Assignment 3

EE 553 Power System Economics

Due April 19, 2017 at 8pm. Email to ywang11@uw.edu

Problem 1. Consider the following optimization problem:

$$\begin{aligned} &\text{minimize } x_1^2 + 0.5x_2^2 + 0.25x_3^2 && \text{(1a)}\\ &\text{subject to } 0.5x_1 + x_2 = 10 && \text{(1b)}\\ &0.5x_1 + x_3 = 20 && \text{(1c)} \end{aligned}$$

Solve this optimization problem and find the multipliers associated with each of the equality constraints. Give a physical interpretation for both of the multipliers. Hint: think about an example with 3 power plants and two loads.

Solution. We form the Lagrangian as

$$\mathcal{L} = x_1^2 + 0.5x_2^2 + 0.25x_3^2 + \lambda_1(10 - 0.5x_1 - x_2) + \lambda_2(20 - 0.5x_1 - x_3),$$

taking the partial derivatives of $\mathcal{L}$ and setting all of them to 0 gives

$$\begin{aligned} 2x_1 - 0.5\lambda_1 - 0.5\lambda_2 &= 0\\ x_2 - \lambda_1 &= 0\\ 0.5x_3 - \lambda_2 &= 0\\ 10 - 0.5x_1 - x_2 &= 0\\ 20 - 0.5x_1 - x_3 &= 0 \end{aligned}$$

Solving, we get $x_1 = 4.21, x_2 = 7.89, x_3 = 17.89, \lambda_1 = 7.89, \lambda_2 = 8.94$. The interpretation of the multipliers is that the optimization problem can be thought having 3 power plants supplying two loads, with half of the power from plant 1 supplies load 1 and 2, respectively, and plant 2 only serves load 2, plant 3 only serves load 3. Then the multipliers are the marginal cost of supplying supplying one more unit of power at load 1 and 2, respectively.

Note: It is important to write the constraints like $\lambda_1(10 - 0.5x_1 - x_2)$ and not $\lambda_1(0.5x_1 + x_2 - 10)$. This is an important convention that allows for a correct interpretation, otherwise the price would be negative, which is incorrect in this case. We did not deduct marks in the homework for this, but the correct convention must be followed in the future.

Problem 2. Consider the following optimization problem:

$$\begin{aligned} &\text{minimize } x_1 + x_2 && \text{(3a)}\\ &\text{subject to } 2x_1 + x_2 \geq 10 && \text{(3b)}\\ &x_1 + 2x_2 \geq 5 && \text{(3c)} \end{aligned}$$

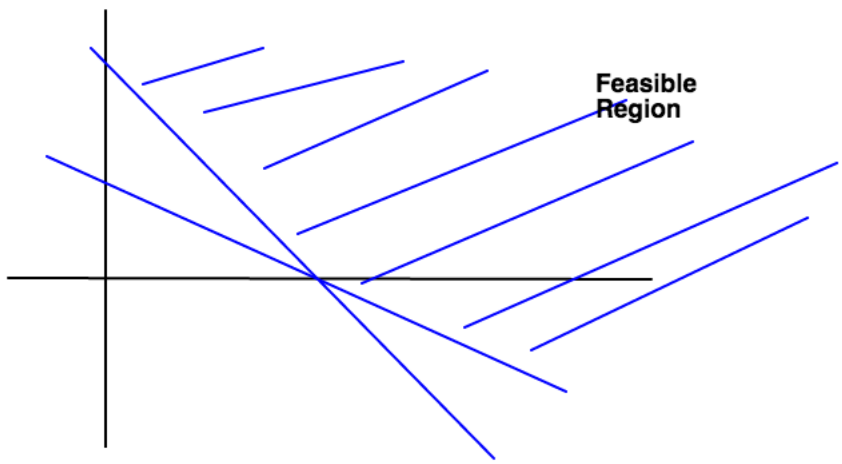

Figure 1: The feasible region for problem 2.

Solve for x_1 and x_2. Graphically present your solution: draw the region represented by the two inequality constraints and indicate the optimal point.
Solution. The region of possible solutions is given in Fig. 1 The optimal solution is $(5, 0)$.

Problem 3. A small power system is supplied by four generators. The cost characteristics of these generators are given by (the powers are expressed in MW):

$$\begin{aligned}
C_1 &= 300 + 12P_1 + 0.05P_1^2 \;\; [\$/h] \\
C_2 &= 250 + 13P_2 + 0.06P_2^2 \;\; [\$/h] \\
C_3 &= 150 + 11P_3 + 0.08P_3^2 \;\; [\$/h] \\
C_4 &= 200 + 10P_4 + 0.07P_4^2 \;\; [\$/h]
\end{aligned}$$

Calculate the economic dispatch (find the output of the generators) for the case where the total load on the system is equal to 800 MW.

1. Suppose that the first generator is limited between 100MW and 200MW ($100 \le P_1 \le 200$) and the second generator is limited between 50MW and 150MW ($50 \le P_2 \le 150$). Find the optimal solution by hand using the techniques of multipliers. Indicate which constraints are tight at optimum.

2. Suppose generator 3 and 4 have constraints $100 \le P_3 \le 300$ and $150 \le P_4 \le 300$, respectively. Using Julia (see tutorial on course website) or Matlab (google linear constrained QP in Matlab if you're not familiar with the programming) to solve this. Hand in a copy of your (commented) code.

Solution.

1. For this part, we have 4 inequality constraints and one equality constraint. Forming the Lagrangian, we have

$$
\begin{aligned}
\mathcal{L} &= 300 + 12P_1 + 0.05P_1^2 + 250 + 13P_2 + 0.06P_2^2 + 150 + 11P_3 + 0.08P_3^2 + 200 + 10P_4 + 0.07P_4^2 \\
&= \lambda(800 - P_1 - P_2 - P_3 - P_4) \\
&= \mu_1(100 - P_1) + \mu_2(P_1 - 200) + \mu_3(50 - P_2) + \mu_4(P_2 - 150)
\end{aligned}
$$

   Since we don't know which constraints are tight, we need to guess then verify. Let's guess $P_1 = 200$ and $P_2 = 150$. This means that $\mu_1 = \mu_3 = 0$. Then taking partial derivatives of $\mathcal{L}$ and solving for the rest of the variables, we get $\lambda = 44.07$, $\mu_2 = 12.07$, and $\mu_3 = 13.07$. This means that all of the complementary slackness conditions are satisfied. We have $P_1 = 200$, $P_2 = 150$, $P_3 = 206.67$ and $P_4 = 243.33$.

2. See code on the course website.