Surds

1. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
2. $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

Indices

1. $a^m \times a^n = a^{m+n}$
2. $a^m / a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^0 = 1$
5. $a^{-n} = \frac{1}{a^n}$
6. $a^{\frac{1}{n}} = \sqrt[n]{a}$
7. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Rules of rationalizing

- Fractions in the form $\frac{1}{\sqrt{a}}$, multiply the top and bottom by $\sqrt{a}$.
- Fractions in the form $\frac{1}{\sqrt{a}+\sqrt{b}}$, multiply the top and bottom by $\sqrt{a} - \sqrt{b}$.
- Fractions in the form $\frac{1}{\sqrt{a}-\sqrt{b}}$, multiply the top and bottom by $\sqrt{a} + \sqrt{b}$.

Logarithms

1. $\log a + \log b = \log ab$
2. $\log a - \log b = \log \frac{a}{b}$
3. $a \log_x y = \log_x y^a$
4. $\log_a a = 1$
5. $\log_a x = \frac{\log b^x}{\log b^a}$
6. $\log_a 1 = 0$
7. $\log_a b = \frac{1}{\log b} a$

Quadratic Equation

Solving quadratic equation

Quadratic equation can be solved by:

1. factorization
2. completing the square:

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

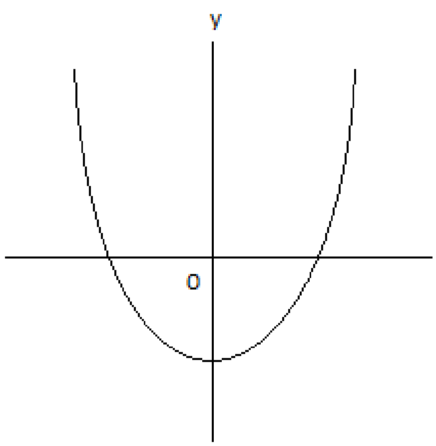
3. using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

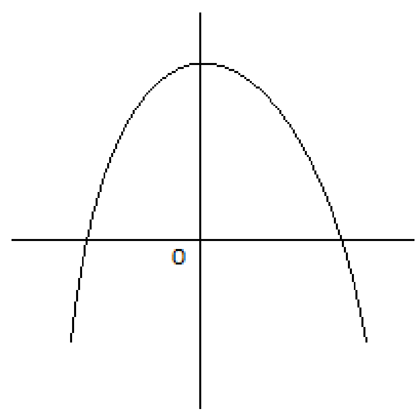
Nature of roots

- $ax^2 + bx + c = 0$

1. If $b^2 - 4ac > 0$, roots are real & different / real and distinct and the curve $y = ax^2 + bx + c$ will cut the x axis at two real and distinct points

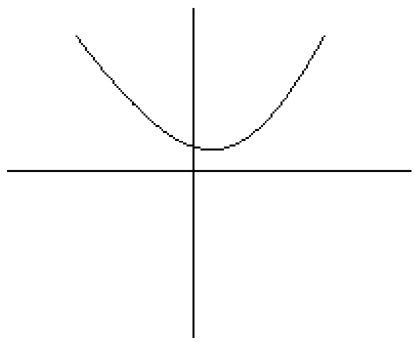


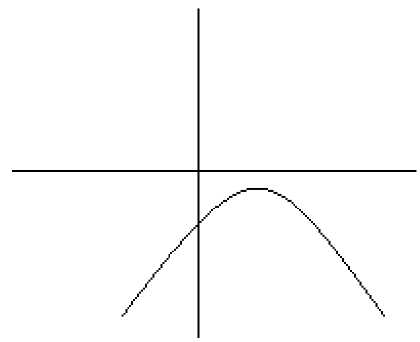
a > 0



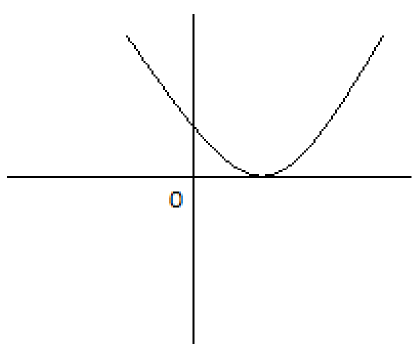
a < 0

2. If $b^2 - 4ac < 0$, roots are not real/ imaginary / complex and the curve $y = ax^2 + bx + c$ will lie entirely above the x axis if $a > 0$ and entirely below the x axis if $a < 0$.

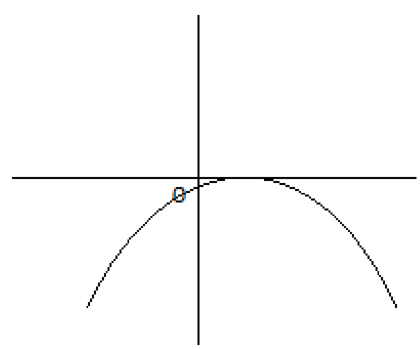




3. If $b^2 - 4ac = 0$, roots are real and equal / repeated / coincident and the curve $y = ax^2 + bx + c$ touches the x-axis.



a > 0



a < 0

4. If $b^2 - 4ac \geq 0$, roots are real.

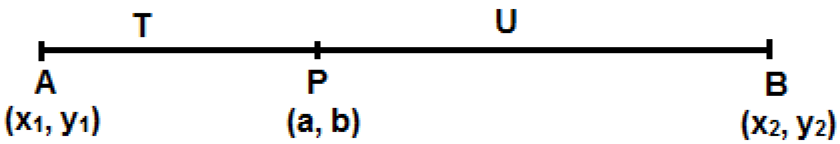
Solving Quadratic Inequality

When α and β (α<β) are two roots of $ax^2 + bx + c = 0$ (a>0) and

1. If $ax^2 + bx + c > 0$, range of values of x: $x < \alpha,\ x > \beta$
2. If $ax^2 + bx + c \geq 0$, range of values of x: $x \leq \alpha,\ x \geq \beta$
3. If $ax^2 + bx + c < 0$, range of values of x: $\alpha < x < \beta$
4. If $ax^2 + bx + c \leq 0$, range of values of x: $\alpha \leq x \leq \beta$

Co – ordinate Geometry

1. The distance between two points A(x_1 , y_1) and B(x_2 , y_2) is $\sqrt{(x_2 - x_1) + (y_2 - y_1)}$
2. The gradient of the line joining A(x_1, y_1) and B(x_2 , y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$
3. The coordinates of the mid-point of the line joining A(x_1 , y_1) and B(x_2 , y_2) are $\left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$.
4. Finding coordinates when a point divides a line internally.



$$a = \frac{(U \times x_1) + (T \times x_2)}{T + U}$$
$$b = \frac{(U \times y_1) + (T \times y_2)}{T + U}$$

5. The equation of the straight line having a gradient m and passing through the point (x_1, y_2) is given by : $y - y_1$ = m $(x - x_1)$.
6. Two lines are parallel if their gradients are equal.
7. Two lines are perpendicular to each other if the product of their gradients is -1.

Equation of circle

Centre(a, b) and radius = r

$$(x - a)^2 + (y - b)^2 = r^2$$

Arithmetic Progression (A.P)

1. nth term = $a + (n-1)d$
2. $S_n = \frac{n}{2}\{2a + (n-1)d\}$

Geometric Progression (G.P)

1. nth term = ar^{n-1}
2. $S_n = \frac{a\,(r^n-1)}{r-1}$, r > 1
3. $S_n = \frac{a\,(1-r^n)}{1-r}$, r < 1

$-1 < r < 1$ or $|r| < 1$.
The series is convergent. It has sum to infinity.

1. $S_\alpha = \frac{a}{1-r}$

Otherwise the series is divergent. It has does not have sum to infinity.

Differentiation

1. For a curve $y = f(x)$ represents the gradient of the tangent to the curve at any point x.
2. If $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$, where a and n are constants.
3. $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$
4. If y is a function of u, and u us a function of x, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (chain rule).
5. If y, u and v are functions of x and $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ (product rule).
6. If y, u and v are functions of x and $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (quotient rule).

The following are true only when x is in radians:

7. $\frac{d}{dx}(\sin x) = \cos x$
8. $\frac{d}{dx}(\cos x) = -\sin x$

Other formulae

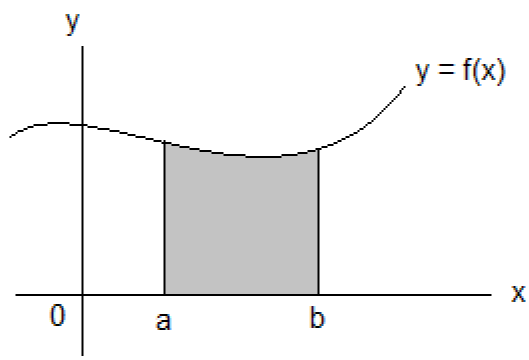
9. $\frac{d}{dx}(\sin^n x) = n \sin^{n-1} x\, (cosx)$
10. $\frac{d}{dx}(\cos^n x) = n \cos^{n-1} x\, (-\sin x)$

Application of Differentiation

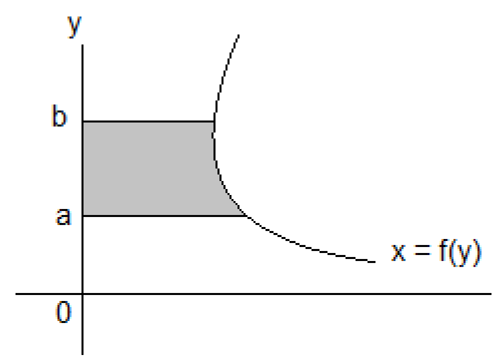
11. For an increasing function $f(x)$ in the interval (a, b) $f'(x) > 0$ in the interval $a \leq x \leq b$.
12. For an decreasing function $f(x)$ in the interval (a, b) $f'(x) < 0$ in the interval $a \leq x \leq b$.
13. Stationary points or turning points of a function $y = f(x)$ occur when $\frac{dy}{dx} = 0$.
14. The second derivative ($\frac{d^2y}{dx^2}$) determines the nature of the stationary points:
    (a) If $\frac{d^2y}{dx^2}$ is negative, the stationery point is a maximum point.
    (b) If $\frac{d^2y}{dx^2}$ is positive, the stationary point is a minimum point.
    (c) If $\frac{d^2y}{dx^2}$ is zero, the point could be either a maximum or a minimum point or a point of inflexion.
    (d) If $\frac{d^2y}{dx^2}$ is zero $\frac{d^3y}{dx}$ is not equal to zero, then the stationary point is point of inflexion.
15. To sketch a curve, note
    (i) the points where $x = 0$ or $y = 0$
    (ii) the nature and position of the stationary points
    (iii) the direction of the curve as x and y approach infinity.
    (iv) the interval on which the gradient is positive or negative.

Integration

1. $\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c \qquad n \neq -1$
2. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c$
3. $\int \cos x \, dx = \sin x + c$
4. $\int \sin x \;\, dx = \; -cosx + c$
5. $\int \cos bx \;\, dx = \frac{1}{b} \sin bx + c$
6. $\int \sin bx \;\, dx = \; -\frac{1}{b} \cos bx + c$
7. $\int \cos(ax+b) \, dx = \frac{1}{a} \sin(ax+b) + c$
8. $\int \sin(ax+b) \, dx = -\frac{1}{a} \cos(ax+b) + c$
9. The area bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is given by $\int_a^b y \, dx$.

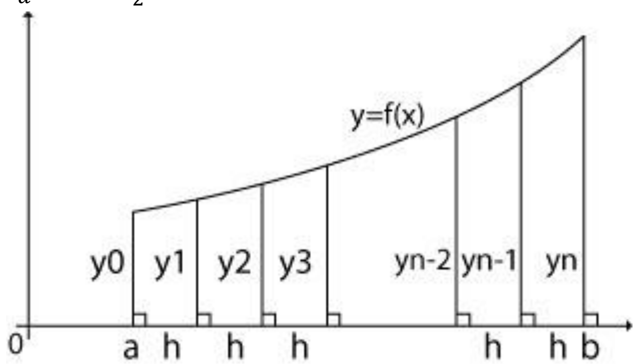


10. The area bounded by the curve $x = f(y)$, the y-axis and the lines $y = a$ and $y = b$ is given by $\int_a^b x \, dy$.



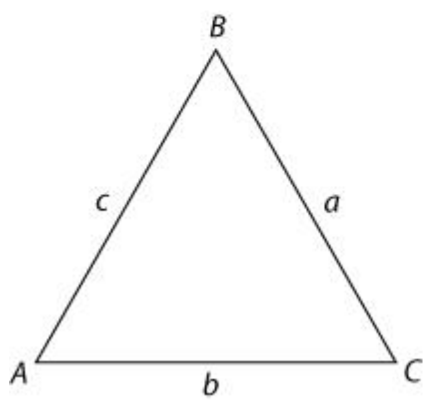
11. Area between $g(x)$ and $f(x)$ = $\int_a^b |g(x) - \; f(x)| \, dx$

12. When the area bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is rotated through 360° about the x-axis, the volume of solid of revolution is given by $\pi \int_a^b y^2 \, dx$.
13. When the area bounded by $y = f(x)$, the y-axis and the lines $y = a$ and $y = b$ is rotated through 360° about the y-axis, the volume of solid of revolution is given by $\pi \int_a^b x^2 \, dy$.
14. **The trapezium rule**: $\int_a^b y \, \mathrm{dx} = \frac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + \cdots y_{n-1})\}$, where $h = \frac{b-a}{n}$



Triangle

Sine rule



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$\cos A = \frac{c^2 + b^2 - a^2}{2bc}$$

Area of triangle

area = $\frac{1}{2}ab\sin C$

Circular Measure

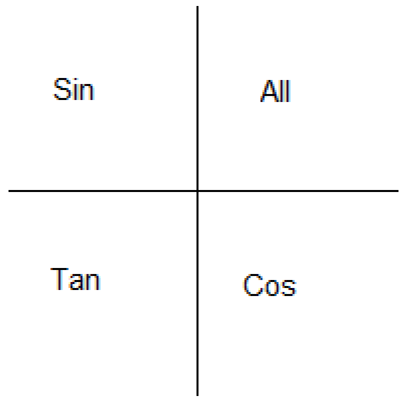
1. ∏ radian = 180^0
2. For a sector of a circle enclosed by two radii that subtend an angle of θ radians at the centre, the arc length s is given by
   s = rθ
   and the area of the sector A is given by
   A = $\frac{1}{2}r^2\,\theta$
   where r is the radius of the circle.

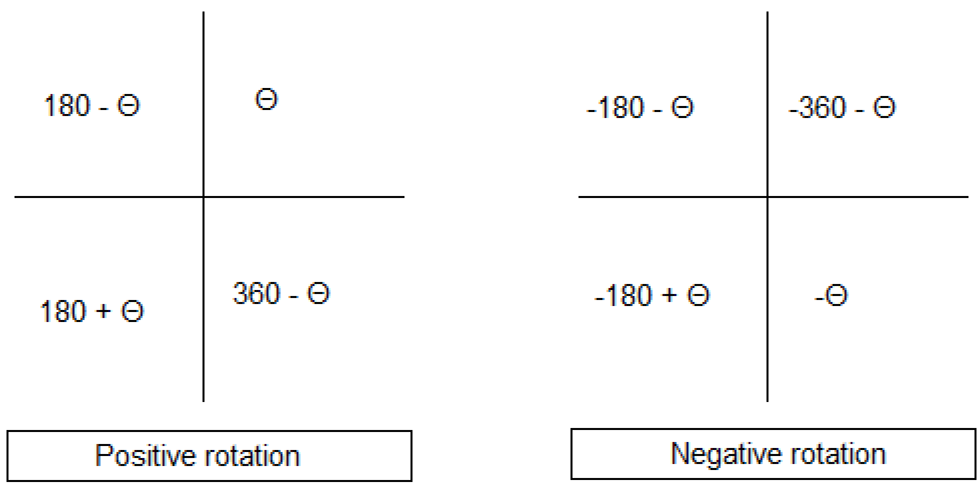
Binomial Expansion

1. $n! = n(n-1)(n-2)(n-3)\ldots$
2. $\frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)}{(n-2)} = n(n-1)$
3. $n_{c_1} = n$
4. $n_{c_2} = \frac{n(n-1)}{2!}$
5. $n_{c_3} = \frac{n(n-1)(n-2)(n-3)}{3!}$
6. $(a+x)^n = a^n + n_{c_1}a^{n-1}x + n_{c_2}a^{n-2}x^2 + n_{c_3}a^{n-3}x^3 + \ldots$
7. $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$
8. $(r + 1)^{th}$ term = ($^n c_r x^{n-r} y^r$)

Trigonometry

Rotation





1. $\sin\theta = \frac{opp}{hyp}$
2. $\cos\theta = \frac{adj}{hyp}$
3. $\tan\theta = \frac{opp}{adj}$
4. $\sin^2 + \cos^2 x = 1$
5. $\tan A = \frac{\sin A}{\cos A}$

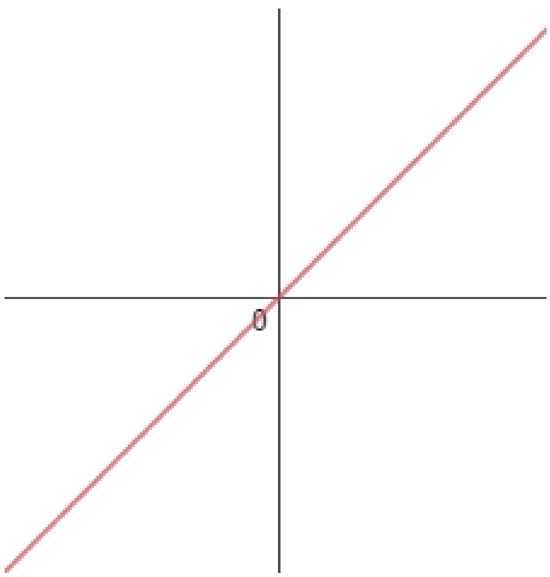
Ratios

The trigonometric ratios of 30^o, 45^o and 60^o have exact forms, given below:

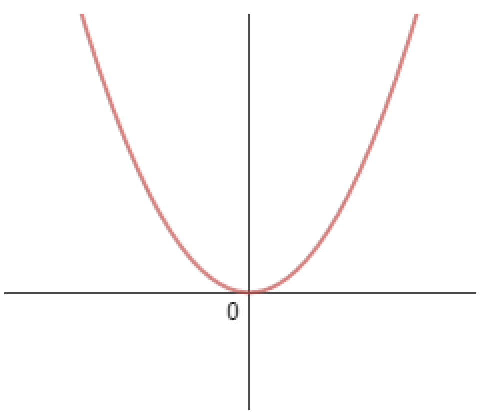
$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{\sqrt{3}}{3}$
$\sin 45^\circ = \frac{\sqrt{2}}{2}$	$\cos 45^\circ = \frac{\sqrt{2}}{2}$	$\tan 45^\circ = 1$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \sqrt{3}$

Graphs

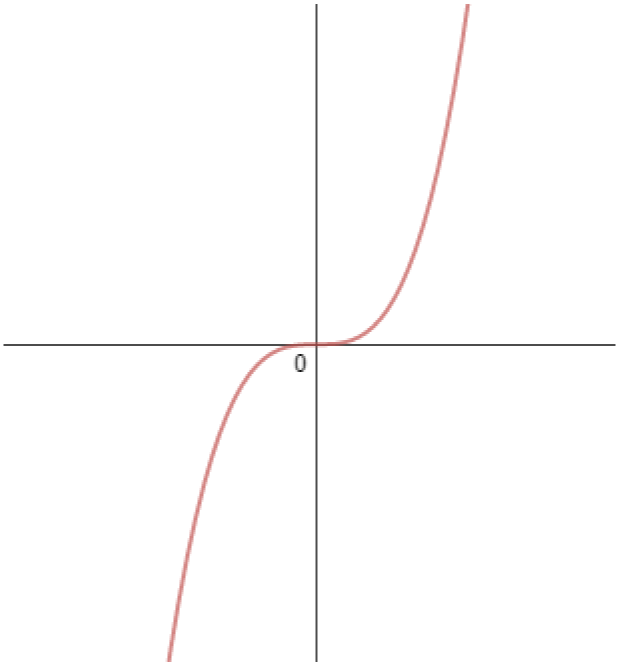
1. $y = x$



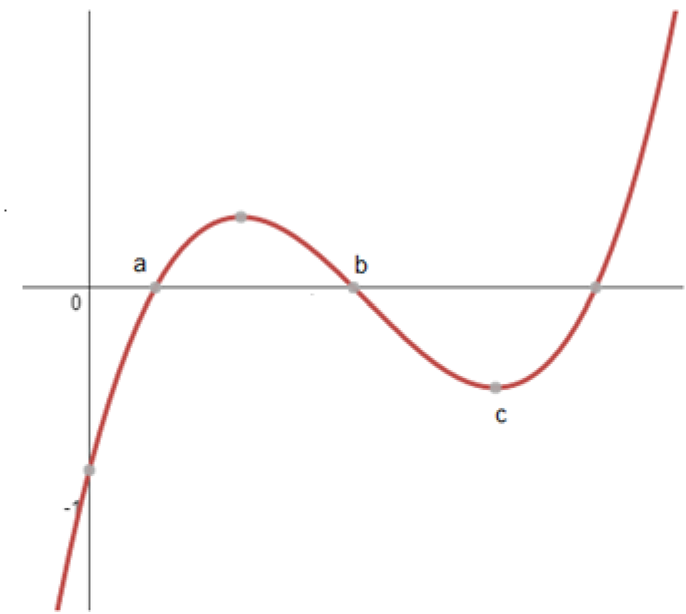
2. $y = x^2$



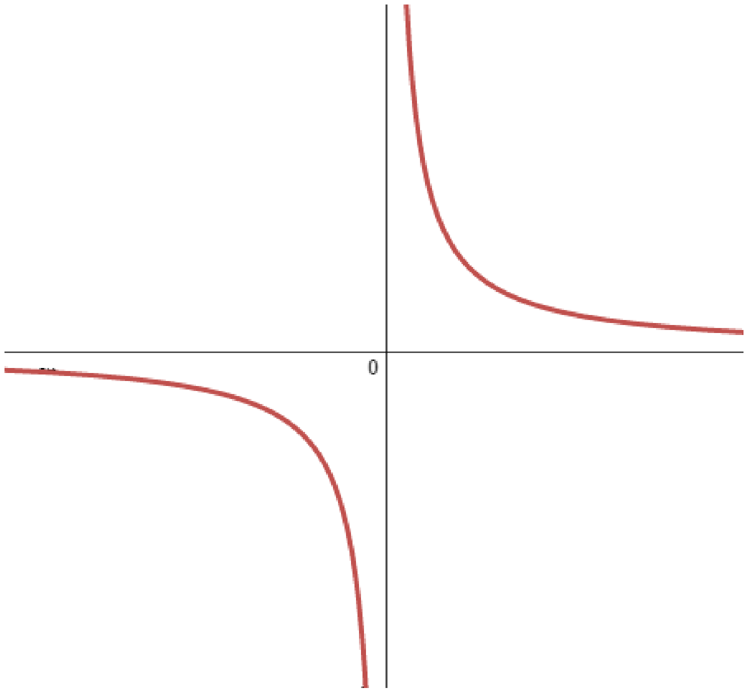
3. $y = x^3$



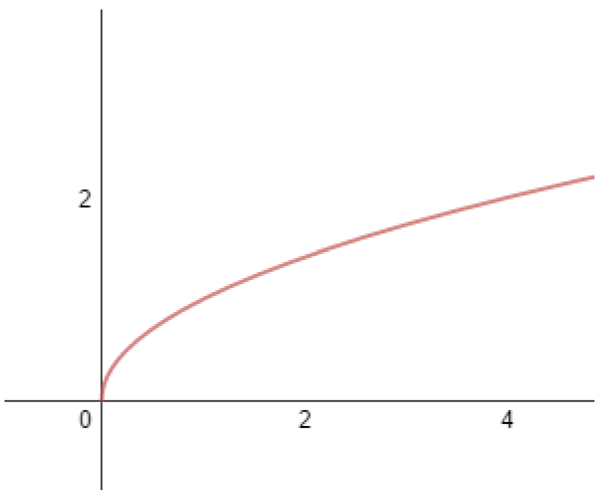
4. $y = (x - a)(x - b)(x - c)$



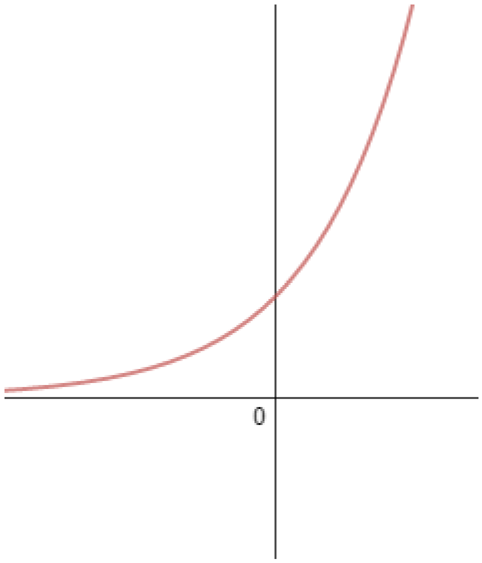
4. $y = \frac{1}{x}$



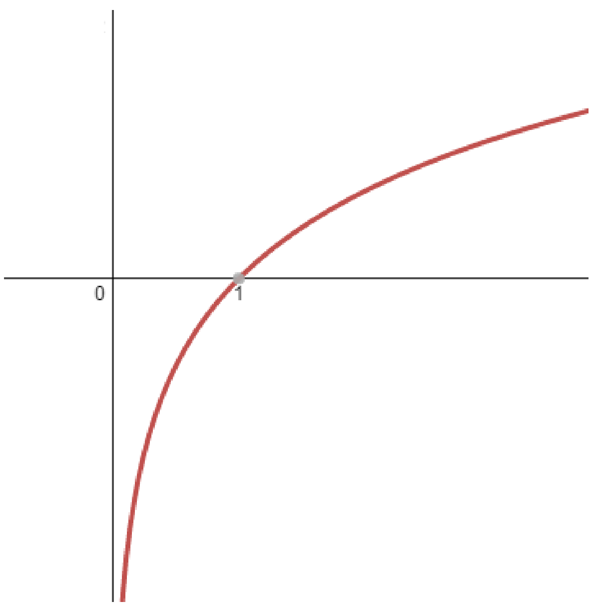
5. $y = \sqrt{x}$



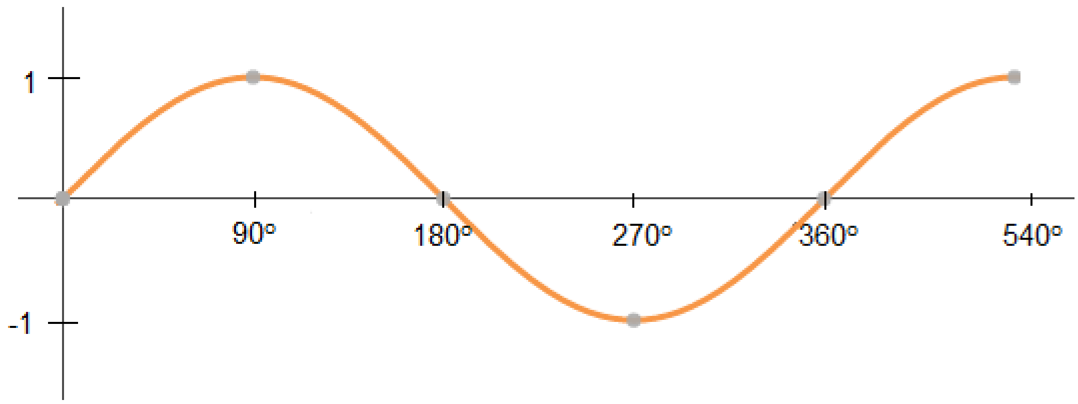
6. $y = e^x$



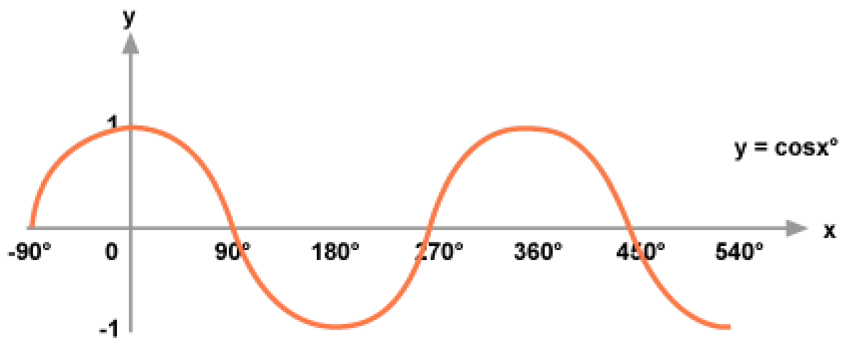
7. $y = \ln x$



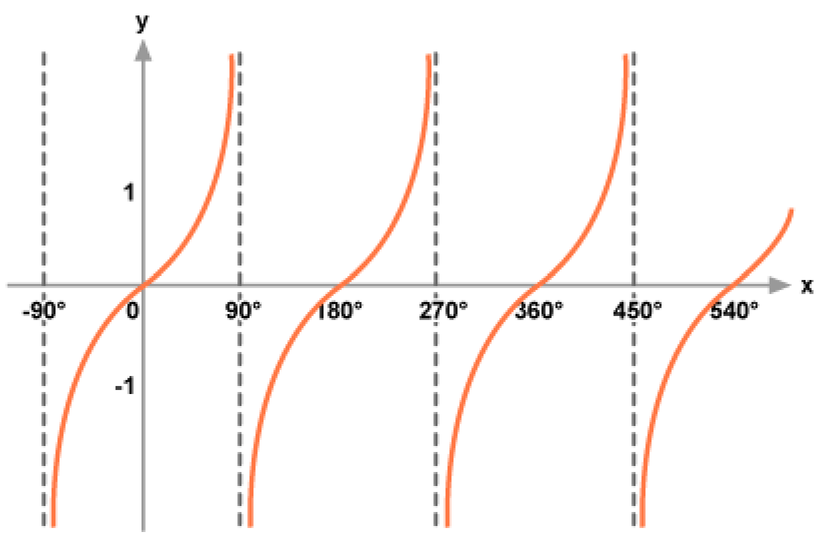
8. $y = \sin x$



9. $y = \cos x$



10. $y = \tan x$



Transformation

$f(x+a)$ is a translation of $-a$ in the x-direction.

$f(x)+a$ is a translation of $+a$ in the y-direction.

$f(ax)$ is a stretch of $\frac{1}{a}$ in the x-direction (multiply x-coordinates by $\frac{1}{a}$).

$af(x)$ is a stretch of a in the y-direction (multiply y-coordinates by a).