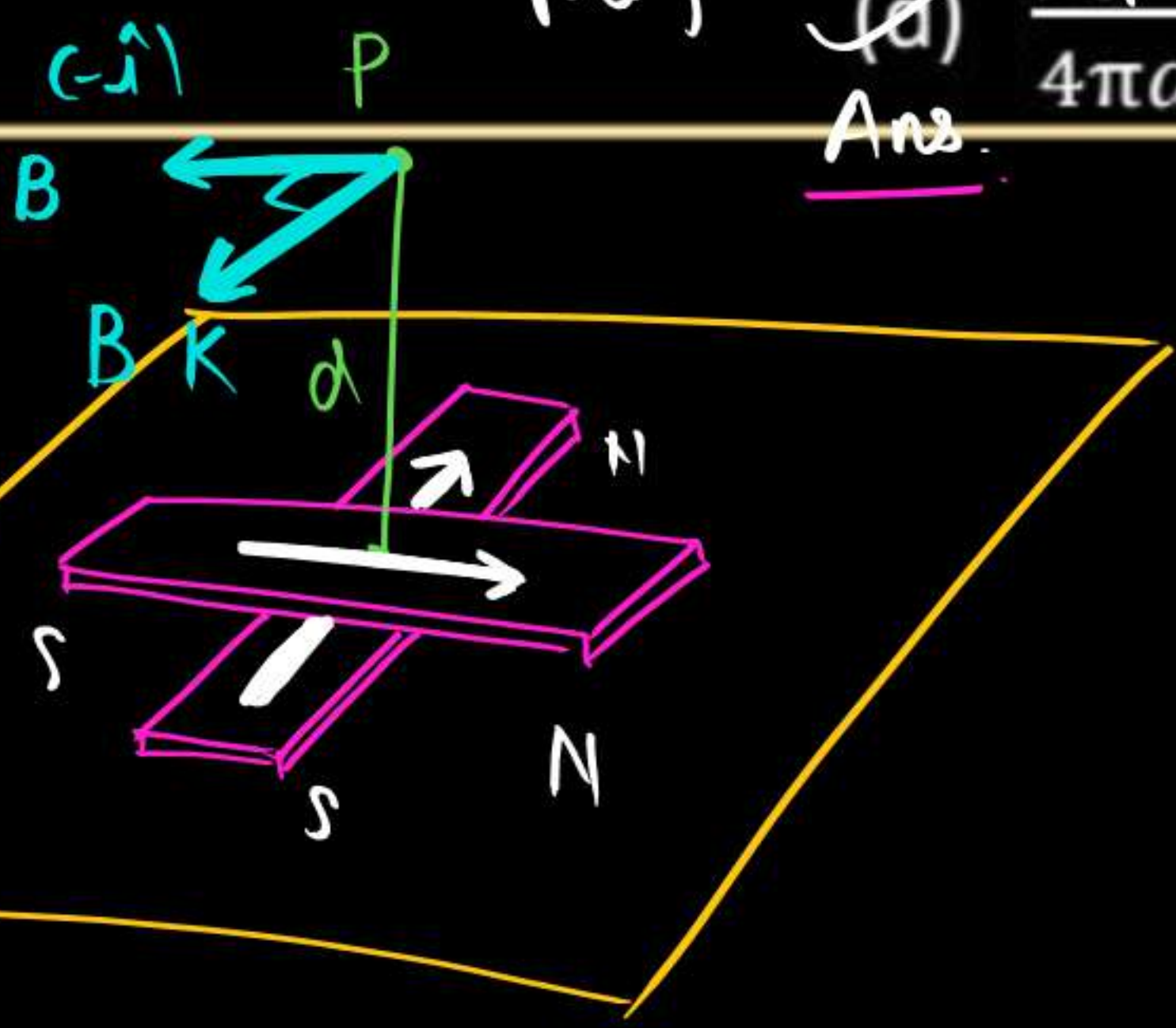
Equatorial Pt for both magnets



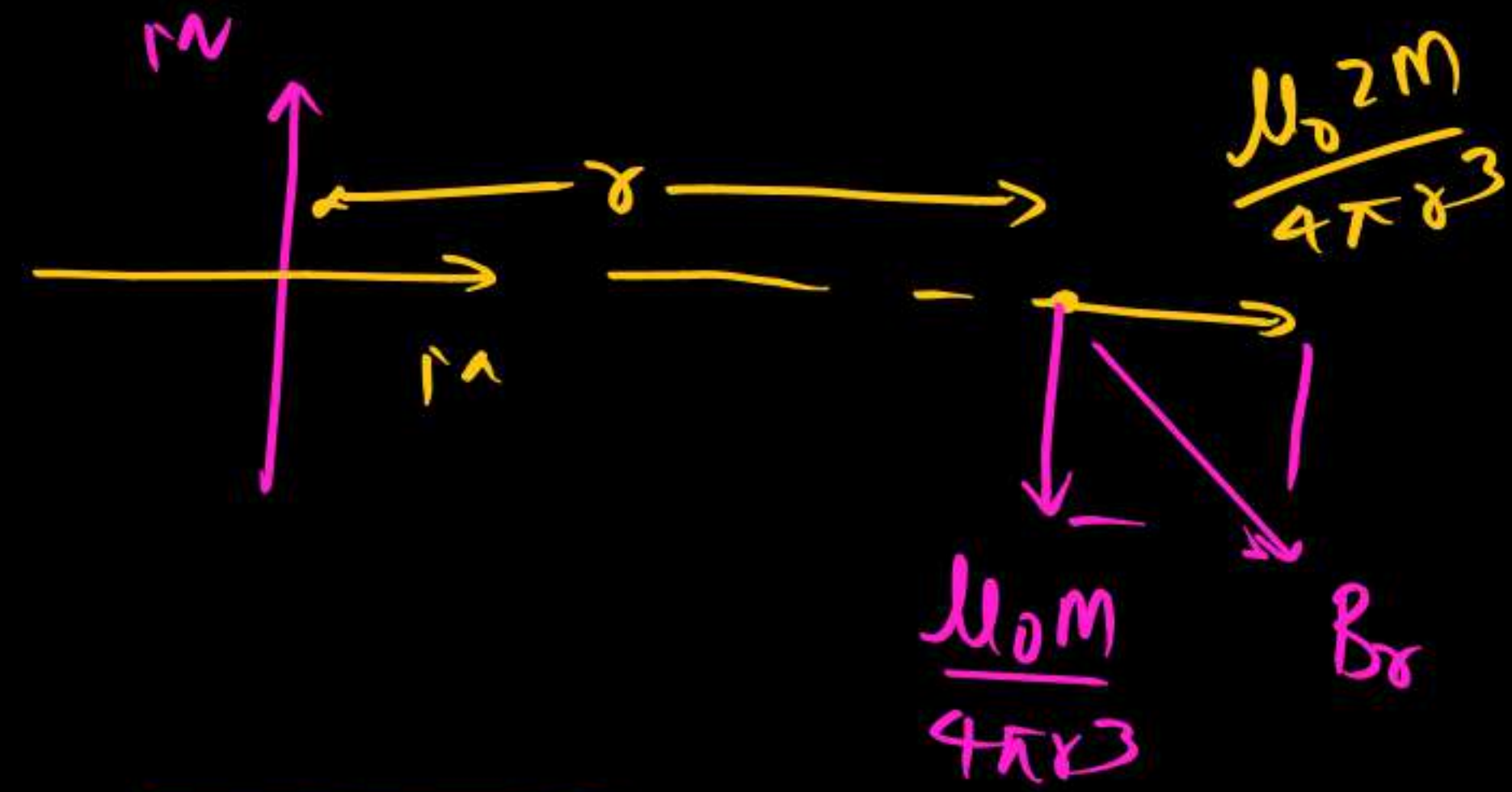
Ans



$\frac{\mu_0 M \sqrt{2}}{4\pi r^3}$ Ans



After



$$B_r = \sqrt{B_x^2 + B_y^2}$$
$$= \frac{\sqrt{5} \mu_0 M}{4\pi r^3}$$

M and $M/\sqrt{3}$ are the magnetic dipole moments of the two magnets, which are joined to form a cross figure. The inclination of the system with the field, if their combination is suspended freely in a uniform external magnetic field B is :

(a) $\theta = 30^\circ$ Ans

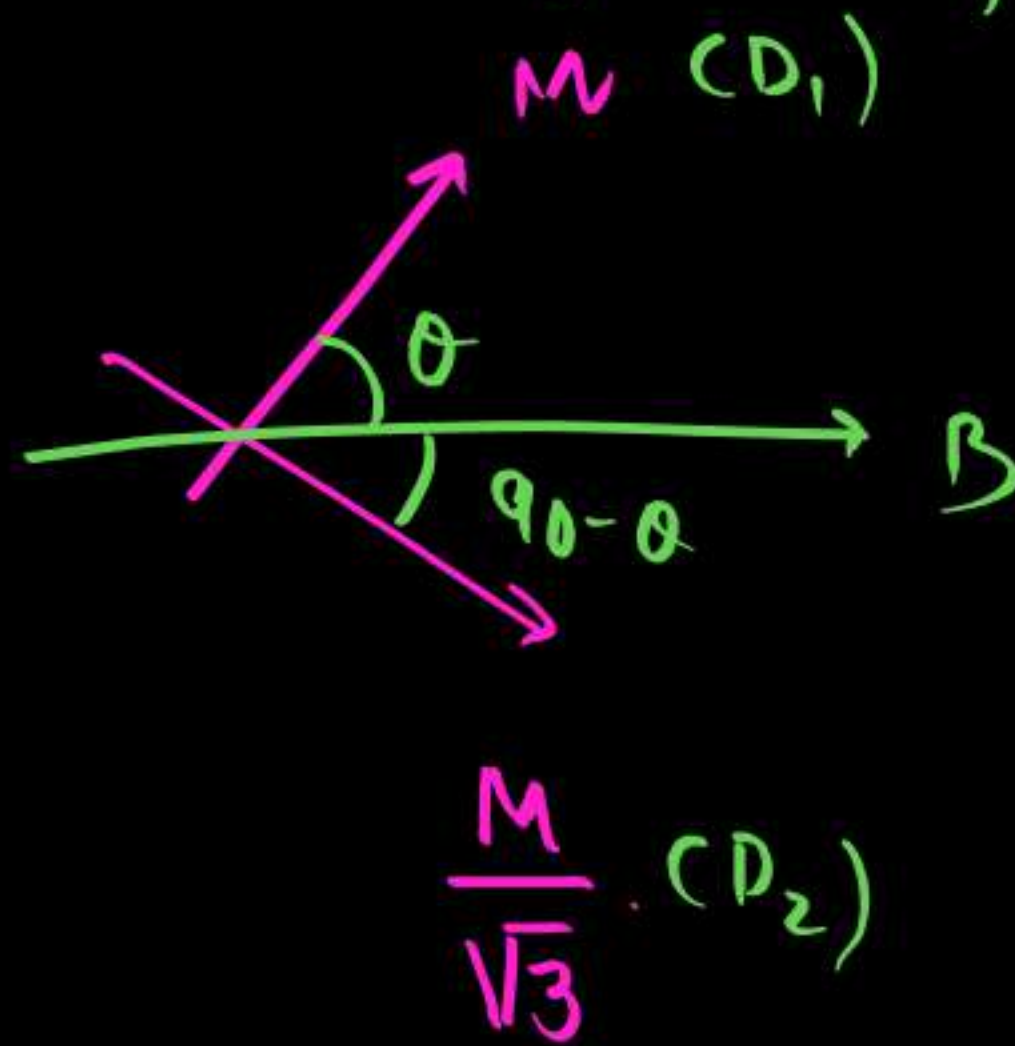
(c) $\theta = 60^\circ$

(b) $\theta = 45^\circ$

(d) $\theta = 15^\circ$

(JEE Main 25 Shift 2)
but in terms of Electro.

Since they are forming 90°

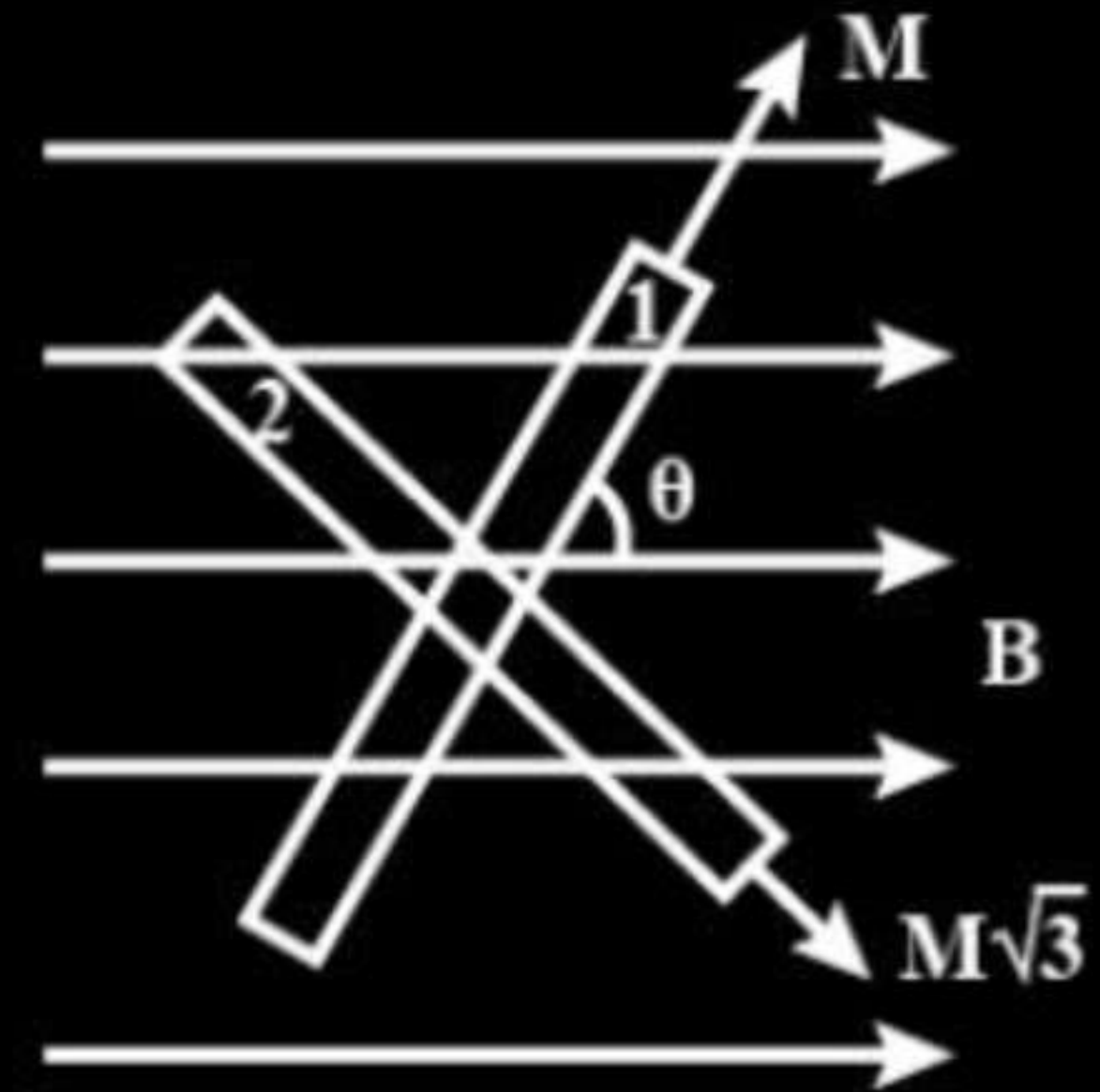


$|\tau_{D1}| = |\tau_{D2}|$

$M B \sin \theta = \frac{M}{\sqrt{3}} B \sin(90 - \theta)$

$\tan \theta = \frac{1}{\sqrt{3}}$

$\theta = 30^\circ$





Dipole is shifted from $(d, 0)$ to $(0, d)$ find work done

Same can be replicated in Magnetics.

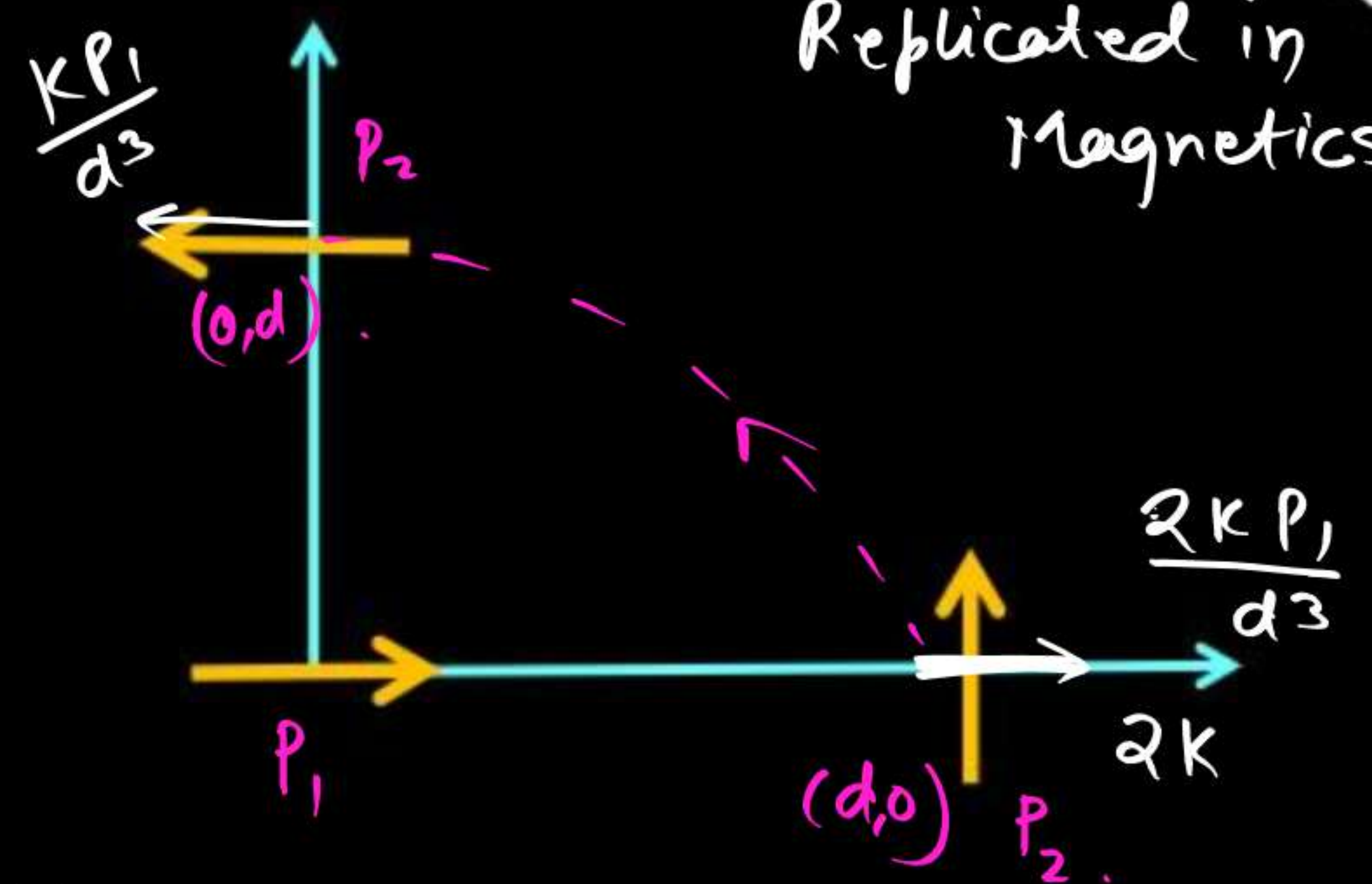
$$PE = -\vec{P} \cdot \vec{E}$$

$$\text{Initial } PE = U_i = -P_2 \left(\frac{2kP_1}{d^3} \right) \cos 90^\circ = 0$$

$$\begin{aligned} \text{final } PE &= -\vec{P}_2 \cdot \vec{E}_f \\ &= -P_2 \frac{kP_1}{d^3} \cos 0 \\ &= -\frac{kP_1 P_2}{d^3} \end{aligned}$$

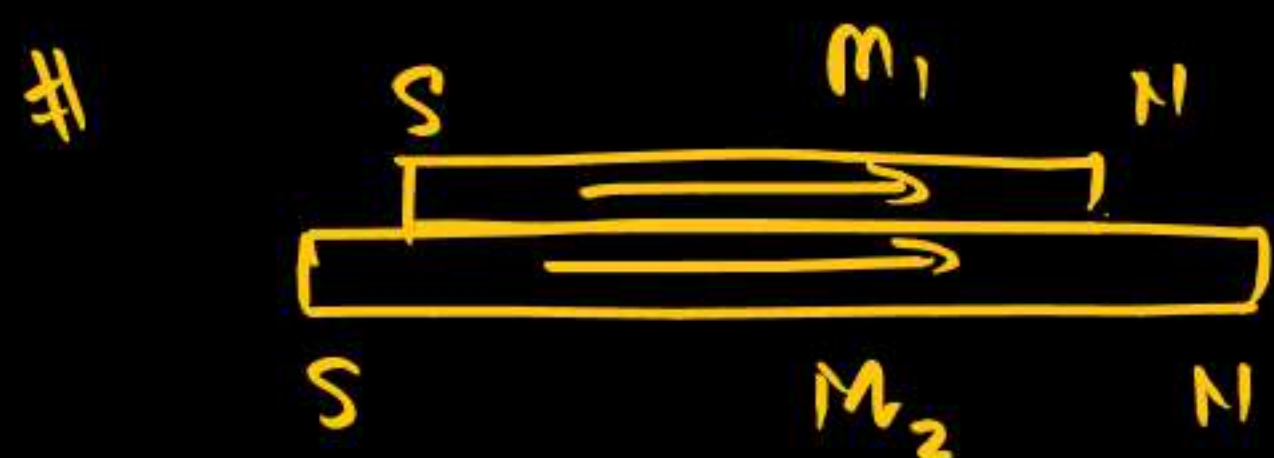
$$W_{\text{ext}} = U_f - U_i$$

$$W = -\frac{kP_1 P_2}{d^3} - 0$$



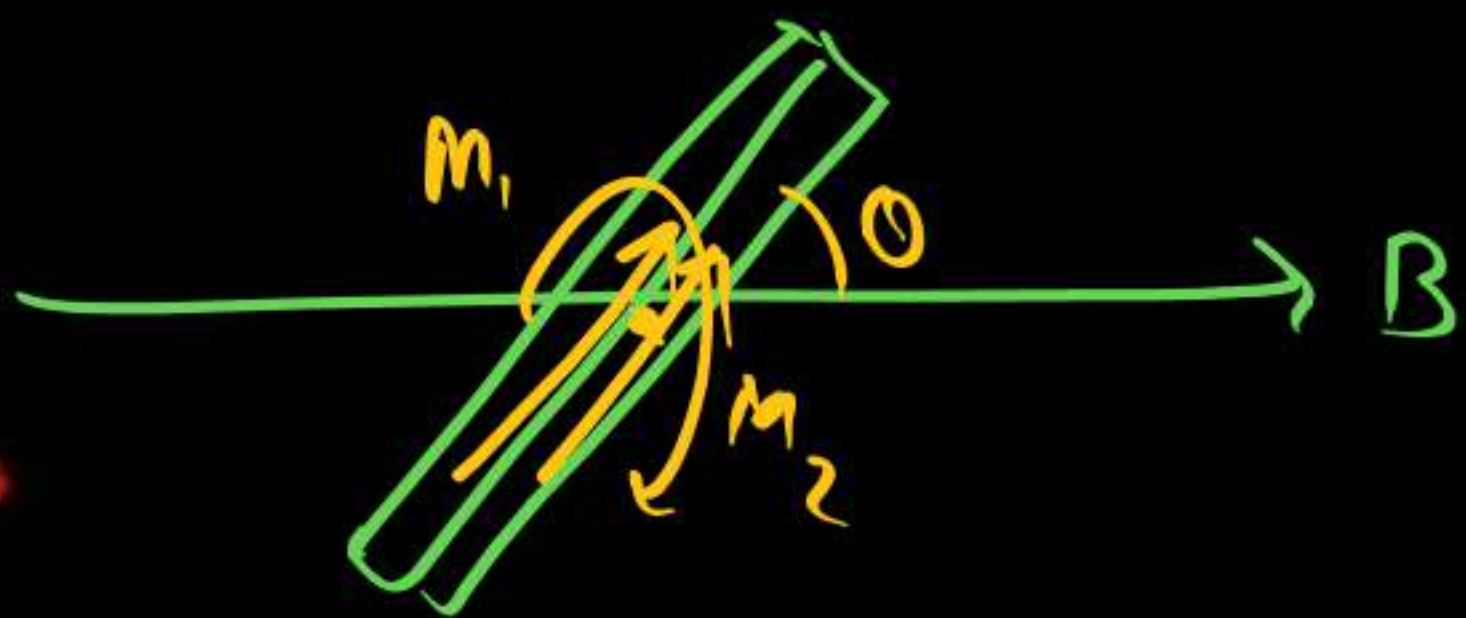
Coupled Dipoles Oscillation

Let $|m_1| > |m_2|$



$$|\vec{M}_T| = m_1 + m_2$$

They are attached
Oscillated in \vec{B}

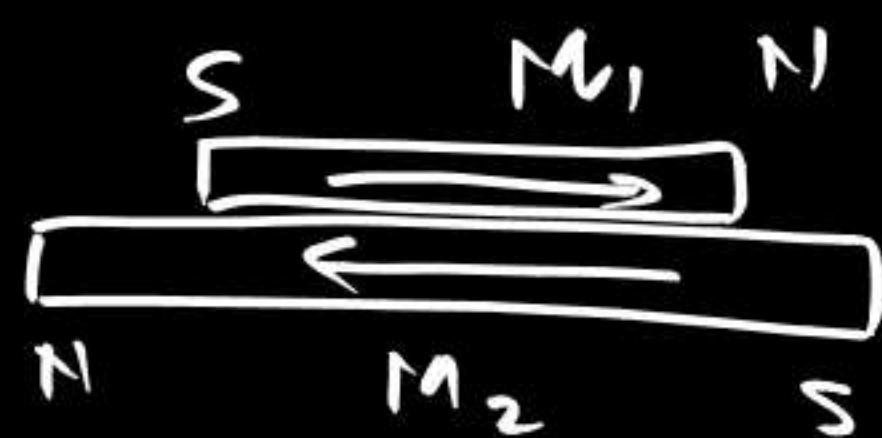
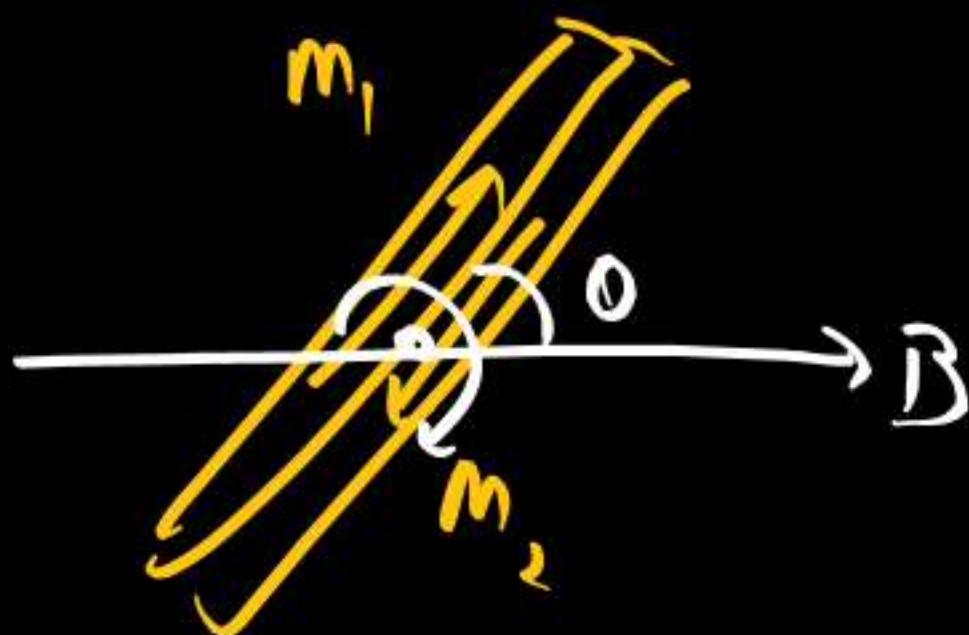


$\theta = 0^\circ$

$$T = 2\pi \sqrt{\frac{I}{M_B}}$$

$$I_T = I_1 + I_2 \quad T_S = 2\pi \sqrt{\frac{I_1 + I_2}{(m_1 + m_2)B}}$$

In second scenario
also $I_T = I_1 + I_2$



$$|\vec{M}_T| = m_1 - m_2$$

$$T_0 = 2\pi \sqrt{\frac{(I_1 + I_2)}{(m_1 - m_2)B}}$$

$$\frac{T_S}{T_0} = \sqrt{\frac{m_1 + m_2}{m_1 - m_2}} \quad \text{Ans}$$



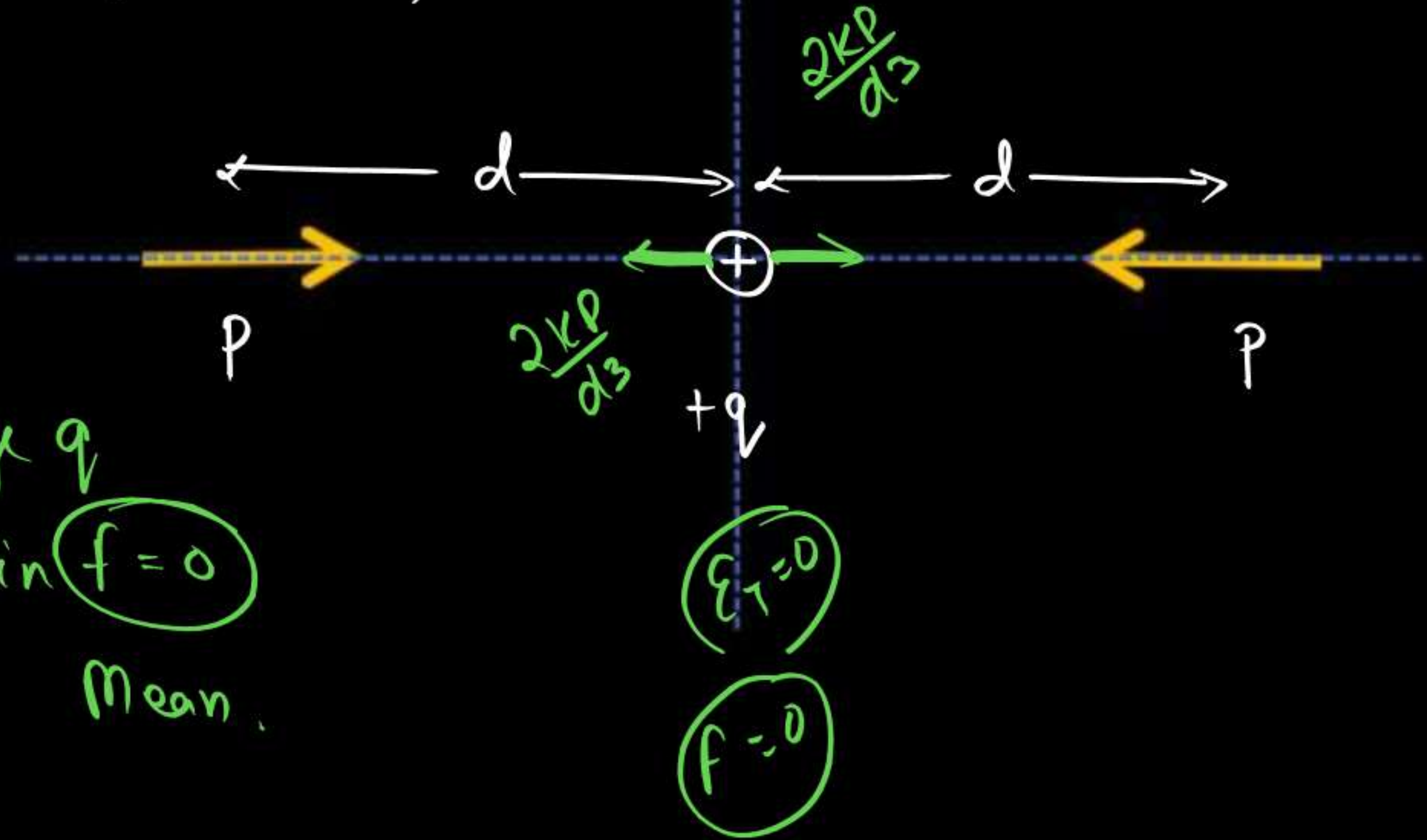
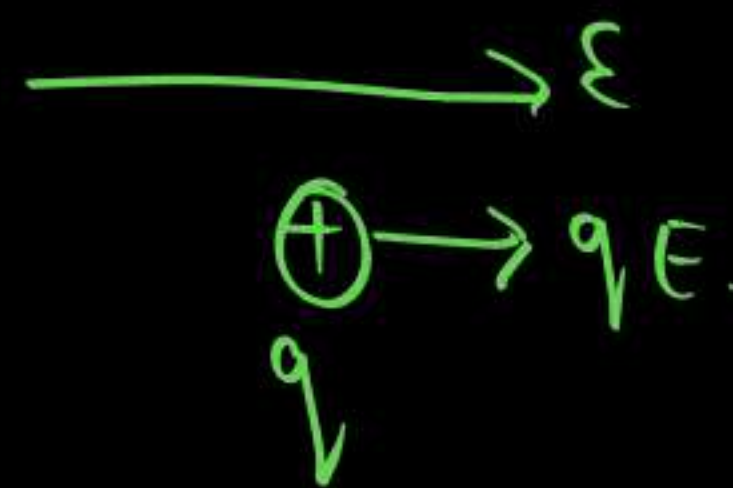
Adv.

Time period of oscillation of a charge between two dipoles

Two dipoles P are separated by distance d on axis as shown. A point charge q is placed at origin & shifted by small distance ' x ' towards right of mass of charge is m_0 . Find Time period of oscillation.

Known Info

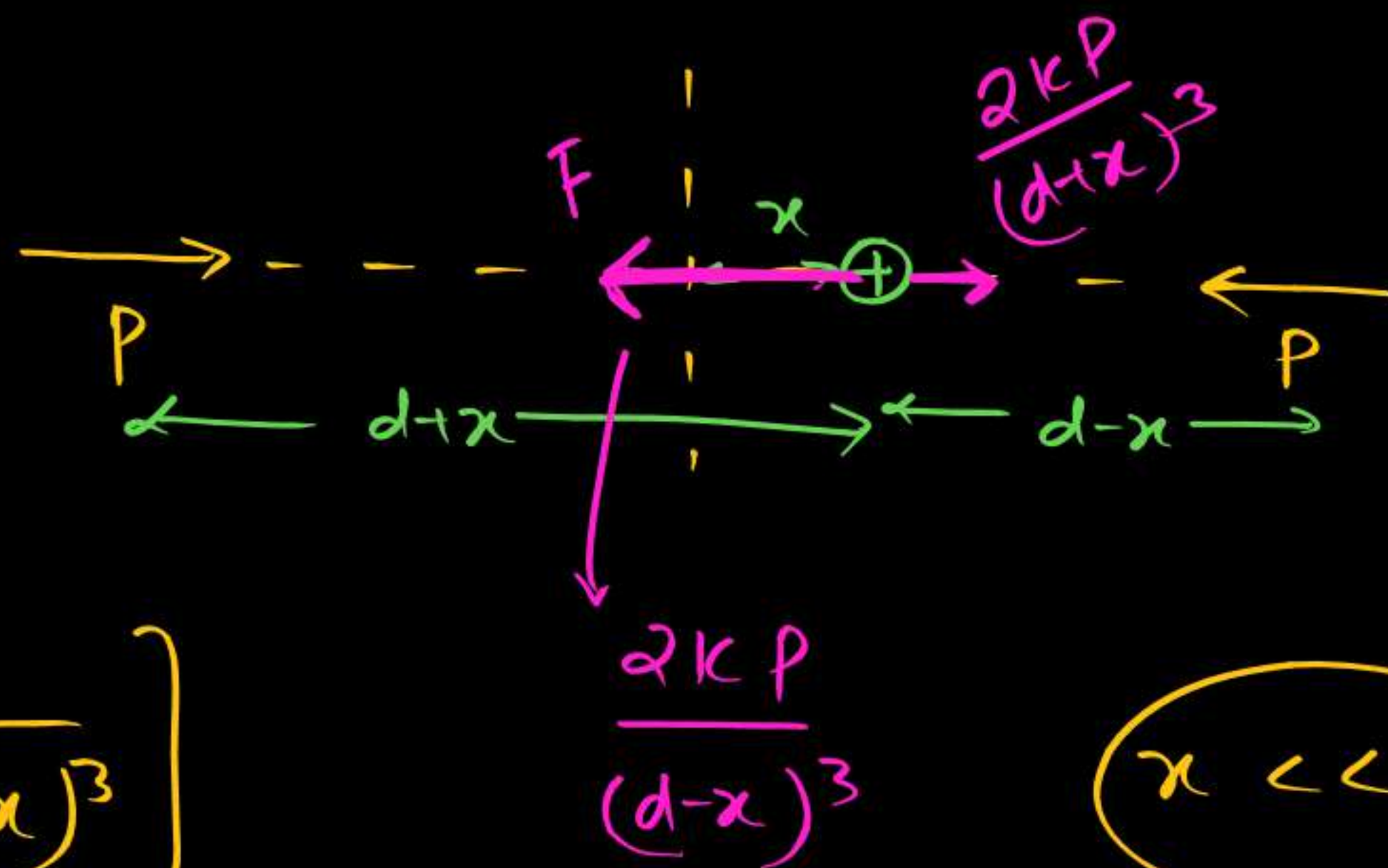
$$E = \frac{2kP}{r^3}$$



When charge q is at origin $f=0$ Mean.

$$\epsilon_{axial} \propto \frac{1}{r^3}$$

$$f_{restoring} = q \left(\frac{2kP}{(d-x)^3} - \frac{2kP}{(d+x)^3} \right)$$



$$\frac{2kPq}{d^3} \frac{6x}{d}$$

$$= 2kPq \left[\frac{1}{(d-x)^3} - \frac{1}{(d+x)^3} \right]$$

$$f_s = \frac{12kPq}{d^4} x$$

$$= \frac{2kPq}{d^3} \left[\frac{1}{\left(1 - \frac{x}{d}\right)^3} - \frac{1}{\left(1 + \frac{x}{d}\right)^3} \right]$$

$$(1 + \Delta)^n = 1 + n\Delta$$

$\Delta \rightarrow 0$

$$= \frac{2kPq}{d^3} \left[\left(1 - \frac{x}{d}\right)^{-3} - \left(1 + \frac{x}{d}\right)^{-3} \right] = \frac{2kPq}{d^3} \left[\sqrt[3]{1 + \frac{3x}{d}} - \sqrt[3]{1 - \frac{3x}{d}} \right]$$

When $f_{rest} \propto x$

$$f_{rest} = m\omega^2 x$$

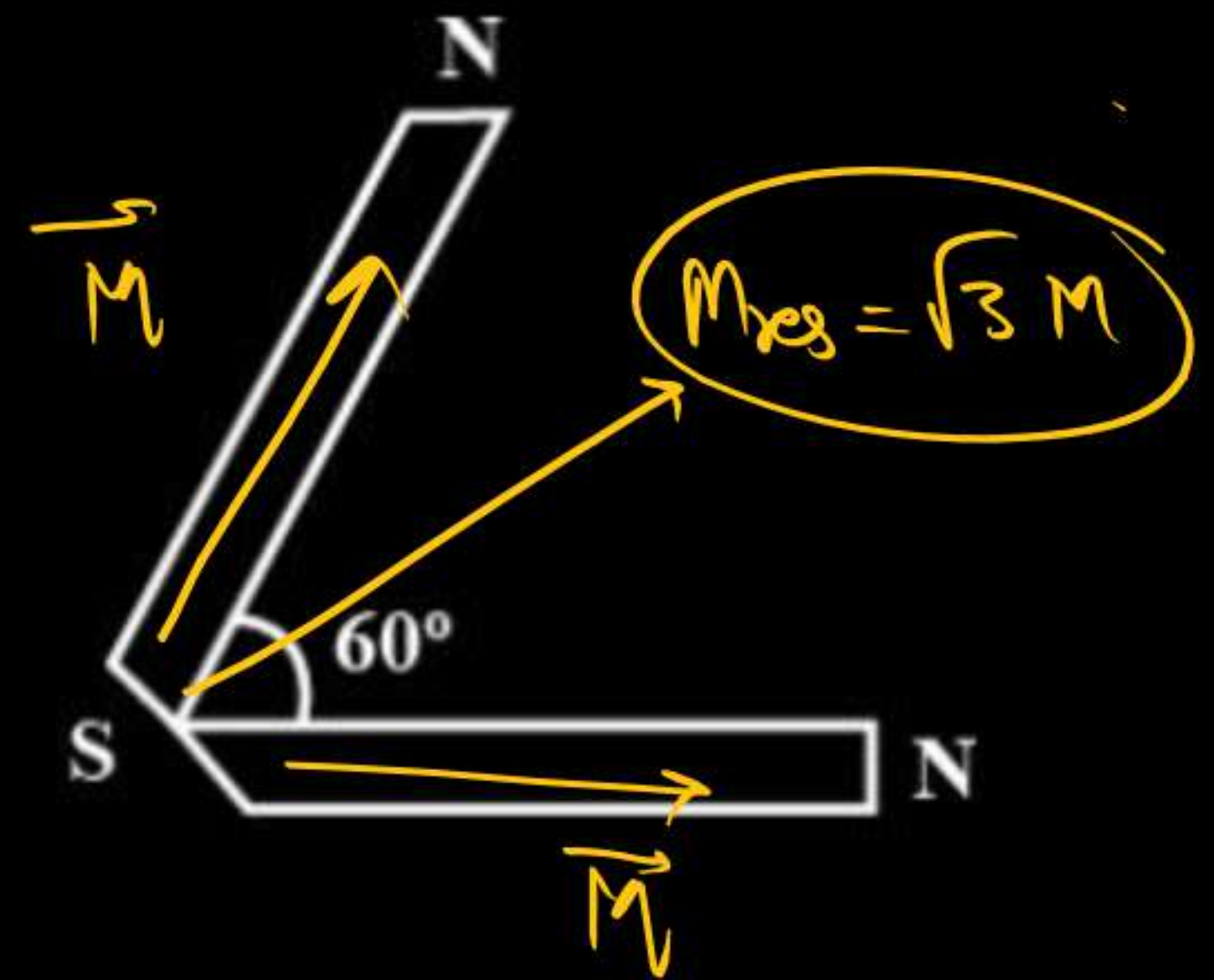
$$\frac{12KPa}{d^4} x = m\omega^2 x$$

$$\frac{2\pi}{T} = \omega = \sqrt{\frac{12KPa}{md^4}}$$

$$T = 2\pi \sqrt{\frac{md^4}{12KPa}}$$

Two magnets of equal magnetic moments M each are placed as shown in figure. The resultant magnetic moment is

- (a) M
- (b) $\sqrt{3} M$ *Ans*
- (c) $\sqrt{2} M$
- (d) $M/2$



Time period for a magnet is T . If it is divided in two equal parts perpendicular to its axis, then time period for each part will be:

- (a) $4T$
- (b) $T/4$
- (c) $T/2$ Ans
- (d) T

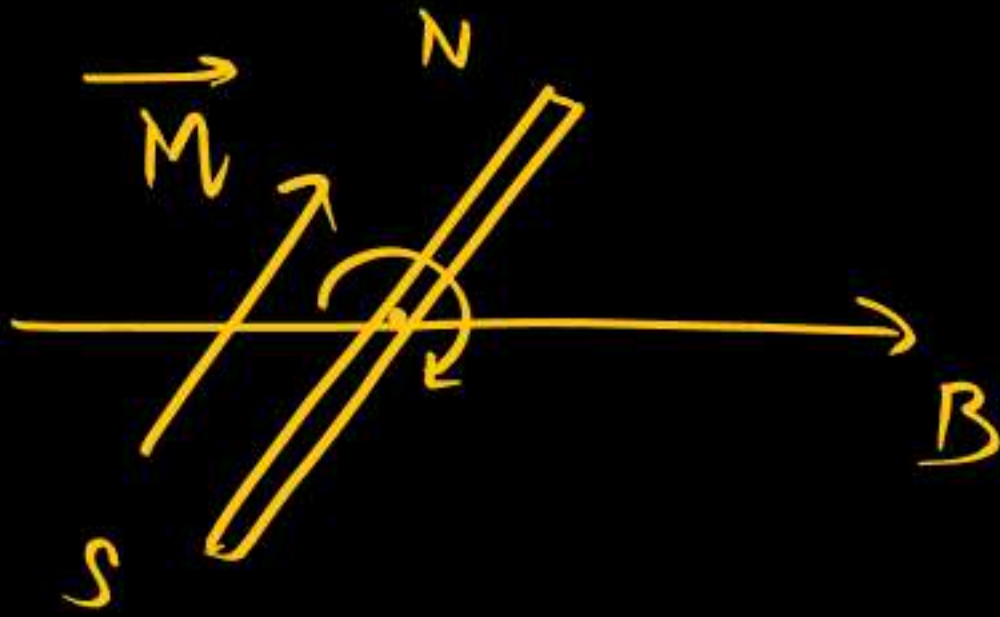
$\frac{Mass}{2}$

$\vec{M} = \frac{M}{2}$



$$T_0 = 2\pi \sqrt{\frac{I}{MB}}$$

$$I = \frac{1}{12} (Mass) L^2$$



$$T_0 = 2\pi \sqrt{\frac{\frac{1}{12} Mass L^2}{MB}}$$



$$T' = 2\pi \sqrt{\frac{\frac{1}{12} (\frac{Mass}{2}) (\frac{L}{2})^2}{\frac{M}{2} \cdot B}} = \sqrt{\frac{1}{4} \frac{(\frac{1}{12} Mass L^2)}{MB}}$$

$T' = \frac{T_0}{2}$

$$= \frac{1}{2} \sqrt{\frac{\frac{1}{12} Mass L^2}{MB}}$$



A bar magnet of magnetic moment 2.5 J/T , is placed in magnetic field 0.2 T . What work is done in turning the magnet from parallel to antiparallel position relative to field direction ?

(a) 1 J

(b) 2 J

(c) 3 J

(d) 4 J



Two dissimilar poles of strength x mWb and 2 mWb are separated by a distance 12 cm. If the null point is at a distance of 4 cm from 2 mWb, then the value of x is :

(a) 5

(b) 6

(c) 7

(d) 8



Thank You Lakshyians