Solutions to NYA Problem Set #2

Vectors 2D Motion Relative Velocity

Question 1:

The magnitude increases by 3 but the direction remains the same:

$$3\vec{W} = 3(4\hat{i} + 7\hat{j}) = 12\hat{i} + 21\hat{j}$$

Question 2:

The magnitude changes but the direction remains the same:

$$0.75\vec{P} = 0.75(32m) \text{ at } 150^\circ = 24m \text{ at } 150^\circ$$

Question 3:

$$A_x = A\cos\theta_A = 3.0\cos 110 = -1.03$$
$$A_y = A\sin\theta_A = 3.0\sin 110 = 2.82$$
$$B_x = B\cos\theta_B = 5.0\cos 200 = -4.69$$
$$B_y = B\sin\theta_B = 5.0\sin 200 = -1.71$$
$$C_x = C\cos\theta_C = 6.0\cos 295 = 2.54$$
$$C_y = C\sin\theta_C = 6.0\sin 295 = -5.44$$

a) in unit vector form, thus $\hat{i}$ and $\hat{j}$ notation:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} = (-1.03\hat{i} + 2.82\hat{j})m$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} = (-4.70\hat{i} - 1.71\hat{j})m$$
$$\vec{C} = C_x\hat{i} + C_y\hat{j} = (2.54\hat{i} - 5.44\hat{j})m$$

b) $R_x = A_x + B_x + C_x = -3.19m$ $R_y = A_y + B_y + C_y = -4.33m$

the resultant in unit vector form: $\vec{R} = \vec{A} + \vec{B} + \vec{C} = \boxed{(-3.2\hat{i} - 4.3\hat{j})m}$

c) in polar form

$$\|\vec{R}\| = \sqrt{R_x^2 + R_y^2} = \sqrt{3.19^2 + 4.33^2} = 5.38m$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-4.33}{-3.19} = 54^\circ + 180 = 234^\circ$$

$$\boxed{\vec{R} = 5.38m \text{ at } \theta = 234^\circ}$$

Question 4:

a) $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_o}{\Delta t} = \dfrac{2\hat{i} + 6\hat{j}}{10s} = \boxed{(0.2\hat{i} + 0.6\hat{j})m/s^2}$

b) $\Delta\vec{r} = \vec{v}_o(\Delta t) + \dfrac{1}{2}\vec{a}(\Delta t)^2 = (v_{ox}\Delta t + \dfrac{1}{2}a_x(\Delta t)^2)\hat{i} + (v_{oy}\Delta t + \dfrac{1}{2}a_y(\Delta t)^2)\hat{j}$

$$\Delta\vec{r} = (3\cdot 10 + \frac{1}{2}0.2\cdot 10^2)\hat{i} + (-4\cdot 10 + \frac{1}{2}0.6\cdot 10^2)\hat{j} = \boxed{(40\hat{i} - 10\hat{j})m}$$

Question 5:

$$\vec{v}_o = (60.2\hat{i} + 79.9\hat{j}`)m/s$$

$$\vec{a} = -10\hat{j}\ m/s^2$$

a)

$$\vec{r}_f = \vec{v}_o\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2 = (722\hat{i} + 958\hat{j}) + (-720\hat{j})$$

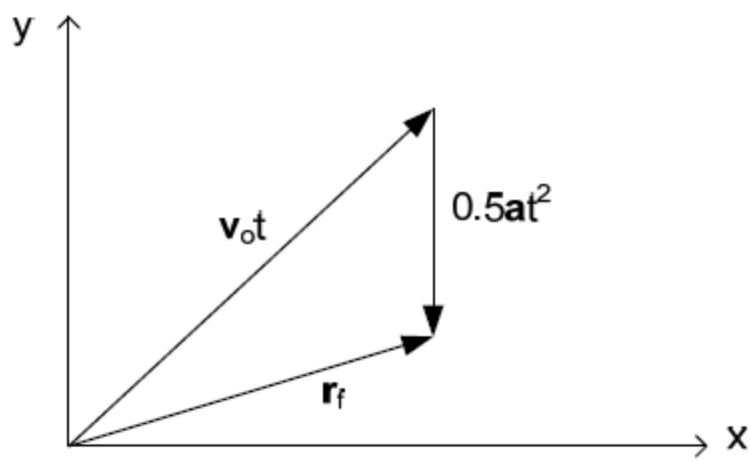
$$\boxed{\vec{r}_f = (722\hat{i} + 238\hat{j})m}$$

b)

$$\vec{v}_f = \vec{v}_o + a(\Delta t) = (60.2\hat{i} + 79.9\hat{j}) + (-120\hat{j}`)$$

$$\boxed{\vec{v}_f = (60.2\hat{i} - 40.1\hat{j})m/s}$$

c)



Question 6:

$$v_{ox} = v_o \cos\theta = 35\cos 45 = 24.75 m/s$$
$$v_{oy} = v_o \sin\theta = 35\sin 45 = 24.75 m/s$$

$$\Delta x = 100m = 24.75t \rightarrow t = 4.04s$$

$$\Delta y = 24.75 \cdot 4.04 + \frac{1}{2}(-9.81)(4.04)^2 = 20.0m$$
$$y_f = y_i + \Delta y = 1.5m + 20.0m = \boxed{21.5m}$$

Since the fence is 8.0m high but the ball is 21.5m above the ground, it will definitively clear the fence and it's a home run!

Question 7:

$$v_{ox} = v_o \cos\theta = 40\cos 30 = 34.6 m/s$$
$$v_{oy} = v_o \sin\theta = 40\sin 30 = 20.0 m/s$$

a)
$$\Delta y = y_f - y_i = 50m - 35m = 15m$$
$$15m = 20 \cdot t + \frac{1}{2}(-9.81)t^2 \rightarrow 4.9t^2 - 20t + 15$$

$$\text{so } t = \frac{20m/s \pm \sqrt{400m^2/s^2 - 294m^2/s^2}}{2(4.9m/s^2)} = 3.09s \text{ or } 0.989s$$

but since the problem state the boulder is on its way <u>down</u> the time must be <u>3.09s</u>.

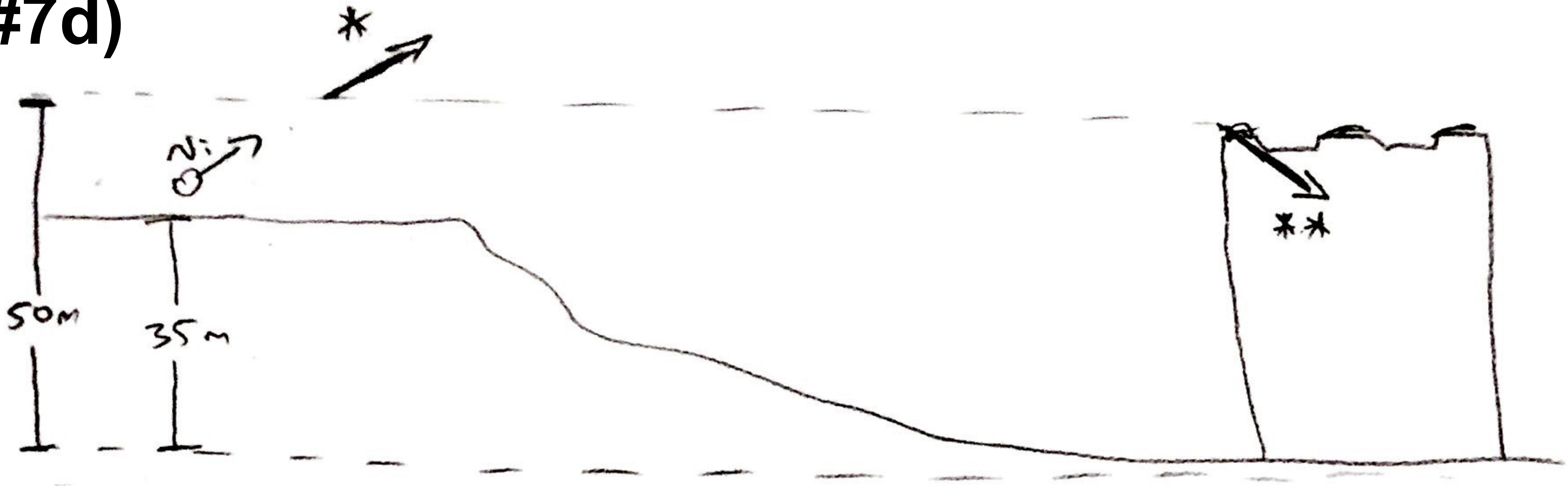
b) $\Delta x = (34.6m/s)(3.09s) = \boxed{107m}$

c) Δy_{max} occurs when $v_y = 0$

$$v_y = 0 = v_{oy} + a_y t = 20m/s - 9.81 \cdot t \rightarrow t = \frac{20}{9.81} = 2.04s$$

$$\text{so } y_{max} = 20m/s \cdot 2.04s - (4.9m/s^2)(2.04)^2 = 20.4m$$
$$y_{max} = y_i + \Delta y = 35m + 20.4m = \boxed{55.4m}$$

#7d)



$$\vec{v}_{fy}^{\,2} = \vec{v}_{iy}^{\,2} + 2\vec{a}\Delta y$$

$$\vec{v}_{fy}^{\,2} = 20^2 + 2(-9.81)(15)$$

$$\vec{v}_{fy}^{\,2} = 105.7$$

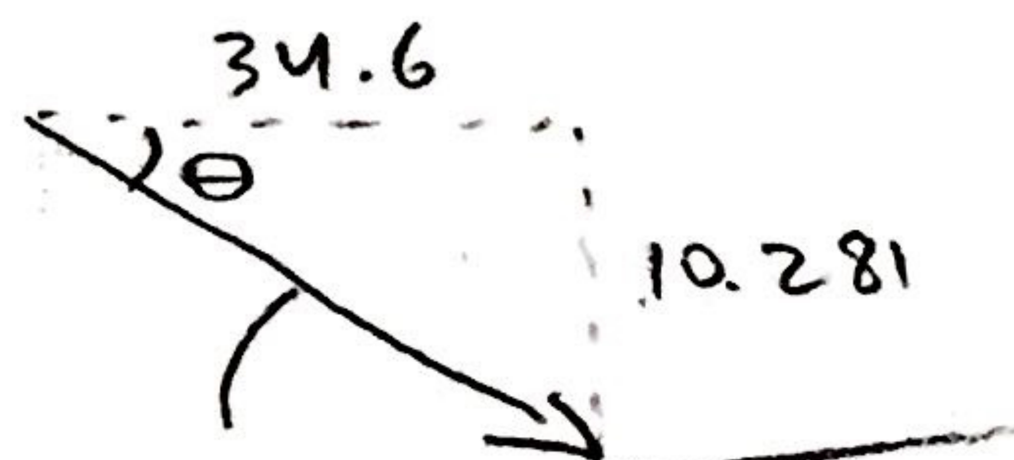
$$\vec{v}_{fy} = \pm 10.281$$

* positive is on its way up

** negative is on its way down

$$\vec{v}_{fx} = \vec{v}_{ix} = 34.6 \text{ m/s}$$

(because no accel. in x-dir'n)



$$|v_f| = \sqrt{34.6^2 + 10.281^2}$$
$$= 36.1 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{10.281}{34.6}\right)$$
$$= 16.5^\circ \text{ below x-axis}$$

$$\vec{v}_f = 36.1 \text{ m/s} @ -16.5^\circ$$

OR

$$= 36.1 \text{ m/s} @ 343.5^\circ$$

OR

$$= (34.6\hat{i} - 10.3\hat{j}) \text{ m/s}$$

Question 8:

$v_{oy} = ? = v_o \sin 45$

$v_{ox} = ? = v_o \cos 45$

$\Delta x = 120m = 0.707 v_o t \qquad (1)$

$\Delta y = 3m - 1.2m = 1.8m = 0.707 v_o t - 4.9m/s^2 \cdot t^2 \quad (2)$

from eq. (1): $0.707 v_o t = 120m$

substituting into eq (2) $1.8m = 120m - 4.9m/s^2 \cdot t^2 \rightarrow t = \sqrt{\dfrac{118.2m}{4.9m/s^2}} = 4.91s$

then substituting this value into eq (1) $v_o = \dfrac{120m}{(0.707)(4.91s)} = \boxed{34.6m/s}$

Question 9:

a) The winning car has travelled a bit more distance than the other car, so it has a greater average speed:

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

b) The losing car is at a different place then where it started; the winning car is at the same place it started, so its displacement is zero. Since:

$$\text{average velocity} = \frac{\text{displacement}}{time}$$

the losing car has the greatest average velocity.