

PROBLEM SET 6

QUANTUM PHYSICS

Part A

1. C

2. B $p = \frac{h}{\lambda}$ 2 x energy $\Rightarrow \frac{1}{2} \lambda$ $\left[E = \frac{hc}{\lambda} \right] \therefore 2 \times p$

3. D x-rays travelling straight through (or reflected) are unchanged; the scattered x-rays (λ_3) have given some of their energy to electrons with which they collided.

4. C

5. B

6. C

7. D

8. E $E_1 = \frac{n^2 h^2}{8mL^2}$ $E_2 = \frac{n^2 h^2}{8(2m)(2L)^2} = \frac{1}{8} E_1$

9. A

10. C Longest $\lambda \Rightarrow$ smallest energy

$E_{\text{photon}} = -\Delta E_{\text{atom}}$ Smallest ΔE_{atom} : Look for smallest 'jump' (but must finish on level $n = 2$ for Balmer series).

11. a) Vibrating electrons, atoms, molecules, etc. can only have discrete values of energy

$$E_n = nhf$$

where f is the natural frequency of oscillation.

b) These oscillators can only emit or absorb energy in discrete packets.

12. An electron accelerating around a nucleus should continuously emit EM radiation (and with it, energy). As it loses energy it should spiral into the nucleus (just like a satellite losing energy due to atmospheric friction would spiral inward and eventually fall to the ground). As it falls into lower orbits it will revolve with greater and greater angular frequencies and therefore the radiation emitted will also have greater and greater frequencies. Result: As it spirals inward to the nucleus, a whole spectrum of EM frequencies will be emitted. (Another result: the lifetime of the orbit, and therefore of the atom, would only be a tiny fraction of a second. Hmmm...)

13. If an object is comparable in size (or smaller) to the wavelength of the waves that are hitting it, it will not be detected. The wavelengths of electrons at the energies used in the electron microscope are orders of magnitude smaller than the wavelengths of visible light and can therefore discern smaller details.

Part B

1. a) The kinetic energy of the photoelectrons is related to the energy of the photon and the work function :

$$K = \frac{hc}{\lambda} - \phi = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{3.5 \times 10^{-7}} - 2.24 = 1.31 \text{ eV}$$

(Note that ϕ does not correspond to the phase constant!!)

b) When wavelengths are longer than the cutoff wavelength, the wavelengths will not have enough energy to extract any electrons. So $K = 0$.

$$\frac{hc}{\lambda} = \phi \Rightarrow \lambda = \frac{hc}{\phi} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{2.24} = 5.54 \times 10^{-7} \text{ m}$$

2. a) The Plank constant can be found from the information given in graph

$$\begin{aligned} K &= hf - \phi \\ \frac{K}{e} &= \frac{hf}{e} - \frac{\phi}{e} && \left(\frac{K}{e} = V_o \right) \\ V_o &= \left(\frac{h}{e} \right) (f) - \frac{\phi}{e} \quad \Rightarrow \quad \text{linear graph of } V_o \text{ vs } f \\ &&& \text{where the slope corresponds to } \frac{h}{e} \end{aligned}$$

$$\text{From graph: } \text{slope} = \frac{4.0 \text{ V}}{1.1 \times 10^{15} \text{ Hz}} = 3.6 \times 10^{-15} \text{ V} \cdot \text{s}$$

(Remember: Volts are Joules/Coulomb (J/C) or electron volts/elementary charge (eV/e))

$$h = (\text{slope})(e) = 3.6 \times 10^{-15} \text{ eV} \cdot \text{s}$$

b) At $V_o = 0$, $hf = \phi$

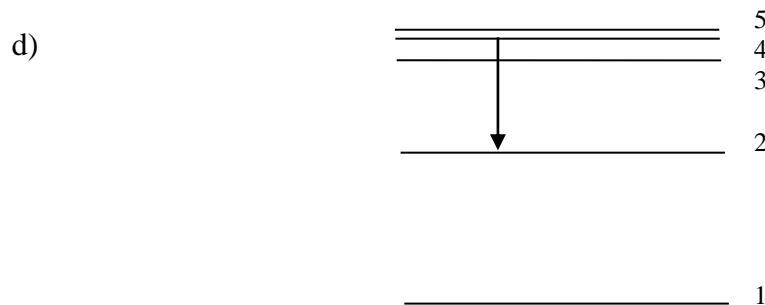
$$\begin{aligned} \phi &= (3.6 \times 10^{-15} \text{ eV} \cdot \text{s})(1.05 \times 10^{15} \text{ Hz}) \quad (\text{from graph}) \\ &= 3.8 \text{ eV} \end{aligned}$$

3. a) $E_{initial} = \frac{-13.6}{n_i^2} = -0.85 \Rightarrow n_i = 4$ and $E_{final} = \frac{-13.6}{n_f^2} = -3.4 \Rightarrow n_f = 2$

b) $E_{photon} = -\Delta E_{atom} = -(-3.4 - (-0.85)) = 2.55 eV$

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{2.55} = 4.87 \times 10^{-7} m$$

c) Balmer



4. In the Balmer series for hydrogen the final state $n_f = 2$

$$E_{photon} = -\Delta E_{atom}$$

from $n = 3$ to $n = 2$:

$$E_{photon} = \frac{13.6}{2^2} - \frac{13.6}{3^2} = 1.89 eV$$

$$\lambda = \frac{hc}{E} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{1.89} = 6.57 \times 10^{-7} m$$

from $n = 4$ to $n = 2$:

$$E_{photon} = \frac{13.6}{2^2} - \frac{13.6}{4^2} = 2.55 eV$$

$$\lambda = \frac{hc}{E} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{2.55} = 4.87 \times 10^{-7} m$$

from $n = 5$ to $n = 2$:

$$E_{\text{photon}} = \frac{13.6}{2^2} - \frac{13.6}{5^2} = 2.86 \text{ eV}$$

$$\lambda = \frac{hc}{E} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{2.86} = 4.35 \times 10^{-7} \text{ m}$$

b) (i) $\Delta E_{\text{atom}} = \frac{-13.6}{5^2} - \frac{-13.6}{4^2} = 0.306 \text{ eV}$ a photon with this energy can cause this transition

(ii) photon energy: $\frac{-13.6}{6^2} - \frac{-13.6}{5^2} = 0.166 \text{ eV}$

(iii) $\lambda = \frac{hc}{E}$ for $\lambda_{4 \rightarrow 5} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{0.306} = 4.06 \times 10^{-6} \text{ m}$

for $\lambda_{5 \rightarrow 6} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{0.166} = 7.48 \times 10^{-6} \text{ m}$

5. a) $\Delta\lambda = \frac{h}{m_e c} (1 - \cos\phi)$

$$\Delta\lambda = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} (3 \times 10^8)} (1 - \cos 90) = 2.43 \times 10^{-12} \text{ m}$$

(Note that h is in units of J·s)

(Note that ϕ does not correspond to the phase constant, nor the work function!!)

b) The kinetic energy of recoiling electron is proportional to the change in energy of the photon

$$E_{\text{photon initial}} = \frac{hc}{\lambda_i} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{2 \times 10^{-10}} = 6.21 \times 10^3 \text{ eV}$$

$$E_{\text{photon final}} = \frac{hc}{\lambda_f} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{2.0243 \times 10^{-10}} = 6.135 \times 10^3 \text{ eV}$$

(note that h is in eVs, make sure you how convert one form to the other and when you use it... check your units)

$$K_{electron} = -\Delta E_{photon} = 6.210 \times 10^3 - 6.135 \times 10^3 = 74.5 eV$$

6. The kinetic energy of recoiling electron is proportional to the change in energy of the photon

$$K_{electron} = \frac{1}{2} m_e v^2 = \frac{1}{2} (9.11 \times 10^{-31}) (1.5 \times 10^6)^2 = 1.02 \times 10^{-18} J$$

$$E_{photon\ initial} = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{8.5 \times 10^{-10}} = 2.34 \times 10^{-16} J$$

$$E_{photon\ final} = 2.34 \times 10^{-16} - 1.02 \times 10^{-18} = 2.33 \times 10^{-16} J \quad (\text{gave energy to electron})$$

$$\lambda_f = \frac{hc}{E_{final}} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{2.33 \times 10^{-16}} = 8.54 \times 10^{-10} m$$

b) To find the angle, we need to apply the Compton's scattering equation

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos\phi)$$

$$\phi = \cos^{-1} \left(1 - \frac{(8.54 \times 10^{-10} - 8.5 \times 10^{-10}) \cdot 9.11 \times 10^{-31} \cdot 3 \times 10^8}{6.63 \times 10^{-34}} \right) = 130 \text{ degrees}$$

7. a) a photon travels at speed of light! So $v_{photon} = c = 3 \times 10^8 \text{ m/s}$

$$\text{electron: } p = mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{m_e \lambda} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} (10^{-10})} = 7.3 \times 10^6 \text{ m/s}$$

baseball with estimate mass of 0.2 kg: $v = \frac{h}{m_b \lambda} = \frac{6.63 \times 10^{-34}}{0.2(10^{-10})} = 3 \times 10^{-23} \text{ m/s}$

b) the energy of the photon is inversely proportional to its wavelength

$$E = \frac{hc}{\lambda} = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{10^{-10}} = 1.2 \times 10^4 \text{ eV}$$

c) the kinetic energy of the electron:

$$K_{electron} = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31})(7.3 \times 10^6)^2 = 2.43 \times 10^{-17} \text{ J}$$

$$K_{electron} = \frac{2.43 \times 10^{-17} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 150 \text{ eV}$$

the kinetic energy of the baseball:

$$K_{baseball} = \frac{1}{2}mv^2 = \frac{1}{2}(0.2)(3 \times 10^{-23})^2 = 9 \times 10^{-47} \text{ J}$$

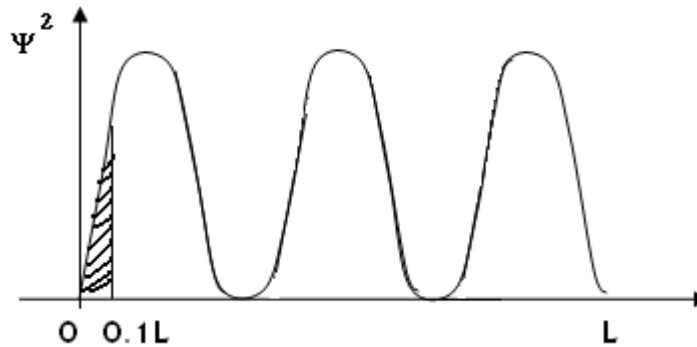
$$K_{baseball} = \frac{9 \times 10^{-47} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 6 \times 10^{-28} \text{ eV}$$

8. $E_{photon} = -\Delta E_{electron} = -(E_1 - E_2)$

$$E_{photon} = (2^2 - 1^2) \left(\frac{h^2}{8mL^2} \right) = \frac{3(6.63 \times 10^{-34})^2}{8(9.11 \times 10^{-31})(2.5 \times 10^{-9})^2} = 2.90 \times 10^{-20} \text{ J}$$

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{2.90 \times 10^{-20}} = 6.87 \times 10^{-6} \text{ m}$$

9. a)



b) we first have to define the work function and the probability function

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$\Psi^2(x) = \frac{2}{L} \sin^2 \frac{n\pi x}{L}$$

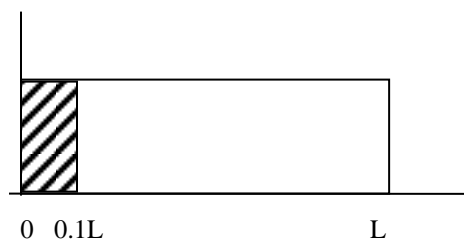
$$P_{0-0.1L} = \int_0^{0.1L} \Psi^2(x) dx \quad (\text{corresponds to the area of shaded region})$$

$$P_{0-0.1L} = \frac{2}{L} \int_0^{0.1L} \sin^2 \left(\frac{n\pi}{L} \right) x dx$$

$$P_{0-0.1L} = \frac{2}{L} \left[\frac{x}{2} - \frac{\sin 2 \left(\frac{n\pi}{L} \right) x}{4 \left(\frac{n\pi}{L} \right)} \right]_0^{0.1L} \quad \text{with } n = 3 \text{ (why? Explain!)}$$

$$P_{0-0.1L} = \frac{2}{L} \left[\frac{0.1L}{2} - \frac{L \sin(0.6\pi)}{12\pi} \right] = 0.0495 = 4.95\%$$

c) Classically, even probability everywhere in box. 1/10 chance of being between 0 and 0.1L



10. From the Heisenberg uncertainty principle for position and momentum

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta v \geq \frac{\hbar}{2\Delta x(m)} = \frac{6.63 \times 10^{-34}}{4\pi(1 \times 10^{-11})(1.67 \times 10^{-27})} = 3.15 \times 10^3 \text{ m/s}$$