SOLUTIONS: PROBLEM SET 2

ELECTRIC POTENTIAL AND ELECTRIC POTENIAL ENERGY

PART A: CONCEPTUAL QUESTIONS

A. Electrons are free to move in a conductor. If there was a potential difference between two points, then an electric field must exist. Charges will be pushed in this electric field, and will redistributed themselves until the electric field is zero. Once there is no electric field, charges do not gain or lose potential energy if they move from point to point. All points are at the same potential.

B. It might be frightening to think you are at 5000V, but remember: potential is *energy per charge*. So if there's not a lot of charge, there's not really a lot of energy involved. Each coulomb of charge may have a potential energy of 5000J relative to the ground, but if you only have 10μC on your body, that's only 0.05J! The story is different if you have a source of charge that is significantly larger. Like a lightning bolt of Hydro-Quebec…

Consider an analogous situation with gravity: the gravitational potential at a point 500 m above the ground is 5000J/kg (taking the ground as 0), but it makes a big difference if a grain of chalk dust is held over your head at that point of it a car is!

C. a)

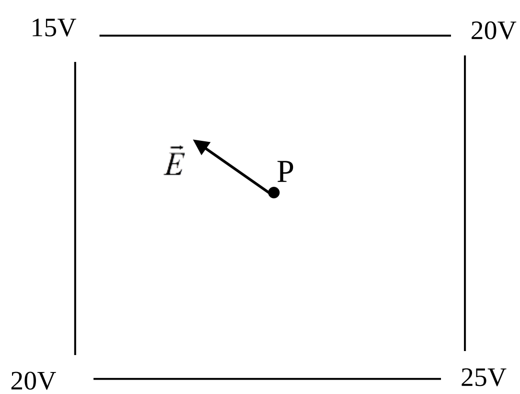
D. a)

E. b)

F. b)

G. b)

H.



E = 1000 N/C = 1000 V/m

I. E (although D is pretty close)

J. A

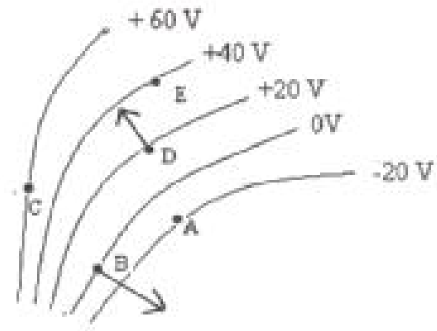
K. 80 V

L. C

M. $2x10^{-5}J$ **N. & O**. (diagram)

P. $8x10^{-5}J$

Q. $4x10^{-5}J$



PART B: NUMERICAL QUESTIONS

QUESTION 1

The potential energy between charges is given by: $U = \frac{kq_1q_2}{r}$

We can find the distance between the charges

$$r = \frac{kq_1q_2}{U} = \frac{k(2.2\times10^{-6})(3.48\times10^{-6})}{0.4}$$
$$= 0.172m$$

b) The potential energy is related to kinetic energy: $\Delta K + \Delta U = 0$

$$K_f - K_i = U_i - U_f$$

$$\frac{1}{2}mv_f^2 - K_i = -(U_f - 0.4)$$

solve for the speed:

$$v = \sqrt{\frac{2(0.4)}{1.5\times10^{-5}}}$$

$$\boxed{v = 231m/s}$$

QUESTION 2

a) The electric potential energy of several charges is the sum of contribution of individual charge.

$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$U = k\left(\frac{(2\times10^{-6})(-4\times10^{-6})}{0.04} + \frac{(2\times10^{-6})(3\times10^{-6})}{0.05} + \frac{(-4\times10^{-6})(3\times10^{-6})}{0.03}\right)$$

$$\boxed{U = -4.32\,J}$$

b) The electric potential of several charges is the algebraic sum of the individual contribution.

$$V_p = k\left[\frac{q_1}{r_{1p}} + \frac{q_2}{r_{2p}} + \frac{q_3}{r_{3p}}\right]$$

$$= k\left[\frac{2\times10^{-6}}{0.03} + \frac{-4\times10^{-6}}{0.05} + \frac{3\times10^{-6}}{0.04}\right]$$

$$\boxed{V_p = 5.55\times10^{5} V}$$

c) The charge *q* at point P would have a potential energy

$$U = qV_p$$

$$\Delta K + \Delta U = 0$$

$$K_f - \cancel{K_i} = -(\cancel{U_f} - U_i)$$

$$\frac{1}{2}mv^2 = qV_p$$

$$\Rightarrow v = \sqrt{\frac{2qV_p}{m}}$$

$$= \sqrt{\frac{2(3\times10^{-10})(5.55\times10^{5})}{4.2\times10^{-20}}}$$

$$\boxed{v = 8.90\times10^{7}\,m/s}$$

QUESTION 3

The kinetic energy and potential energy are related to the electric potential.

$$\Delta K + \Delta U = 0 = qV_{BA}$$

$$K_f = K_i + \Delta K = K_i + (-\Delta U)$$

$$K_f = \frac{1}{2}(2\times10^{-4})5^2 + [-(5\times10^{-6})(800-200)]$$

$$K_f = 2.5\times10^{-3} + 3\times10^{-3} = 5.5\times10^{-3} J$$

$$K_f = \frac{1}{2}mv_f^2 = 5.5\times10^{-3} J$$

$$\Rightarrow v_f = \sqrt{\frac{2K_f}{m}}$$

$$= \sqrt{\frac{2(5.5\times10^{-3})}{2\times10^{-4}}}$$

$$\boxed{v_f = 7.42 m/s}$$

QUESTION 4

The kinetic energy and potential energy are related to the electric potential

$$\Delta U = -\Delta K$$

$$qV_{3,0} = -(K_f - K_i)$$

$$\Rightarrow V_{3,0} = \frac{-(K_f - K_i)}{q} = -\left[\frac{(1.5\times10^{-2}) - (5\times10^{-2})}{-6.5\times10^{-7}}\right]$$

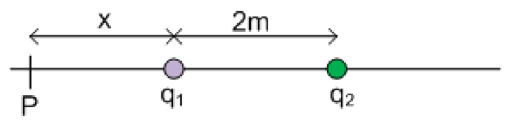
$$\boxed{V_{3,0} = -5.38\times10^{4} V}$$

V at origin is $5.38\text{x}10^4$ V higher than *V* at 3 cm.

A positive charge going from 0 to 3 would lose potential energy and gain kinetic energy. The reverse happened for this negatively charged particle.

QUESTION 5

Consider point P, x meter to the left of q_1:



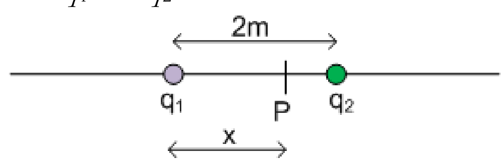
$$V_p = \frac{\not{k} q_1}{x} + \frac{\not{k} q_2}{(x+2)} = 0 \quad \rightarrow \quad \frac{q_1}{x} = -\frac{q_2}{(x+2)}$$

$$(q_1 + q_2)x = -2q_1$$

$$\Rightarrow x = -\frac{2(2\mu C)}{-1\mu C}$$

$\boxed{x = 4m}$ to the left of q_1

Consider a different point P, in between q_1 and q_2



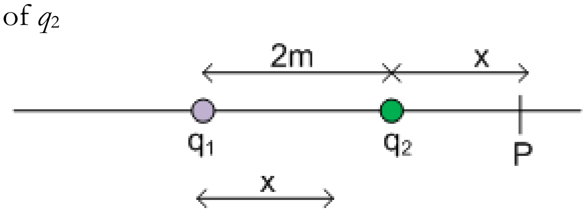
$$V_p = \frac{\not{k} q_1}{x} + \frac{\not{k} q_2}{(2-x)} = 0 \quad \rightarrow \quad \frac{q_1}{x} = -\frac{q_2}{(2-x)}$$

$$(-q_1 + q_2)x = -2q_1$$

$$\Rightarrow x = \frac{-2(2\mu C)}{-5\mu C}$$

$\boxed{x = 0.8m}$ to the right of q_1

What about point P to the right of q_2



$$V_p = \frac{\not{k} q_1}{2+x} + \frac{\not{k} q_2}{x} = 0 \quad \rightarrow \quad \frac{q_1}{2+x} = -\frac{q_2}{x}$$

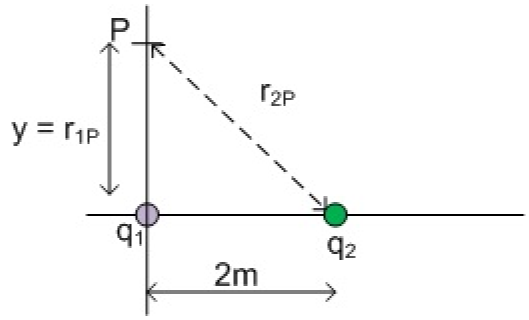
$$(q_1 + q_2)x = -2(q_2)$$

$$\Rightarrow x = \frac{-2(3\mu C)}{-1\mu C}$$

$$\boxed{x = -6m} \quad \text{to the right of } q_2 \text{ !?}$$

Answers: 4 m to the left of q_1 ad 0.8m to the right of q_1

b)



$$V_A = \frac{\not{k} q_1}{y} + \frac{\not{k} q_2}{\sqrt{4 + y^2}} = 0 \qquad \rightarrow \qquad \frac{q_1}{y} = -\frac{q_2}{\sqrt{4 + y^2}}$$

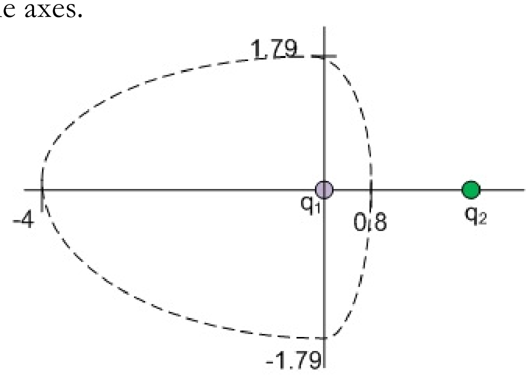
$$q_1^2(4 + y^2) = q_2^2 y^2$$

$$y^2(q_1^2 - q_2^2) = -4q_1^2$$

$$y = \sqrt{\frac{-4(2\mu C)^2}{(2\mu C)^2 - (-3\mu C)^2}}$$

$$\boxed{y = \pm 1.79\, m}$$

N.B. The equipotential *V* = 0 is a continuous surface (I'm not sure what its actual shape is); what we've done is find the 4 points where it cuts through the axes.



c) No.

First of all, by inspection, it is clear that **E** cannot be 0 at any of the points except, maybe (-4, 0)m. and a quick calculation shows that it is not equal to 0 here as well. More generally, the value for V anywhere depend on the arbitrary choice of a reference zero and does not depend upon the **E** at the location in question. For example, a table top might be chosen as a level of 0 gravitational potential energy; that doesn't mean that the gravitational field is 0 at the level.

QUESTION 6

How close it will get?

The energy conservation states that:

$$(E_{tot})_{far} = (E_{tot})_{near}$$

$$Vq = U + \cancel{K} = \frac{kQq}{r}$$

$$r = \frac{kQ}{V}$$

$$= \frac{k(79 \times 1.6 \times 10^{-19})}{3 \times 10^{6}}$$

$$\boxed{r = 3.79 \times 10^{-14} m}$$

The force:

$$F = \frac{kqQ}{r^2}$$

$$= \frac{k(2 \times 1.6 \times 10^{-19})(79 \times 1.6 \times 10^{-19})}{(3.79 \times 10^{-14})^2}$$

$$\boxed{F = 25.3N} \quad \text{WOW!}$$

The acceleration:

$$a = \frac{F}{m}$$

$$= \frac{25.3}{4 \times 1.67 \times 10^{-27}}$$

$$\boxed{a = 3.79 \times 10^{27} m/s^2}$$

QUESTION 7

a) Recall your physics NYA and energy stored in a string. The energy conservation states that

$$(E_{tot})_A = (E_{tot})_B$$

$$0 = \frac{1}{2}kx^2 + (-V_{AB}q)$$

$$V_{AB} = \frac{kx^2}{2q} = \frac{2(0.05)^2}{2(6\times10^{-5})}$$

$$\boxed{V_{AB} = 41.7V}$$

b) The electric field is related to the electric potential if the electric field is uniform by:

$$E = \frac{V_{AB}}{\Delta x} = \frac{41.7}{0.05}$$

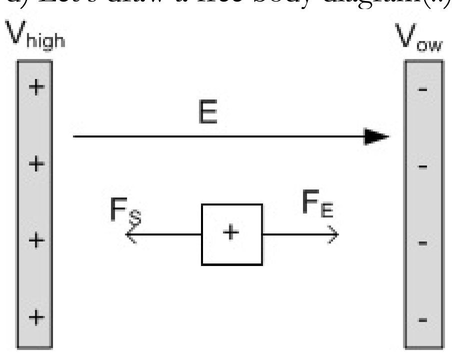
$$\boxed{E = 834V/m}$$

c) for a uniform electric field:

$$V = Ed = 834V/m \times 0.2m$$

$$\boxed{V = 167V}$$

d) Let's draw a free body diagram(!!)



$$\Sigma F = F_E - F_S = 0 \qquad \rightarrow F_E = F_S$$

$$qE = k\Delta x \qquad \rightarrow \Delta x = \frac{Eq}{k} = \frac{834V/m \cdot 6\times10^{-5}C}{2N/m} = \boxed{0.025m}$$

Answer: 2.5 cm. Note that this is the midpoint of the block's oscillation.