Assignment 5

EE 553 Power System Economics

Due May 10th, 2017 at 8pm. Email to ywang11@uw.edu

Problem 1. The demand curve, in terms of quantity q (kWh) brought by a consumer, in a given period, as a function of price p (¢) is given by the following:

$$p = -0.01q + 5.$$

1. Suppose the price is $p = 3.5¢/kWh$. Find the total payment made by the consumer and the demand surplus.

2. If the price is increased by 10%, find the reduction in consumption and the new payment.

3. Now suppose the **cost** of production is given by

   $$C = 0.0075q^2 + 1.3q.$$

   Find the marginal cost (the supply curve) and find the equilibrium price and demand at this price.

4. At the equilibrium point, find the demand surplus, the *net revenue (profit)* of the supplier, and the social welfare.

Solution.

1. Solving for q gives $q = 150$ kWh. The total payment made by the demand is $150 \cdot 3.5 =$ \$5.25. The demand surplus is (area of the triangle) $150 \cdot 1.5/2 = 112.5¢ =$ \$1.125.

2. The new price is $1.1 \cdot 3.5 = 3.85¢/kWh$. The new consumption is 115kWh, so a reduction of 35kWh, and the new payment is $115 \cdot 3.85 =$ \$4.4.

3. The marginal cost (supply) curve is

   $$p = 0.015q + 1.3.$$

   Solving for the intersection between supply and demand curves, we get the equilibrium price is $p = 3.52¢/kWh$ and $q = 148kWh$.

4. The demand surplus is $148 \cdot 1.48/2 = ¢109.5 =$ \$1.095, the net revenue to the supplier is $(3.52 - 1.3) \cdot 148/2 = ¢164 =$ \$1.64. The social welfare is \$1.095+\$1.64=\$2.74.

Seller/Buyer	Amount (MW)	price ($/MW)
Gen/Trader 1	1 MW	45
Gen/Trader 2	2 MW	55
Gen/Load	1 MW	48
Trader 2/Trader 1	1 MW	60
Trader 1/Load	2 MW	50

Problem 2. Consider a system of a physical generator, a physical load, and two virtual traders (speculators). The agree on the following forward contract:

At the time of delivery, the spot market has a price of $\$50/MW$. Find the total revenue of the generator, the traders, and the total payment of the load.

Solution. We need to calculate the profit/payment across to time periods for each entity:

- Generator: total profit=$45 + 2 \cdot 55 + 48 = 203$.
- Trader 1: total profit=$-45 - 60 + 2 \cdot 50 = -5$.
- Trader 2: total profit=$-2 \cdot 55 + 60 + 50 = 0$.
- Load: total payment=$48 + 2 \cdot 50 + 50 = 198$.

Problem 3. A regional operator runs the following electricity market. The total cost of energy is:

$$C = 1.5q^2 + 100q$$

where q is in MWh and C is in \$. The demand curve is:

$$p = -20q + 4000.$$

1. Find the supply curve in terms of price and quantity.
2. Calculate the equilibrium clearing price and quantity.
3. Find the consumers' surplus and the producer's surplus.
4. Now suppose the operate wants to impose a tax of \$20/MWh. This is accomplished by building it into the supply function: raising supply curve by \$20/MWh. Find the new supply function and the new equilibrium point. How much revenue would it raise?

Solution.

1. The supply curve is $p = 3q + 100$.
2. The equilibrium price and quantity are found by finding the intersection of the demand and supply curves. So we solve

   $$3q + 100 = -20q + 4000$$

   to get $q = 169.57$ and then $p = 608.7$.

3. The consumer surplus is $(4000 - 608.7) \cdot 169.57/2 = 287,531$. The producer's surplus is $(608.7 - 100) \cdot 169.57/2 = 43,130$.

4. The supply curve moves up by 20 to $p = 3q + 120$. The new equilibrium price is $p = 626$ and quantity is $q = 168.7$. The revenue from the tax is $20 \cdot 168.7 = 3,374$.