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ECON 715

Major Results from Theory of the Firm

1 Cost of Inputs and Profit

Q. What happens to firm's profit and demand for inputs when input prices go up? Let the profit of some firm be $\pi(w, x) = R(x) - wx$, where w is $1 \times n$ vector of input prices, x is $n \times 1$ vector of inputs and R is revenue function. We can prove mathematically that it is impossible that rising cost can increase profit. This is true for any profit maximizing firm and any industry.

Theorem 1 *Let w_1, w_2 be two input price vectors, with $w_2 > w_1$. This means that for any input i we have $w_{2i} \geq w_{1i}$, with $w_{2i} > w_{1i}$ for at least one input. Let x_1 and x_2 be profit maximizing inputs under w_1, w_2. Then we must have*

$$\pi(w_1, x_1) \geq \pi(w_2, x_2)$$

Proof.

$$\begin{aligned} \pi(w_1, x_1) &\geq \pi(w_1, x_2) && (x_1 \text{ is profit maximizing quantity under } w_1) \\ &\geq \pi(w_2, x_2) && (\text{firm's cost are higher at } w_2 \text{ than at } w_1) \end{aligned}$$

■

Notice that in the first step, the object $\pi(w_1, x_2)$ is well defined, since the firm can afford to buy x_2 under w_2, so it can definitely afford x_2 when inputs are cheaper under w_1. Also notice the generality of the last theorem. The only assumption we made about the firm is that it maximizes profit. The firm can use any number of inputs, produce any number of products, and operate under any possible market structure and government regulation. We proved that the firm's profit cannot increase when the price of inputs goes up. We can also prove that input demand is decreasing in their prices.

Theorem 2 *Let w_1, w_2 be two input price vectors, with $w_2 > w_1$ and let x_1 and x_2 be profit maximizing inputs under w_1, w_2. Then we must have*

$$(w_2 - w_1)(x_1 - x_2) \geq 0$$

Proof.

$$\begin{aligned} R(x_1) - w_1 x_1 &\geq R(x_2) - w_1 x_2 && (x_1 \text{ is profit maximizing quantity under } w_1) \\ R(x_2) - w_2 x_2 &\geq R(x_1) - w_2 x_1 && (x_2 \text{ is profit maximizing quantity under } w_2) \end{aligned}$$

Adding and rearranging, gives

$$\begin{aligned} -w_2 x_2 - w_1 x_1 &\geq -w_2 x_1 - w_1 x_2 \\ w_2(x_1 - x_2) - w_1(x_1 - x_2) &\geq 0 \\ (w_2 - w_1)(x_1 - x_2) &\geq 0 \end{aligned}$$

This means that if for input i we have $w_{2i} \geq w_{1i}$, then we must have $x_1 \leq x_2$. That is, the firm's response to a higher price of an input is to buy less of that input. ■

2 Cost of Inputs and Cost of Output

Q. If the cost of some input increasing by $\%\Delta p_i$, what is the $\%$ change in the firm's marginal cost and total cost?

Suppose the total cost of inputs is:

$$TC = p_1 x_1 + p_2 x_2 + ... + p_2 x_2$$

Suppose that the firm has constant marginal cost, i.e.

$$TC = MC \cdot Q$$

where Q is the total output. It can be shown that

$$\%\Delta MC \approx \frac{p_i x_i}{TC} \%\Delta p_i$$

That is, if the fraction of total cost spent on input i is $p_i x_i / TC = 7\%$, and the price of input i increased by 1%, then the marginal cost increase by $7\% \cdot 1\% = 0.07\%$. This approximation assumes that the fraction of total cost spent on input i remains constant. As another example, suppose that transportation cost of some company double, and the company spends 7% of its total cost on delivery. Then the marginal cost of the company, assuming that it continues to spend 7% of cost on transportation, will increase by $7\% \cdot 2 = 14\%$.

3 Passing Higher Cost to Consumers/Buyers

Q. Can the rising marginal cost be passed to consumers?

A. Depends on how inelastic the demand is for the product.

We showed that for any firm with market power, the price charged is given by

$$p = \left[1 - \frac{1}{1+\varepsilon}\right] MC = [1 + \mu] MC \tag{1}$$

where ε is the price elasticity of demand and μ is the markup rate. Thus, is MC increase by some percent, $\%\Delta MC$, if the markup rate does not change, the price will increase by the same percentage as the marginal cost:

$$\%\Delta p = \%\Delta MC$$

However, if the elasticity increases, and the markup rate drops, then the price will not increase by the same percent.

Example 3 *Suppose demand for a product is $p = 100 - Q$, with marginal revenue $MR = 100 - 2Q$. Suppose that $MC = 50$ initially, and after rising cost of transportation, $MC = 60$. Find the output, price and profit before and after the change in cost, ignoring the firm's fixed cost.*

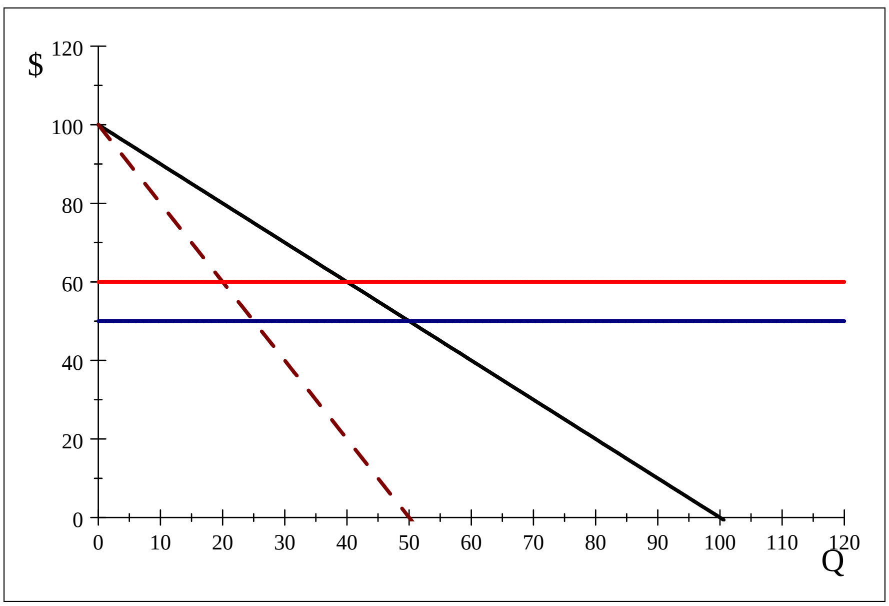
Solution 4 *Profit maximization before the change:*

$$\begin{aligned}
MR &= MC \\
100 - 2Q &= 50 \Rightarrow Q^* = 25 \\
P^* &= 100 - Q^* = 75 \\
\pi^* &= P^*Q^* - MC \cdot Q^* = 75 \cdot 25 - 50 \cdot 25 = 625 \\
[\textit{Markup rate}] &: \mu = \frac{P}{MC} - 1 = \frac{75}{50} - 1 = 50\%
\end{aligned}$$

Profit maximization after the change:

$$\begin{aligned}
MR &= MC \\
100 - 2Q &= 60 \Rightarrow Q^* = 20 \\
P^* &= 100 - Q^* = 80 \\
\pi^* &= P^*Q^* - MC \cdot Q^* = 80 \cdot 20 - 60 \cdot 20 = 400 \\
[\textit{Markup rate}] &: \mu = \frac{P}{MC} - 1 = \frac{80}{60} - 1 = 33\frac{1}{3}\%
\end{aligned}$$

The next figure illustrates the equilibrium before the rise in MC.



Summary:

	Before	*After*	$\%\Delta$
MC	50	60	20%
Q^*	25	20	-20%
P^*	75	80	$10\frac{2}{3}\%$
π^*	625	400	-36%
$\mu = \frac{P}{MC} - 1$	50%	$33\frac{1}{3}\%$	-33%

Notice that the firm cost per unit increased by 20%, but the market price increased only by $10\frac{2}{3}\%$. The profit fell by 36%! According to (1), the firm could maintain the same markup, if the elasticity (and markup rate) remain constant.

4 Uncertainty

Q. When should firms be worried about future uncertainty?

A. If the optimal actions today depend on the realization of uncertainty in the future, then the firm's need to try and forecast key variables and/or insure themselves against future changes in key variables. For example, if the firm considers expanding its productive capacity (e.g. building new plants), and the future success of such expansion depends on realization of future variables (e.g. demand, cost of inputs), the decision to expand depends on the future uncertainty. If instead, the company can make choices after the uncertainty is realized, then the future uncertainty does not affect current actions. For example, if the company makes production choices after the transportation costs are known, then the best course of action is to wait until the costs are known.

Key result: only risk-averse people will ever buy insurance. Therefore, if the owners of the firm are risk averse, and insurance is not too expensive, the managers can consider buying insurance. For example futures contracts or other financial derivatives, which pay to the holder when the price of diesel goes up, can be used as insurance.