

CHAPTER

14



A.C. Distribution

- 14.1** A.C. Distribution Calculations
- 14.2** Methods of Solving A.C. Distribution Problems
- 14.3** 3-Phase Unbalanced Loads
- 14.4** Four-Wire Star-Connected Unbalanced Loads
- 14.5** Ground Detectors

Introduction

In the beginning of electrical age, electricity was generated, transmitted and distributed as direct current. The principal disadvantage of d.c. system was that voltage level could not readily be changed, except by the use of rotating machinery, which in most cases was too expensive. With the development of transformer by George Westinghouse, a.c. system has become so predominant as to make d.c. system practically extinct in most parts of the world. The present day large power system has been possible only due to the adoption of a.c. system.

Now-a-days, electrical energy is generated, transmitted and distributed in the form of alternating current as an economical proposition. The electrical energy produced at the power station is transmitted at very high voltages by 3-phase, 3-wire system to step-down sub-stations for distribution. The distribution system consists of two parts viz. primary distribution and secondary distribution. The primary distribution circuit is 3-phase, 3-wire and operates at voltages (3.3 or 6.6 or 11kV) somewhat higher than general utilisation levels. It delivers power to the secondary distribution circuit through distribution transformers

situated near consumers' localities. Each distribution transformer steps down the voltage to 400 V and power is distributed to ultimate consumers' by 400/230 V, 3-phase, 4-wire system. In this chapter, we shall focus our attention on the various aspects of a.c. distribution.

14.1 A.C. Distribution Calculations

A.C. distribution calculations differ from those of d.c. distribution in the following respects :

- (i) In case of d.c. system, the voltage drop is due to resistance alone. However, in a.c. system, the voltage drops are due to the combined effects of resistance, inductance and capacitance.
- (ii) In a d.c. system, additions and subtractions of currents or voltages are done arithmetically but in case of a.c. system, these operations are done vectorially.
- (iii) In an a.c. system, power factor (p.f.) has to be taken into account. Loads tapped off from the distributor are generally at different power factors. There are two ways of referring power factor *viz*
 - (a) It may be referred to supply or receiving end voltage which is regarded as the reference vector.
 - (b) It may be referred to the voltage at the load point itself.

There are several ways of solving a.c. distribution problems. However, symbolic notation method has been found to be most convenient for this purpose. In this method, voltages, currents and impedances are expressed in complex notation and the calculations are made exactly as in d.c. distribution.

14.2 Methods of Solving A.C. Distribution Problems

In a.c. distribution calculations, power factors of various load currents have to be considered since currents in different sections of the distributor will be the vector sum of load currents and not the arithmetic sum. The power factors of load currents may be given (i) *w.r.t.* receiving or sending end voltage or (ii) *w.r.t.* to load voltage itself. Each case shall be discussed separately.

(i) Power factors referred to receiving end voltage. Consider an a.c. distributor AB with concentrated loads of I_1 and I_2 tapped off at points C and B as shown in Fig. 14.1. Taking the receiving end voltage V_B as the reference vector, let lagging power factors at C and B be $\cos \phi_1$ and $\cos \phi_2$ *w.r.t.* V_B . Let R_1, X_1 and R_2, X_2 be the resistance and reactance of sections AC and CB of the distributor.

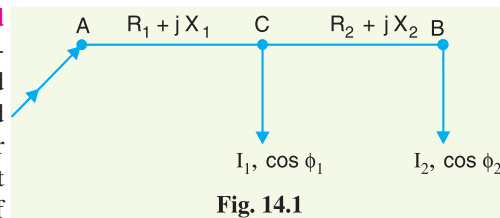


Fig. 14.1

$$\text{Impedance of section } AC, \quad \overrightarrow{Z_{AC}} = R_1 + j X_1$$

$$\text{Impedance of section } CB, \quad \overrightarrow{Z_{CB}} = R_2 + j X_2$$

$$\text{Load current at point } C, \quad \overrightarrow{I_1} = I_1 (\cos \phi_1 - j \sin \phi_1)$$

$$\text{Load current at point } B, \quad \overrightarrow{I_2} = I_2 (\cos \phi_2 - j \sin \phi_2)$$

$$\text{Current in section } CB, \quad \overrightarrow{I_{CB}} = \overrightarrow{I_2} = I_2 (\cos \phi_2 - j \sin \phi_2)$$

$$\begin{aligned} \text{Current in section } AC, \quad \overrightarrow{I_{AC}} &= \overrightarrow{I_1} + \overrightarrow{I_2} \\ &= I_1 (\cos \phi_1 - j \sin \phi_1) + I_2 (\cos \phi_2 - j \sin \phi_2) \end{aligned}$$

$$\text{Voltage drop in section } CB, \quad \overrightarrow{V_{CB}} = \overrightarrow{I_{CB}} \overrightarrow{Z_{CB}} = I_2 (\cos \phi_2 - j \sin \phi_2) (R_2 + j X_2)$$

$$\text{Voltage drop in section } AC, \quad \overrightarrow{V_{AC}} = \overrightarrow{I_{AC}} \overrightarrow{Z_{AC}} = (\overrightarrow{I_1} + \overrightarrow{I_2}) \overrightarrow{Z_{AC}}$$

$$= [I_1(\cos \phi_1 - j \sin \phi_1) + I_2(\cos \phi_2 - j \sin \phi_2)] [R_1 + jX_1]$$

$$\text{Sending end voltage, } \vec{V}_A = \vec{V}_B + \vec{V}_{CB} + \vec{V}_{AC}$$

$$\text{Sending end current, } \vec{I}_A = \vec{I}_1 + \vec{I}_2$$

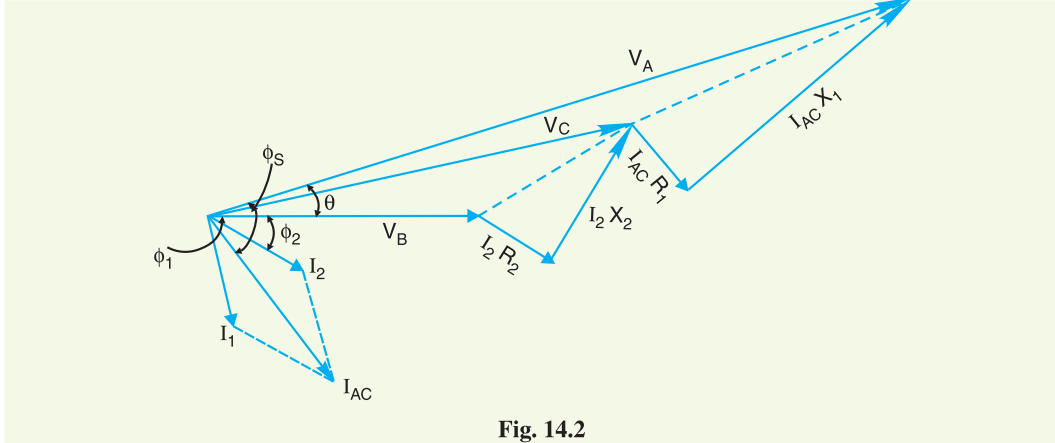


Fig. 14.2

The vector diagram of the a.c. distributor under these conditions is shown in Fig. 14.2. Here, the receiving end voltage V_B is taken as the reference vector. As power factors of loads are given *w.r.t.* V_B , therefore, I_1 and I_2 lag behind V_B by ϕ_1 and ϕ_2 respectively.

(ii) **Power factors referred to respective load voltages.** Suppose the power factors of loads in the previous Fig. 14.1 are referred to their respective load voltages. Then ϕ_1 is the phase angle between V_C and I_1 and ϕ_2 is the phase angle between V_B and I_2 . The vector diagram under these conditions is shown in Fig. 14.3.

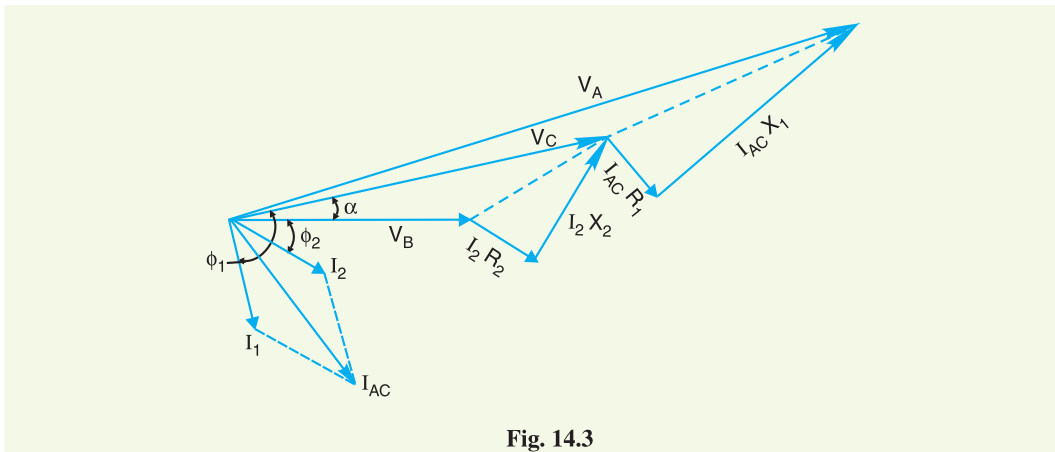


Fig. 14.3

$$\text{Voltage drop in section } CB = \vec{I}_2 \vec{Z}_{CB} = I_2 (\cos \phi_2 - j \sin \phi_2) (R_2 + j X_2)$$

$$\text{Voltage at point } C = \vec{V}_B + \text{Drop in section } CB = V_C \angle \alpha \text{ (say)}$$

$$\text{Now } \vec{I}_1 = I_1 \angle -\phi_1 \text{ w.r.t. voltage } V_C$$

$$\therefore \vec{I}_1 = I_1 \angle -(\phi_1 - \alpha) \text{ w.r.t. voltage } V_B$$

$$\text{i.e. } \vec{I}_1 = I_1 [\cos (\phi_1 - \alpha) - j \sin (\phi_1 - \alpha)]$$

$$\text{Now } \vec{I}_{AC} = \vec{I}_1 + \vec{I}_2$$

$$= I_1 [\cos (\phi_1 - \alpha) - j \sin (\phi_1 - \alpha)] + I_2 (\cos \phi_2 - j \sin \phi_2)$$

$$\text{Voltage drop in section AC} = \overrightarrow{I_{AC}} \overrightarrow{Z_{AC}}$$

$$\therefore \text{Voltage at point A} = V_B + \text{Drop in CB} + \text{Drop in AC}$$

Example 14.1. A single phase a.c. distributor AB 300 metres long is fed from end A and is loaded as under :

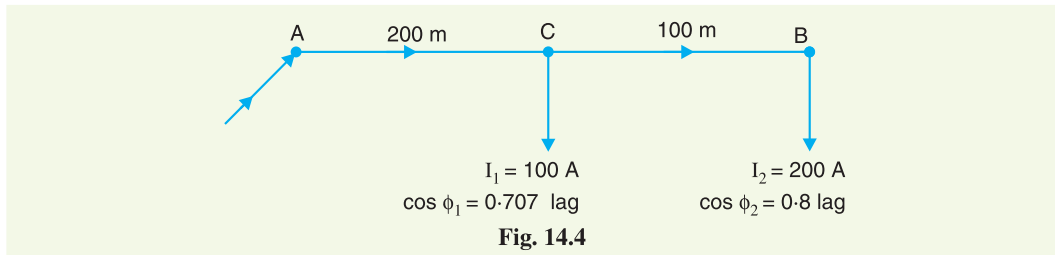
(i) 100 A at 0.707 p.f. lagging 200 m from point A

(ii) 200 A at 0.8 p.f. lagging 300 m from point A

The load resistance and reactance of the distributor is 0.2Ω and 0.1Ω per kilometre. Calculate the total voltage drop in the distributor. The load power factors refer to the voltage at the far end.

Solution. Fig. 14.4 shows the single line diagram of the distributor.

$$\text{Impedance of distributor/km} = (0.2 + j 0.1) \Omega$$



$$\text{Impedance of section AC, } \overrightarrow{Z_{AC}} = (0.2 + j 0.1) \times 200/1000 = (0.04 + j 0.02) \Omega$$

$$\text{Impedance of section CB, } \overrightarrow{Z_{CB}} = (0.2 + j 0.1) \times 100/1000 = (0.02 + j 0.01) \Omega$$

Taking voltage at the far end B as the reference vector, we have,

$$\begin{aligned} \text{Load current at point B, } \overrightarrow{I_2} &= I_2 (\cos \phi_2 - j \sin \phi_2) = 200 (0.8 - j 0.6) \\ &= (160 - j 120) \text{ A} \end{aligned}$$

$$\begin{aligned} \text{Load current at point C, } \overrightarrow{I_1} &= I_1 (\cos \phi_1 - j \sin \phi_1) = 100 (0.707 - j 0.707) \\ &= (70.7 - j 70.7) \text{ A} \end{aligned}$$

$$\text{Current in section CB, } \overrightarrow{I_{CB}} = \overrightarrow{I_2} = (160 - j 120) \text{ A}$$

$$\begin{aligned} \text{Current in section AC, } \overrightarrow{I_{AC}} &= \overrightarrow{I_1} + \overrightarrow{I_2} = (70.7 - j 70.7) + (160 - j 120) \\ &= (230.7 - j 190.7) \text{ A} \end{aligned}$$

$$\begin{aligned} \text{Voltage drop in section CB, } \overrightarrow{V_{CB}} &= \overrightarrow{I_{CB}} \overrightarrow{Z_{CB}} = (160 - j 120) (0.02 + j 0.01) \\ &= (4.4 - j 0.8) \text{ volts} \end{aligned}$$

$$\begin{aligned} \text{Voltage drop in section AC, } \overrightarrow{V_{AC}} &= \overrightarrow{I_{AC}} \overrightarrow{Z_{AC}} = (230.7 - j 190.7) (0.04 + j 0.02) \\ &= (13.04 - j 3.01) \text{ volts} \end{aligned}$$

$$\begin{aligned} \text{Voltage drop in the distributor} &= \overrightarrow{V_{AC}} + \overrightarrow{V_{CB}} = (13.04 - j 3.01) + (4.4 - j 0.8) \\ &= (17.44 - j 3.81) \text{ volts} \end{aligned}$$

$$\text{Magnitude of drop} = \sqrt{(17.44)^2 + (3.81)^2} = \mathbf{17.85 \text{ V}}$$

Example 14.2. A single phase distributor 2 kilometres long supplies a load of 120 A at 0.8 p.f. lagging at its far end and a load of 80 A at 0.9 p.f. lagging at its mid-point. Both power factors are

referred to the voltage at the far end. The resistance and reactance per km (go and return) are 0.05Ω and 0.1Ω respectively. If the voltage at the far end is maintained at 230 V , calculate :

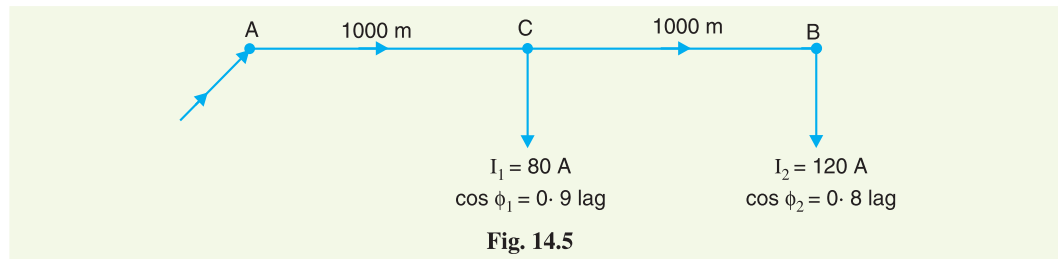
- (i) voltage at the sending end
- (ii) phase angle between voltages at the two ends.

Solution. Fig. 14.5 shows the distributor AB with C as the mid-point

$$\text{Impedance of distributor/km} = (0.05 + j 0.1) \Omega$$

$$\text{Impedance of section AC, } \vec{Z}_{AC} = (0.05 + j 0.1) \times 1000/1000 = (0.05 + j 0.1) \Omega$$

$$\text{Impedance of section CB, } \vec{Z}_{CB} = (0.05 + j 0.1) \times 1000/1000 = (0.05 + j 0.1) \Omega$$



Let the voltage V_B at point B be taken as the reference vector.

$$\text{Then, } \vec{V}_B = 230 + j 0$$

$$(i) \text{ Load current at point B, } \vec{I}_2 = 120 (0.8 - j 0.6) = 96 - j 72$$

$$\text{Load current at point C, } \vec{I}_1 = 80 (0.9 - j 0.436) = 72 - j 34.88$$

$$\text{Current in section CB, } \vec{I}_{CB} = \vec{I}_2 = 96 - j 72$$

$$\begin{aligned} \text{Current in section AC, } \vec{I}_{AC} &= \vec{I}_1 + \vec{I}_2 = (72 - j 34.88) + (96 - j 72) \\ &= 168 - j 106.88 \end{aligned}$$

$$\begin{aligned} \text{Drop in section CB, } \vec{V}_{CB} &= \vec{I}_{CB} \vec{Z}_{CB} = (96 - j 72) (0.05 + j 0.1) \\ &= 12 + j 6 \end{aligned}$$

$$\begin{aligned} \text{Drop in section AC, } \vec{V}_{AC} &= \vec{I}_{AC} \vec{Z}_{AC} = (168 - j 106.88) (0.05 + j 0.1) \\ &= 19.08 + j 11.45 \end{aligned}$$

$$\begin{aligned} \therefore \text{ Sending end voltage, } \vec{V}_A &= \vec{V}_B + \vec{V}_{CB} + \vec{V}_{AC} \\ &= (230 + j 0) + (12 + j 6) + (19.08 + j 11.45) \\ &= 261.08 + j 17.45 \end{aligned}$$

$$\text{Its magnitude is } = \sqrt{(261.08)^2 + (17.45)^2} = \mathbf{261.67 \text{ V}}$$

(ii) The phase difference θ between V_A and V_B is given by :

$$\tan \theta = \frac{17.45}{261.08} = 0.0668$$

$$\therefore \theta = \tan^{-1} 0.0668 = \mathbf{3.82^\circ}$$

Example 14.3. A single phase distributor one km long has resistance and reactance per conductor of 0.1Ω and 0.15Ω respectively. At the far end, the voltage $V_B = 200 \text{ V}$ and the current is 100 A at a p.f. of 0.8 lagging. At the mid-point M of the distributor, a current of 100 A is tapped at a p.f.

of 0.6 lagging with reference to the voltage V_M at the mid-point. Calculate :

- (i) voltage at mid-point
- (ii) sending end voltage V_A
- (iii) phase angle between V_A and V_B

Solution. Fig. 14.6 shows the single line diagram of the distributor AB with M as the mid-point.

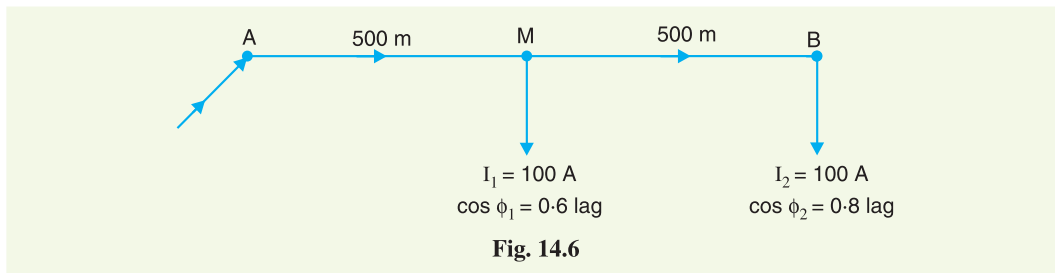
Total impedance of distributor = $2(0.1 + j 0.15) = (0.2 + j 0.3) \Omega$

Impedance of section AM, $\overrightarrow{Z_{AM}} = (0.1 + j 0.15) \Omega$

Impedance of section MB, $\overrightarrow{Z_{MB}} = (0.1 + j 0.15) \Omega$

Let the voltage V_B at point B be taken as the reference vector.

Then, $\overrightarrow{V_B} = 200 + j 0$



(i) Load current at point B, $\overrightarrow{I_2} = 100 (0.8 - j 0.6) = 80 - j 60$

Current in section MB, $\overrightarrow{I_{MB}} = \overrightarrow{I_2} = 80 - j 60$

Drop in section MB, $\overrightarrow{V_{MB}} = \overrightarrow{I_{MB}} \overrightarrow{Z_{MB}}$
 $= (80 - j 60) (0.1 + j 0.15) = 17 + j 6$

\therefore Voltage at point M, $\overrightarrow{V_M} = \overrightarrow{V_B} + \overrightarrow{V_{MB}} = (200 + j 0) + (17 + j 6)$
 $= 217 + j 6$

Its magnitude is $= \sqrt{(217)^2 + (6)^2} = \mathbf{217.1 \text{ V}}$

Phase angle between V_M and V_B , $\alpha = \tan^{-1} 6/217 = \tan^{-1} 0.0276 = 1.58^\circ$

(ii) The load current I_1 has a lagging p.f. of 0.6 w.r.t. V_M . It lags behind V_M by an angle $\phi_1 = \cos^{-1} 0.6 = 53.13^\circ$

\therefore Phase angle between I_1 and V_B , $\phi'_1 = \phi_1 - \alpha = 53.13^\circ - 1.58^\circ = 51.55^\circ$

Load current at M, $\overrightarrow{I_1} = I_1 (\cos \phi'_1 - j \sin \phi'_1) = 100 (\cos 51.55^\circ - j \sin 51.55^\circ)$
 $= 62.2 - j 78.3$

Current in section AM, $\overrightarrow{I_{AM}} = \overrightarrow{I_1} + \overrightarrow{I_2} = (62.2 - j 78.3) + (80 - j 60)$
 $= 142.2 - j 138.3$

Drop in section AM, $\overrightarrow{V_{AM}} = \overrightarrow{I_{AM}} \overrightarrow{Z_{AM}} = (142.2 - j 138.3) (0.1 + j 0.15)$
 $= 34.96 + j 7.5$

Sending end voltage, $\overrightarrow{V_A} = \overrightarrow{V_M} + \overrightarrow{V_{AM}} = (217 + j 6) + (34.96 + j 7.5)$

$$= 251.96 + j 13.5$$

Its magnitude is

$$= \sqrt{(251.96)^2 + (13.5)^2} = \mathbf{252.32 \text{ V}}$$

(iii) The phase difference θ between V_A and V_B is given by :

$$\tan \theta = 13.5/251.96 = 0.05358$$

\therefore

$$\theta = \tan^{-1} 0.05358 = \mathbf{3.07^\circ}$$

Hence supply voltage is 252.32 V and leads V_B by 3.07° .

Example 14.4. A single phase ring distributor ABC is fed at A. The loads at B and C are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively ; both expressed with reference to the voltage at A. The total impedance of the three sections AB, BC and CA are $(1 + j 1)$, $(1 + j 2)$ and $(1 + j 3)$ ohms respectively. Find the total current fed at A and the current in each section. Use Thevenin's theorem to obtain the results.

Solution. Fig. 14.7 (i) shows the ring distributor ABC. Thevenin's theorem will be used to solve this problem. First, let us find the current in BC. For this purpose, imagine that section BC is removed as shown in Fig. 14.7 (ii).

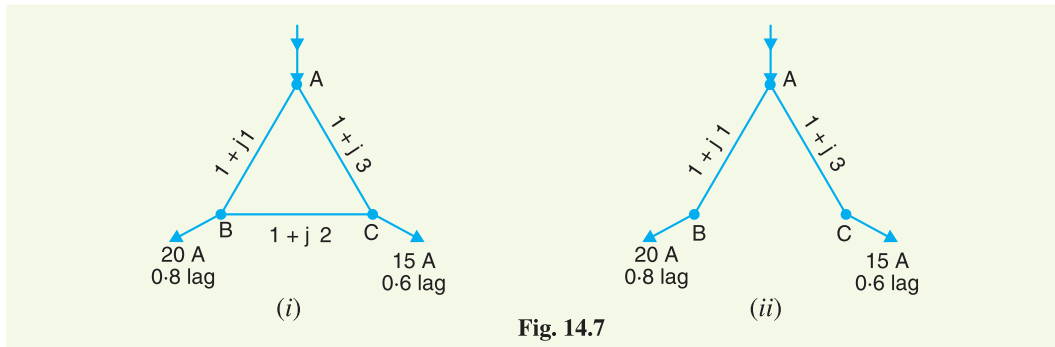


Fig. 14.7

Referring to Fig. 14.7 (ii), we have,

$$\text{Current in section AB} = 20 (0.8 - j 0.6) = 16 - j 12$$

$$\text{Current in section AC} = 15 (0.6 - j 0.8) = 9 - j 12$$

$$\text{Voltage drop in section AB} = (16 - j 12) (1 + j 1) = 28 + j 4$$

$$\text{Voltage drop in section AC} = (9 - j 12) (1 + j 3) = 45 + j 15$$

Obviously, point B is at higher potential than point C. The p.d. between B and C is Thevenin's equivalent circuit e.m.f. E_0 i.e.

$$\text{Thevenin's equivalent circuit e.m.f., } E_0 = \text{p.d. between B and C}$$

$$= (45 + j 15) - (28 + j 4) = 17 + j 11$$

Thevenin's equivalent impedance Z_0 can be found by looking into the network from points B and C.

$$\text{Obviously, } Z_0 = (1 + j 1) + (1 + j 3) = 2 + j 4$$

\therefore

$$\begin{aligned} \text{Current in BC} &= \frac{E_0}{Z_0 + \text{Impedance of BC}} \\ &= \frac{17 + j 11}{(2 + j 4) + (1 + j 2)} = \frac{17 + j 11}{3 + j 6} \\ &= 2.6 - j 1.53 = \mathbf{3\angle -30.48^\circ \text{ A}} \end{aligned}$$

$$\text{Current in AB} = (16 - j 12) + (2.6 - j 1.53)$$

$$\begin{aligned}
 &= 18.6 - j 13.53 = \mathbf{23\angle -36.03^\circ \text{ A}} \\
 \text{Current in AC} &= (9 - j 12) - (2.6 - j 1.53) \\
 &= 6.4 - j 10.47 = \mathbf{12.27\angle -58.56^\circ \text{ A}} \\
 \text{Current fed at A} &= (16 - j 12) + (9 - j 12) \\
 &= 25 - j 24 = \mathbf{34.65\angle -43.83^\circ \text{ A}}
 \end{aligned}$$

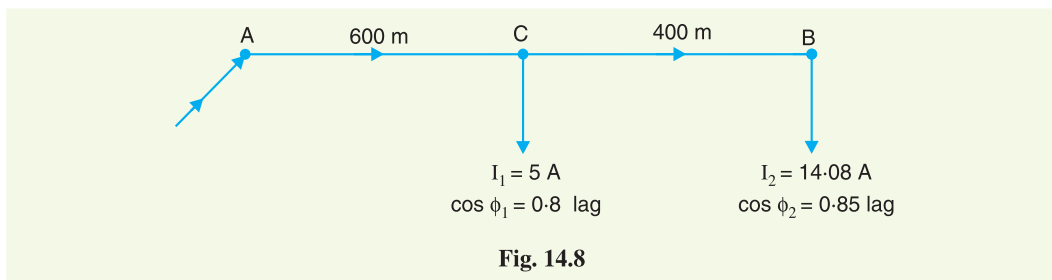
Example 14.5. A 3-phase, 400V distributor AB is loaded as shown in Fig.14.8. The 3-phase load at point C takes 5A per phase at a p.f. of 0.8 lagging. At point B, a 3-phase, 400 V induction motor is connected which has an output of 10 H.P. with an efficiency of 90% and p.f. 0.85 lagging.

If voltage at point B is to be maintained at 400 V, what should be the voltage at point A ? The resistance and reactance of the line are 1Ω and 0.5Ω per phase per kilometre respectively.

Solution. It is convenient to consider one phase only. Fig.14.8 shows the single line diagram of the distributor. Impedance of the distributor per phase per kilometre = $(1 + j 0.5) \Omega$.

$$\text{Impedance of section AC, } \overrightarrow{Z_{AC}} = (1 + j 0.5) \times 600/1000 = (0.6 + j 0.3) \Omega$$

$$\text{Impedance of section CB, } \overrightarrow{Z_{CB}} = (1 + j 0.5) \times 400/1000 = (0.4 + j 0.2) \Omega$$



$$\text{Phase voltage at point B, } V_B = 400/\sqrt{3} = 231 \text{ V}$$

Let the voltage V_B at point B be taken as the reference vector.

$$\text{Then, } \overrightarrow{V_B} = 231 + j 0$$

$$\begin{aligned}
 \text{Line current at B} &= \frac{\text{H.P.} \times 746}{\sqrt{3} \times \text{line voltage} \times \text{p.f.} \times \text{efficiency}} \\
 &= \frac{10 \times 746}{\sqrt{3} \times 400 \times 0.85 \times 0.9} = 14.08 \text{ A}
 \end{aligned}$$

$$\therefore \text{ *Current/phase at B, } I_2 = 14.08 \text{ A}$$

$$\text{Load current at B, } \overrightarrow{I_2} = 14.08 (0.85 - j 0.527) = 12 - j 7.4$$

$$\text{Load current at C, } \overrightarrow{I_1} = 5 (0.8 - j 0.6) = 4 - j 3$$

$$\begin{aligned}
 \text{Current in section AC, } \overrightarrow{I_{AC}} &= \overrightarrow{I_1} + \overrightarrow{I_2} = (4 - j 3) + (12 - j 7.4) \\
 &= 16 - j 10.4
 \end{aligned}$$

$$\text{Current in section CB, } \overrightarrow{I_{CB}} = \overrightarrow{I_2} = 12 - j 7.4$$

$$\begin{aligned}
 \text{Voltage drop in CB, } \overrightarrow{V_{CB}} &= \overrightarrow{I_{CB}} \overrightarrow{Z_{CB}} = (12 - j 7.4) (0.4 + j 0.2) \\
 &= 6.28 - j 0.56
 \end{aligned}$$

$$\begin{aligned}
 \text{Voltage drop in AC, } \overrightarrow{V_{AC}} &= \overrightarrow{I_{AC}} \overrightarrow{Z_{AC}} = (16 - j 10.4) (0.6 + j 0.3) \\
 &= 12.72 - j 1.44
 \end{aligned}$$

* In a 3-phase system, if the type of connection is not mentioned, then star connection is understood.

$$\begin{aligned}
 \text{Voltage at A per phase, } \vec{V}_A &= \vec{V}_B + \vec{V}_{CB} + \vec{V}_{AC} \\
 &= (231 + j0) + (6.28 - j0.56) + (12.72 - j1.44) \\
 &= 250 - j2
 \end{aligned}$$

$$\text{Magnitude of } V_A/\text{phase} = \sqrt{(250)^2 + (2)^2} = 250 \text{ V}$$

$$\therefore \text{Line voltage at A} = \sqrt{3} \times 250 = \mathbf{433 \text{ V}}$$

Example 14.6. A 3-phase ring main ABCD fed at A at 11 kV supplies balanced loads of 50 A at 0.8 p.f. lagging at B, 120 A at unity p.f. at C and 70 A at 0.866 lagging at D, the load currents being referred to the supply voltage at A. The impedances of the various sections are :

Section AB = $(1 + j0.6) \Omega$; Section BC = $(1.2 + j0.9) \Omega$

Section CD = $(0.8 + j0.5) \Omega$; Section DA = $(3 + j2) \Omega$

Calculate the currents in various sections and station bus-bar voltages at B, C and D.

Solution. Fig. 14.9 shows one phase of the ring main. The problem will be solved by Kirchhoff's laws. Let current in section AB be $(x + jy)$.

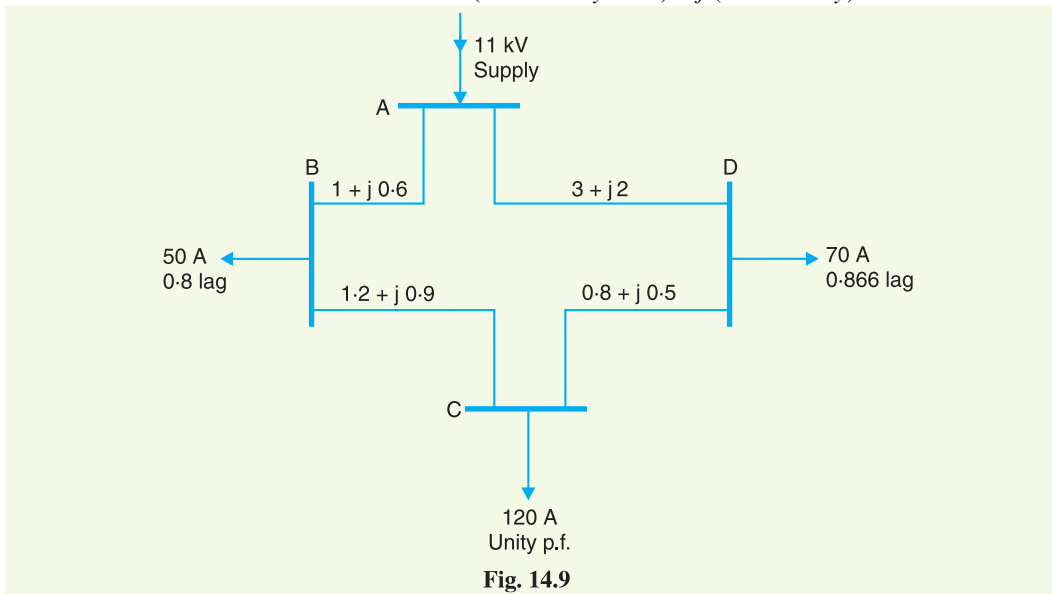
$$\therefore \text{Current in section BC, } \vec{I}_{BC} = (x + jy) - 50(0.8 - j0.6) = (x - 40) + j(y + 30)$$

$$\begin{aligned}
 \text{Current in section CD, } \vec{I}_{CD} &= [(x - 40) + j(y + 30)] - [120 + j0] \\
 &= (x - 160) + j(y + 30)
 \end{aligned}$$

$$\begin{aligned}
 \text{Current in section DA, } \vec{I}_{DA} &= [(x - 160) + j(y + 30)] - [70(0.866 - j0.5)] \\
 &= (x - 220.6) + j(y + 65)
 \end{aligned}$$

$$\begin{aligned}
 \text{Drop in section AB} &= \vec{I}_{AB} \vec{Z}_{AB} = (x + jy)(1 + j0.6) \\
 &= (x - 0.6y) + j(0.6x + y)
 \end{aligned}$$

$$\begin{aligned}
 \text{Drop in section BC} &= \vec{I}_{BC} \vec{Z}_{BC} \\
 &= [(x - 40) + j(y + 30)][(1.2 + j0.9)] \\
 &= (1.2x - 0.9y - 75) + j(0.9x + 1.2y)
 \end{aligned}$$



$$\begin{aligned}
 \text{Drop in section } CD &= \overrightarrow{I_{CD}} \overrightarrow{Z_{CD}} \\
 &= [(x - 160) + j(y + 30)] [(0.8 + j0.5)] \\
 &= (0.8x - 0.5y - 143) + j(0.5x + 0.8y - 56)
 \end{aligned}$$

$$\begin{aligned}
 \text{Drop in section } DA &= \overrightarrow{I_{DA}} \overrightarrow{Z_{DA}} \\
 &= [(x - 220.6) + j(y + 65)] [(3 + j2)] \\
 &= (3x - 2y - 791.8) + j(2x + 3y - 246.2)
 \end{aligned}$$

Applying Kirchhoff's voltage law to mesh $ABCD$, we have,

$$\text{Drop in } AB + \text{Drop in } BC + \text{Drop in } CD + \text{Drop in } DA = 0$$

$$\begin{aligned}
 \text{or } [(x - 0.6y) + j(0.6x + y)] + [(1.2x - 0.9y - 75) + j(0.9x + 1.2y)] \\
 + [(0.8x - 0.5y - 143) + j(0.5x + 0.8y - 56)]
 \end{aligned}$$

$$+ [(3x - 2y - 791.8) + j(2x + 3y - 246.2)] = 0$$

$$\text{or } (6x - 4y - 1009.8) + j(4x + 6y - 302.2) = 0$$

As the real (or active) and imaginary (or reactive) parts have to be separately zero,

$$\therefore 6x - 4y - 1009.8 = 0$$

$$\text{and } 4x + 6y - 302.2 = 0$$

Solving for x and y , we have,

$$x = 139.7 \text{ A} ; y = -42.8 \text{ A}$$

$$\text{Current in section } AB = (139.7 - j42.8) \text{ A}$$

$$\begin{aligned}
 \text{Current in section } BC &= (x - 40) + j(y + 30) \\
 &= (139.7 - 40) + j(-42.8 + 30) = (99.7 - j12.8) \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 \text{Current in section } CD &= (x - 160) + j(y + 30) \\
 &= (139.7 - 160) + j(-42.8 + 30) \\
 &= (-20.3 - j12.8) \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 \text{Current in section } DA &= (x - 220.6) + j(y + 65) \\
 &= (139.7 - 220.6) + j(-42.8 + 65) \\
 &= (-80.9 + j22.2) \text{ A}
 \end{aligned}$$

$$\text{Voltage at supply end } A, V_A = 11000/\sqrt{3} = 6351 \text{ V/phase}$$

$$\begin{aligned}
 \therefore \text{Voltage at station } B, \overrightarrow{V_B} &= \overrightarrow{V_A} - \overrightarrow{I_{AB}} \overrightarrow{Z_{AB}} \\
 &= (6351 + j0) - (139.7 - j42.8)(1 + j0.6) \\
 &= (6185.62 - j41.02) \text{ volts/phase}
 \end{aligned}$$

$$\begin{aligned}
 \text{Voltage at station } C, \overrightarrow{V_C} &= \overrightarrow{V_B} - \overrightarrow{I_{BC}} \overrightarrow{Z_{BC}} \\
 &= (6185.62 - j41.02) - (99.7 - j12.8)(1.2 + j0.9) \\
 &= (6054.46 - j115.39) \text{ volts/phase}
 \end{aligned}$$

$$\begin{aligned}
 \text{Voltage at station } D, \overrightarrow{V_D} &= \overrightarrow{V_C} - \overrightarrow{I_{CD}} \overrightarrow{Z_{CD}} \\
 &= (6054.46 - j115.39) - (-20.3 - j12.8)(0.8 + j0.5) \\
 &= (6064.3 - j95) \text{ volts/phase}
 \end{aligned}$$

TUTORIAL PROBLEMS

1. A single phase distributor AB has a total impedance of $(0.1 + j 0.2)$ ohm. At the far end B , a current of 80 A at 0.8 p.f. lagging and at mid-point C a current of 100 A at 0.6 p.f. lagging are tapped. If the voltage of the far end is maintained at 200 V, determine :

- (i) Supply end voltage V_A
 (ii) Phase angle between V_A and V_B

The load power factors are w.r.t. the voltage at the far end.

[(i) 227.22 V (ii) $2^{\circ}31'$]

2. A single-phase a.c. distributor AB is fed from end A and has a total impedance of $(0.2 + j 0.3)$ ohm. At the far end, the voltage $V_B = 240$ V and the current is 100 A at a p.f. of 0.8 lagging. At the mid-point M , a current of 100 A is tapped at a p.f. of 0.6 lagging with reference to the voltage V_M at the mid-point. Calculate the supply voltage V_A and phase angle between V_A and V_B .

[292 V, 2.6°]

3. A single phase ring distributor ABC is fed at A . The loads at B and C are 40 A at 0.8 p.f. lagging and 60 A at 0.6 p.f. lagging respectively. Both power factors expressed are referred to the voltage at point A . The total impedance of sections AB , BC and CA are $2 + j1$, $2 + j3$ and $1 + j2$ ohms respectively. Determine the current in each section.

[Current in $AB = (39.54 - j 25.05)$ amp ; $BC = (7.54 - j 1.05)$ amp ; $CA = (28.46 - j 46.95)$ amp.]

4. A 3-phase ring distributor $ABCD$ fed at A at 11 kV supplies balanced loads of 40 A at 0.8 p.f. lagging at B , 50 A at 0.707 p.f. lagging at C and 30 A at 0.8 p.f. lagging at D , the load currents being referred to the supply voltage at A .

The impedances per phase of the various sections are :

Section $AB = (1 + j 2) \Omega$; Section $BC = (2 + j 3) \Omega$

Section $CD = (1 + j 1) \Omega$; Section $DA = (3 + j 4) \Omega$

Calculate the currents in various sections and station bus-bar voltages at B , C and D .

[Current in $AB = (53.8 - j 46)$ amp ; $BC = (21.8 - j 22)$ amp.

$CD = (-13.55 + j 13.35)$ amp ; $DA = (-40.55 - j 26.45)$ amp.

$V_B = (6212.5 - j 61.6)$ volts/phase ; $V_C = (6103 - j 83)$ volts/phase

$V_D = (6129.8 - j 82.8)$ volts/phase]



Phase Sequence Indicator

14.3 3-Phase Unbalanced Loads

The 3-phase loads that have the same impedance and power factor in each phase are called balanced loads. The problems on balanced loads can be solved by considering one phase only ; the conditions in the other two phases being similar. However, we may come across a situation when loads are unbalanced *i.e.* each load phase has different impedance and/or power factor. In that case, current and power in each phase will be different. In practice, we may come across the following unbalanced loads :

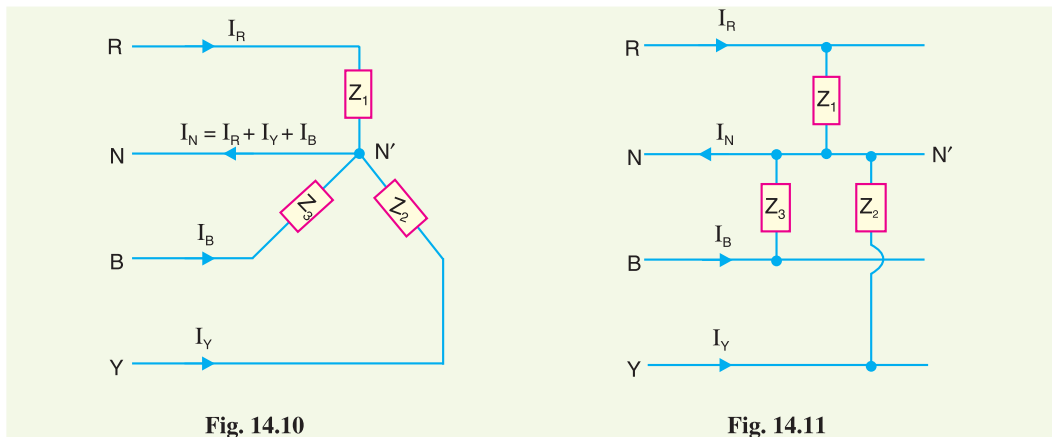
- (i) Four-wire star-connected unbalanced load
- (ii) Unbalanced Δ -connected load
- (iii) Unbalanced 3-wire, Y -connected load

The 3-phase, 4-wire system is widely used for distribution of electric power in commercial and industrial buildings. The single phase load is connected between any line and neutral wire while a 3-phase load is connected across the three lines. The 3-phase, 4-wire system invariably carries *unbalanced loads. In this chapter, we shall only discuss this type of unbalanced load.

14.4 Four-Wire Star-Connected Unbalanced Loads

We can obtain this type of load in two ways. First, we may connect a 3-phase, 4-wire unbalanced load to a 3-phase, 4-wire supply as shown in Fig. 14.10. Note that star point N of the supply is connected to the load star point N' . Secondly, we may connect single phase loads between any line and the neutral wire as shown in Fig. 14.11. This will also result in a 3-phase, 4-wire **unbalanced load because it is rarely possible that single phase loads on all the three phases have the same magnitude and power factor. Since the load is unbalanced, the line currents will be different in magnitude and displaced from one another by unequal angles. The current in the neutral wire will be the phasor sum of the three line currents *i.e.*

$$\text{Current in neutral wire, } I_N = I_R + I_Y + I_B \quad \dots \text{phasor sum}$$



The following points may be noted carefully :

- (i) Since the neutral wire has negligible resistance, supply neutral N and load neutral N' will be at the same potential. It means that voltage across each impedance is equal to the phase voltage of the supply. However, current in each phase (or line) will be different due to unequal impedances.
- (ii) The amount of current flowing in the neutral wire will depend upon the magnitudes of line currents and their phasor relations. In most circuits encountered in practice, the neutral current is equal to or smaller than one of the line currents. The exceptions are those circuits having severe unbalance.

* No doubt 3-phase loads (*e.g.* 3-phase motors) connected to this supply are balanced but when we add single phase loads (*e.g.* lights, fans etc.), the balance is lost. It is because it is rarely possible that single phase loads on all the three phases have the same magnitude and power factor.

** In actual practice, we never have an unbalanced 3-phase, 4-wire load. Most of the 3-phase loads (*e.g.* 3-phase motors) are 3-phase, 3-wire and are balanced loads. In fact, these are the single phase loads on the 3-phase, 4-wire supply which constitute unbalanced, 4-wire Y -connected load.

Example 14.7. Non-reactive loads of 10 kW, 8 kW and 5 kW are connected between the neutral and the red, yellow and blue phases respectively of a 3-phase, 4-wire system. The line voltage is 400V. Calculate (i) the current in each line and (ii) the current in the neutral wire.

Solution. This is a case of unbalanced load so that the line currents (and hence the phase currents) in the three lines will be different. The current in the *neutral wire will be equal to the phasor sum of three line currents as shown in Fig. 14.12.

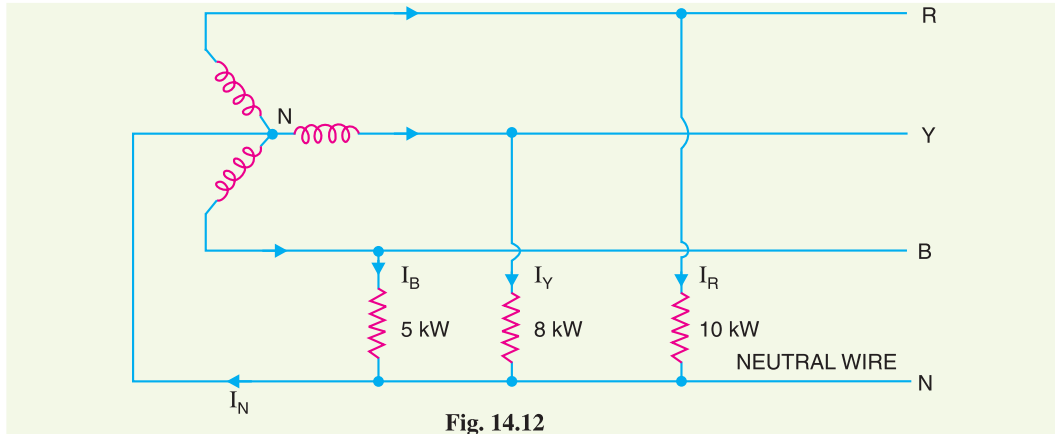


Fig. 14.12

(i) Phase voltage = $400/\sqrt{3} = 231 \text{ V}$
 $I_R = 10 \times 10^3 / 231 = 43.3 \text{ A}$
 $I_Y = 8 \times 10^3 / 231 = 34.6 \text{ A}$
 $I_B = 5 \times 10^3 / 231 = 21.65 \text{ A}$

(ii) The three line currents are represented by the respective phasors in Fig. 14.13. Note that the three line currents are of different magnitude but displaced 120° from one another. The current in the neutral wire will be the phasor sum of the three line currents.

Resolving the three currents along x -axis and y -axis, we have,

Resultant horizontal component = $I_Y \cos 30^\circ - I_B \cos 30^\circ$
 $= 34.6 \times 0.866 - 21.65 \times 0.866 = 11.22 \text{ A}$

Resultant vertical component = $I_R - I_Y \cos 60^\circ - I_B \cos 60^\circ$
 $= 43.3 - 34.6 \times 0.5 - 21.65 \times 0.5 = 15.2 \text{ A}$

As shown in Fig. 14.14, current in neutral wire is

$$I_N = \sqrt{(11.22)^2 + (15.2)^2} = 18.9 \text{ A}$$

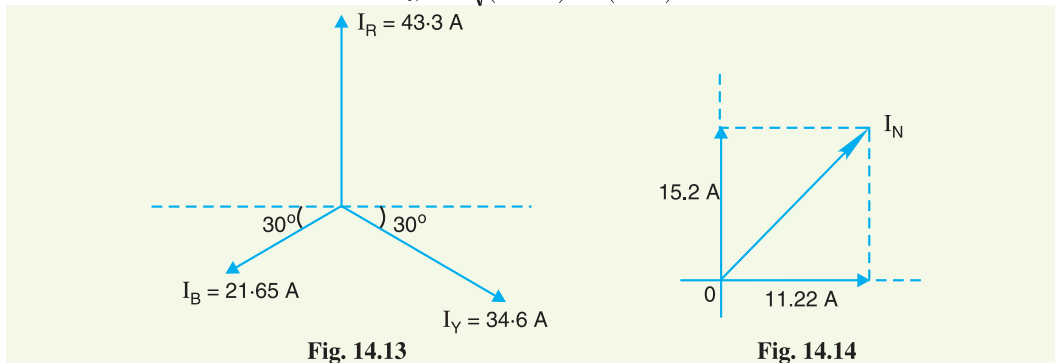


Fig. 14.13

Fig. 14.14

* Had the load been balanced (i.e. each phase having identical load), the current in the neutral wire would have been zero.

Example 14.8. A 3-phase, 4-wire system supplies power at 400 V and lighting at 230 V. If the lamps use require 70, 84 and 33 amperes in each of the three lines, what should be the current in the neutral wire? If a 3-phase motor is now started, taking 200 A from the lines at a p.f. of 0.2 lagging, what should be the total current in each line and the neutral wire? Find also the total power supplied to the lamps and the motor.

Solution. Fig. 14.15 shows the lamp load and motor load on 400 V/230 V, 3-phase, 4-wire supply.

Lamp load alone. If there is lamp load alone, the line currents in phases R, Y and B are 70 A, 84 A and 33 A respectively. These currents will be 120° apart (assuming phase sequence RYB) as shown in Fig. 14.16.

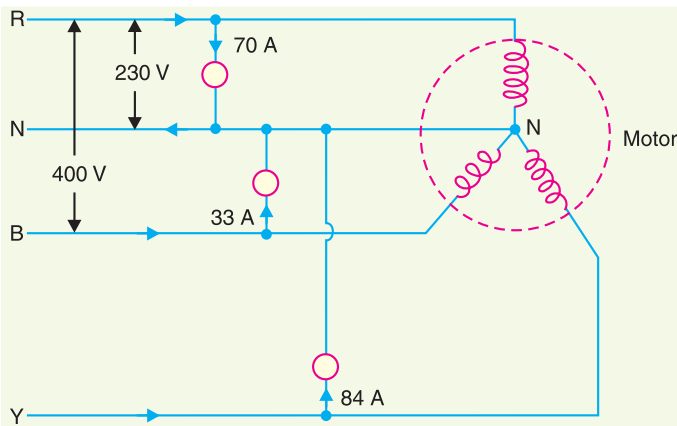


Fig. 14.15

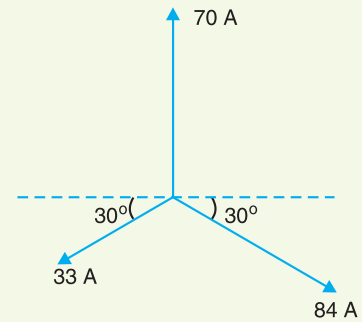


Fig. 14.16

$$\begin{aligned}\text{Resultant H-component} &= 84 \cos 30^\circ - 33 \cos 30^\circ = 44.17 \text{ A} \\ \text{Resultant V-component} &= 70 - 33 \cos 60^\circ - 84 \cos 60^\circ = 11.5 \text{ A}\end{aligned}$$

$$\therefore \text{Neutral current, } I_N = \sqrt{(44.17)^2 + (11.5)^2} = \mathbf{45.64 \text{ A}}$$

Both lamp load and motor load

When motor load is also connected along with lighting load, there will be no change in current in the neutral wire. It is because the motor load is balanced and hence no current will flow in the neutral wire due to this load.

$$\therefore \text{Neutral current, } I_N = \mathbf{45.64 \text{ A}} \quad \dots \text{same as before}$$

The current in each line is the phasor sum of the line currents due to lamp load and motor load.

$$\begin{aligned}\text{Active component of motor current} &= 200 \times \cos \phi_m = 200 \times 0.2 = 40 \text{ A} \\ \text{Reactive component of motor current} &= 200 \times \sin \phi_m = 200 \times 0.98 = 196 \text{ A}\end{aligned}$$

$$\begin{aligned}\therefore I_R &= \sqrt{(\text{sum of active comp.})^2 + (\text{reactive comp.})^2} \\ &= \sqrt{(40 + 70)^2 + (196)^2} = \mathbf{224.8 \text{ A}} \\ I_Y &= \sqrt{(40 + 84)^2 + (196)^2} = \mathbf{232 \text{ A}} \\ I_B &= \sqrt{(40 + 33)^2 + (196)^2} = \mathbf{209.15 \text{ A}}\end{aligned}$$

Power supplied

$$\text{Power supplied to lamps} = 230 (70 + 84 + 33) \times 1 = \mathbf{43010 \text{ W}} \quad (\because \cos \phi_L = 1)$$

$$\begin{aligned}
 \text{Power supplied to motor} &= \sqrt{3} V_L I_L \cos \phi_m \\
 &= \sqrt{3} \times 400 \times 200 \times 0.2 = \mathbf{27712 \text{ W}}
 \end{aligned}$$

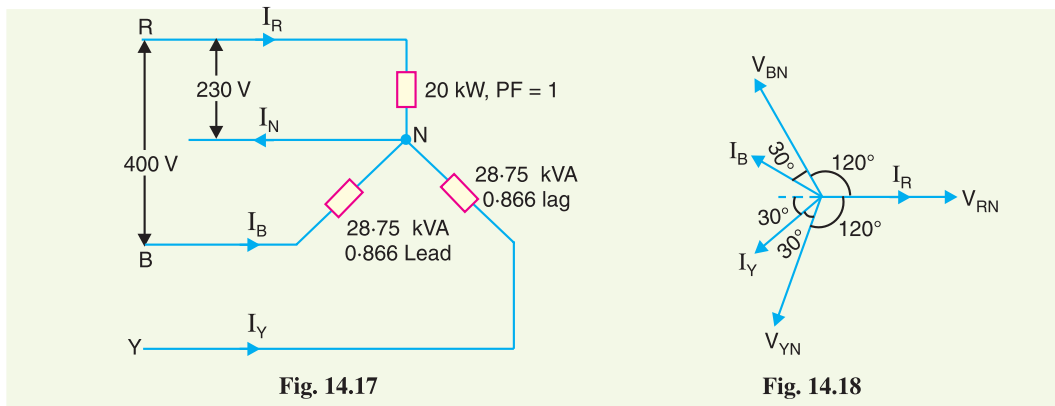
Example 14.9. The three line leads of a 400/230 V, 3-phase, 4-wire supply are designated as R, Y and B respectively. The fourth wire or neutral wire is designated as N. The phase sequence is RYB. Compute the currents in the four wires when the following loads are connected to this supply :

From R to N : 20 kW, unity power factor

From Y to N : 28.75 kVA, 0.866 lag

From B to N : 28.75 kVA, 0.866 lead

If the load from B to N is removed, what will be the value of currents in the four wires ?



Solution. Fig. 14.17 shows the circuit diagram whereas Fig. 14.18 shows its phasor diagram. The current I_R is in phase with V_{RN} , current I_Y lags behind its phase voltage V_{YN} by $\cos^{-1} 0.866 = 30^\circ$ and the current I_B leads its phase voltage V_{BN} by $\cos^{-1} 0.866 = 30^\circ$.

$$I_R = 20 \times 10^3 / 230 = \mathbf{86.96 \text{ A}}$$

$$I_Y = 28.75 \times 10^3 / 230 = \mathbf{125 \text{ A}}$$

$$I_B = 28.75 \times 10^3 / 230 = \mathbf{125 \text{ A}}$$

The current in the neutral wire will be equal to the phasor sum of the three line currents I_R , I_Y and I_B . Referring to the phasor diagram in Fig. 14.18 and resolving these currents along x-axis and y-axis, we have,

$$\begin{aligned}
 \text{Resultant X-component} &= 86.96 - 125 \cos 30^\circ - 125 \cos 30^\circ \\
 &= 86.96 - 108.25 - 108.25 = -129.54 \text{ A}
 \end{aligned}$$

$$\text{Resultant Y-component} = 0 + 125 \sin 30^\circ - 125 \sin 30^\circ = 0$$

$$\therefore \text{Neutral current, } I_N = \sqrt{(-129.54)^2 + (0)^2} = \mathbf{129.54 \text{ A}}$$

When load from B to N removed. When the load from B to N is removed, the various line currents are :

$$I_R = \mathbf{86.96 \text{ A}} \text{ in phase with } V_{RN} ; I_Y = \mathbf{125 \text{ A}} \text{ lagging } V_{YN} \text{ by } 30^\circ ; I_B = \mathbf{0 \text{ A}}$$

The current in the neutral wire is equal to the phasor sum of these three line currents. Resolving the currents along x-axis and y-axis, we have,

$$\text{Resultant X-component} = 86.96 - 125 \cos 30^\circ = 86.96 - 108.25 = -21.29 \text{ A}$$

$$\text{Resultant Y-component} = 0 - 125 \sin 30^\circ = 0 - 125 \times 0.5 = -62.5 \text{ A}$$

$$\therefore \text{Neutral current, } I_N = \sqrt{(-21.29)^2 + (-62.5)^2} = \mathbf{66.03 \text{ A}}$$

Example 14.10. A 3-phase, 4-wire distributor supplies a balanced voltage of 400/230 V to a load consisting of 30 A at p.f. 0.866 lagging for R-phase, 30 A at p.f. 0.866 leading for Y phase and 30 A at unity p.f. for B phase. The resistance of each line conductor is 0.2 Ω . The area of X-section of neutral is half of any line conductor. Calculate the supply end voltage for R phase. The phase sequence is RYB.

Solution. The circuit diagram is shown in Fig. 14.19. Since neutral is half the cross-section, its resistance is 0.4 Ω . Considering the load end and taking V_R as the reference vector, the phase voltages can be written as :

$$\vec{V}_R = 230 \angle 0^\circ \text{ volts} ; \vec{V}_Y = 230 \angle -120^\circ \text{ volts} ; \vec{V}_B = 230 \angle 120^\circ \text{ volts}$$

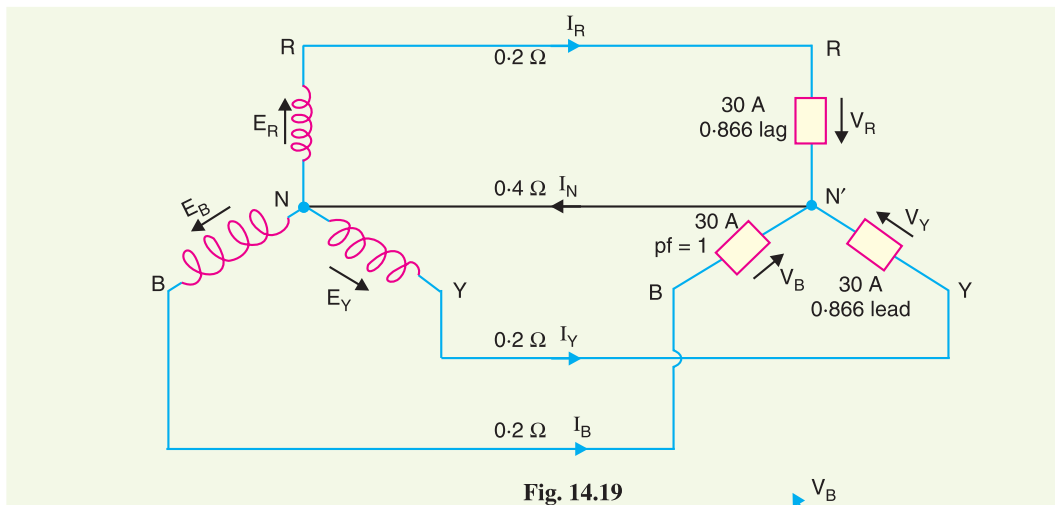


Fig. 14.19

The vector diagram of the circuit is shown in Fig. 14.20. The line current I_R lags behind V_R by an angle $\cos^{-1} 0.866 = 30^\circ$. The current I_Y leads V_Y by 30° and the current I_B is in phase with V_B . Referring to the vector diagram of Fig. 14.20, the line currents can be expressed as :

$$\vec{I}_R = 30 \angle -30^\circ \text{ amperes}$$

$$\vec{I}_Y = 30 \angle -90^\circ \text{ amperes}$$

$$\vec{I}_B = 30 \angle 120^\circ \text{ amperes}$$

Current in neutral wire,

$$\vec{I}_N = \vec{I}_R + \vec{I}_Y + \vec{I}_B$$

$$\begin{aligned} &= 30 \angle -30^\circ + 30 \angle -90^\circ + 30 \angle 120^\circ \\ &= 30 (0.866 - j 0.5) - 30 (j) + 30 (-0.5 + j 0.866) \\ &= 10.98 - j 19.02 \end{aligned}$$

Let the supply voltage of phase R to neutral be \vec{E}_R . Then,

$$\begin{aligned} \vec{E}_R &= \vec{V}_R + \text{Drop in R phase} + \text{Drop in neutral} \\ &= (230 + j 0) + 0.2 \times 30 \angle -30^\circ + (10.98 - j 19.02) \times 0.4 \\ &= 230 + 6 (0.866 - j 0.5) + 0.4 (10.98 - j 19.02) \\ &= 239.588 - j 10.608 \\ &= \mathbf{239.8 \angle -2.54^\circ \text{ volts}} \end{aligned}$$

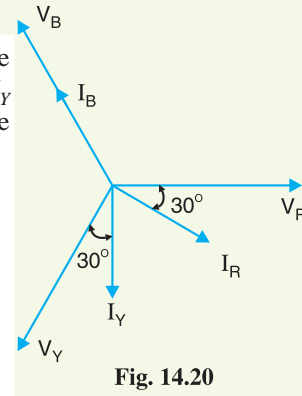


Fig. 14.20

Example 14.11. In a 3-phase, 4-wire, 400/230 V system, a lamp of 100 watts is connected to one phase and neutral and a lamp of 150 watts is connected to the second phase and neutral. If the neutral wire is disconnected accidentally, what will be the voltage across each lamp ?

Solution. Fig. 14.21 (i) shows the lamp connections. The lamp L_1 of 100 watts is connected between phase R and neutral whereas lamp L_2 of 150 watts is connected between phase Y and the neutral.

$$\text{Resistance of lamp } L_1, \quad R_1 = \frac{(230)^2}{100} = 529 \, \Omega$$

$$\text{Resistance of lamp } L_2, \quad R_2 = \frac{(230)^2}{150} = 352.67 \, \Omega$$

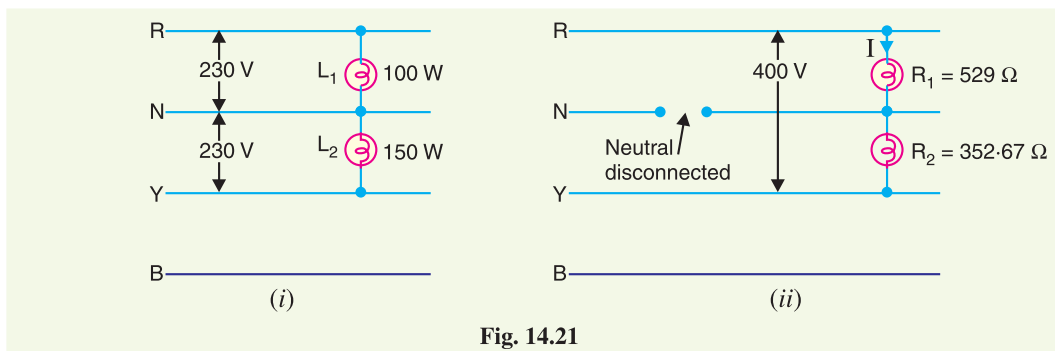


Fig. 14.21

When the neutral wire is disconnected as shown in Fig. 14.21 (ii), the two lamps are connected in series and the p.d. across the combination becomes equal to the line voltage $E_L (= 400 \text{ V})$.

$$\text{Current through lamps, } I = \frac{E_L}{R_1 + R_2} = \frac{400}{529 + 352.67} = 0.454 \text{ A}$$

$$\text{Voltage across lamp } L_1 = I R_1 = 0.454 \times 529 = \mathbf{240 \text{ V}}$$

$$\text{Voltage across lamp } L_2 = I R_2 = 0.454 \times 352.67 = \mathbf{160 \text{ V}}$$

Comments. The voltage across 100-watt lamp is increased to 240 V whereas that across 150-watt is decreased to 160 V. Therefore, 100-watt lamp becomes brighter and 150-watt lamp becomes dim. It may be noted here that if 100-watt lamp happens to be rated at 230 V, it may burn out due to 240 V coming across it.

TUTORIAL PROBLEMS

1. Non-reactive loads of 10 kW, 6 kW and 4 kW are connected between the neutral and red, yellow and blue phases respectively of a 3-phase, 4-wire 400/230V supply. Find the current in each line and in the neutral wire.
[$I_R = 43.3 \text{ A}$; $I_Y = 26 \text{ A}$; $I_B = 17.3 \text{ A}$; $I_N = 22.9 \text{ A}$]
2. A factory has the following loads with a power factor of 0.9 lagging in each case. Red phase 40 A, yellow phase 50 A and blue phase 60 A. If the supply is 400V, 3-phase, 4-wire, calculate the current in the neutral wire and the total power.
[17.3A, 31.2 kW]
3. In a 3-phase, 4-wire system, two phases have currents of 10A and 6A at lagging power factors of 0.8 and 0.6 respectively, while the third phase is open-circuited. Calculate the current in the neutral wire. [7A]
4. A 3-phase, 4-wire system supplies a lighting load of 40A, 30A and 20A respectively in the three phases. If the line voltage is 400 V, determine the current in the neutral wire. [17.32A]

14.5. Ground Detectors

Ground detectors are the devices that are used to detect the ground fault for ungrounded a.c. systems.

When a ground fault occurs on such a system, immediate steps should be taken to clear it. If this is not done and a second ground fault happens, a short circuit occurs.

Fig.14.22 shows how lamps are connected to an ungrounded 3-phase system for the detection of ground fault. If ground fault occurs on any wire, the lamp connected to that wire will be dim and the lamps connected to healthy (ungrounded) wire will become brighter.

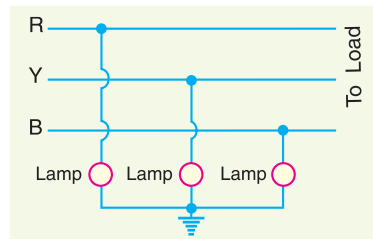


Fig. 14.22

SELF - TEST

1. Fill in the blanks by inserting appropriate words/figures.

- (i) The most common system for secondary distribution is 400/..... V, 3-phase, wire system.
- (ii) In a 3-phase, 4-wire a.c. system, if the loads are balanced, then current in the neutral wire is
- (iii) Distribution transformer links the and systems.
- (iv) The 3-phase, 3-wire a.c. system of distribution is used for loads.
- (v) For combined power and lighting load, system is used.

2. Pick up the correct words/figures from brackets and fill in the blanks.

- (i) 3-phase, 4-wire a.c. system of distribution is used for load. (*balanced, unbalanced*)
- (ii) In a balanced 3-phase, 4-wire a.c. system, the phase sequence is RYB . If the voltage of R phase = $230 \angle 0^\circ$ volts, then for B phase it will be ($230 \angle -120^\circ$ volts, $230 \angle 120^\circ$ volts)
- (iii) In a.c. system, additions and subtractions of currents are done (*vectorially, arithmetically*)
- (iv) The area of X-section of neutral is generally that of any line conductor. (*the same, half*)
- (v) For purely domestic loads, a.c. system is employed for distribution. (*single phase 2-wire, 3-phase 3-wire*)

ANSWERS TO SELF-TEST

1. (i) 230, 4 (ii) zero (iii) primary, secondary (iv) balanced (v) 3-phase 4-wire.
2. (i) unbalanced (ii) $230 \angle 120^\circ$ (iii) vectorially (iv) half (v) single phase 2-wire.

CHAPTER REVIEW TOPICS

1. How does a.c. distribution differ from d.c. distribution ?
2. What is the importance of load power factors in a.c. distribution ?
3. Describe briefly how will you solve a.c. distribution problems ?
4. Write short notes on the following :
 - (i) Difference between d.c. and a.c. distribution
 - (ii) Systems of a.c. distribution

DISCUSSION QUESTIONS

1. What are the undesirable effects of too much voltage variation on a distribution circuit ?
2. What are the effects of diversity factor on the maximum load of a distribution transformer ?
3. Where does the greatest current density occur in a distribution feeder ?
4. What is the controlling factor in determining the size of a distributor ?
5. In which situation is secondary distribution eliminated ?