The Allocation of Scientific Talent*

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Abstract

I consider a model in which firms produce new knowledge by building labs and hiring researchers in a competitive market. I show that the allocation of researchers to labs may be efficient or inefficient depending on the curvature of firms' return on knowledge production. I then argue that the allocation of researchers to labs is likely to be inefficient if firms invest in R&D primarily to increase their *absorptive capacity* and decrease the cost of using knowledge that is publicly available. I conclude by separately introducing researchers' reputation concerns and a university research sector into the model. In particular, I show that the best researchers sort into the university research sector, and that within the university research sector better researchers work in bigger labs.

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1 Introduction

Despite headline-grabbing stories about elite researchers leaving one country for another or one firm for another,[1] very little is known about the efficiency properties of the labor market for researchers. A firm that hires a very talented researcher increases its R&D output and profits, but may cause other firms to settle for someone with lower scientific talent. A frictionless labor market matches each researcher to the firm that values him or her the most. However, in the presence of positive externalities such as knowledge spillovers, there is no presumption that the equilibrium allocation of researchers to firms also maximizes the positive externality that each firm generates. This paper discusses whether and under what conditions a trade-off exists between the assignment of researchers to firms maximizing the private value of knowledge production and the one maximizing the social value of knowledge production.

I consider a theoretical model in which several firms produce new knowledge by building labs and hiring researchers in a frictionless labor market. Researchers are a non-homogeneous input in the production of knowledge and are characterized by their *ability*. Each lab is an aggregate of all homogeneous inputs in the production of new knowledge (e.g., machines, technicians, raw material), and is characterized by its *size* – that is, the size of the investment required to build it. The knowledge produced within a firm increases with the ability of the researcher hired and the size of the lab built.[2] Furthermore, I assume that researcher's ability and the lab size are complements in the knowledge production function, so that total knowledge produced in the economy is maximized under a Positive Assortative Matching (PAM) rule, assigning the most productive researcher to work in the largest lab. Because of knowledge spillovers, each firm benefits from the knowledge produced by all other firms. It follows that each firm captures only a fraction of the social benefit generated by its R&D effort, and therefore builds a lab that is smaller than the socially optimal size.

[1] For example, see "Steal This Scientist" M. Liu, *Newsweek,* 14th of November 2009 (accessed from http://www.thedailybeast.com/newsweek/2009/11/13/steal-this-scientist.html). See also "Climbing Mount Publishable: The Old Scientific Powers Are Starting to Lose Their Grip" *The Economist,* 11th of November 2010 (accessed from http://www.economist.com/node/17460678).

[2] Lab size may be determined by the lab physical size, by the number of people (technicians, post-docs) working in it, but also by the quality and productivity of its machines and staff. However, for the sake of clarity, I use terms related to quantity (i.e., size) when referring to labs, and to quality (i.e., ability) when referring to researchers.

As a first step to my analysis, I show that the market allocation of researchers to firms depends on the curvature of the return on knowledge production, that is, on how fast the firms' marginal return on its research activities decreases with the amount of knowledge produced. When this curvature is sufficiently low (i.e. the marginal return decreases slowly), researchers and labs are complements from the firm's point of view. It follows that the market allocation of researchers to labs is PAM, which also maximizes aggregate knowledge production. In this case, the only source of inefficiency in the economy is the firms' investment in labs. On the other hand, if firms' return on knowledge is sufficiently curved (i.e. the marginal return decreases fast), the two inputs in the production of new knowledge—labs and researchers—may be substitutes in the firms' objective function. In this case, the market allocation of researchers to labs is Negative Assortative Matching (NAM), in which the most productive researcher in the economy is hired by the firm with the smallest lab. It follows that, for any given distribution of labs, the private sector minimizes the amount of knowledge produced: the decentralized allocation of researchers to labs is inefficient. Subsidies to firms' investment in labs cannot restore efficiency because they do not affect the labor market for researchers. The first best can be achieved by simultaneously subsidizing firms' investments in labs and reallocating researchers to firms. Furthermore, under some regularity conditions, the equilibrium allocation is NAM or PAM depending on whether the return on knowledge is bounded or unbounded above.

This last observation motivates the second step of my analysis, in which I argue that the efficiency of the labor market for researchers depends on firms' main motive for knowledge production. It is well understood that firms engage in R&D activities to generate knowledge that can be used in the production of patents and new products, but also to build *absorptive capacity* and decrease the cost of using knowledge produced outside of the firm. For example, R&D provides "a ticket of admission to an information network" (Rosenberg, 1990; p. 170), and allows firms to become immediately aware of the latest discoveries and inventions. Also, new knowledge may be difficult to understand and use in a timely manner, unless a firm employs researchers who are familiar with the current scientific and technological frontier. Producing new knowledge and increasing absorptive capacity are therefore the "two faces of R&D" (Cohen and Levinthal, 1989): two separate outputs for the same activity.[3] Similarly,

[3] On the empirical relevance of absorptive capacity, see Tilton (1971); Cockburn and Henderson (1998); Gambardella (1992); Griffith, Redding, and Van Reenen (2003, 2004).

Kealey and Ricketts (2014) argue that science is a *contribution good*, a good that can be enjoyed by all contributors but not by non-contributors. According to this framework, a firm's R&D activities produce new knowledge and at the same time allow the firm to tap into the pool of knowledge produced by other firms.

Going back to the model, in the limit case in which firms invest in R&D exclusively to build absorptive capacity, the benefit from science production is bounded above by the stock of knowledge produced outside of the firm, and the allocation of researchers to labs is inefficient. As a simple illustration, assume that there is a cost to using outside knowledge, which is constant if a firm produces at least a given amount of new knowledge and is arbitrarily high otherwise. To reach the required threshold, a firm with a large lab will hire a relatively unproductive researcher, while a firm with a small lab will hire a more productive researcher. In this simple example, the competitive market allocation of researchers to labs is NAM, which is inefficient. On the other hand, when firms' objective is to push the knowledge frontier by creating new knowledge, the benefit from science production is unbounded above and the allocation of researchers to firms is PAM, which is efficient. Overall, when both motives for science production are present, the equilibrium of the labor market for scientists depends on which motive is dominant.[4]

The fact that the best researchers may not be hired by the firm with the largest labs finds empirical support in the literature on firms' size and R&D productivity, which shows that small firms are often more productive than large firms in their R&D efforts. In the model, the productivity of a lab is determined by the ability of the researcher hired. Hence, when absorptive capacity is the dominant motive for science production the model predicts a negative relationship between lab size and lab productivity. If large firms have a comparative advantage at building labs (because of, for example, economies of scopes among different research lines), we should therefore expect a negative relationship between lab productivity and firm size. By looking at the number of patents produced in firms of different sizes, several authors find results that support this prediction.[5] More directly, Elfenbein, Hamilton, and

[4] Other mechanisms can generate a private return on knowledge production that is sufficiently curved, and cause a misallocation of scientific talent in equilibrium. For example, when experimenting in a lab failures can be socially valuable because of learning. However, only successes are valuable from the firm point of view, generating a private return on knowledge that can be very curved (see the conclusion of the paper for further details). Here I focus on absorptive capacity because there is ample empirical evidence showing its importance for firms.

[5] See Scherer (1965), Acs and Audretsch (1987), and Cohen and Klepper (1996) who review the empirical

Zenger (2010) examine the allocation of researchers to firms and show that productive R&D workers are more likely to work for small firms than for big firms.

In addition to firms' objectives, other elements are likely to affect the job market for researchers and the equilibrium allocation of scientific talent to labs. In the last part of the paper, I separately consider two such elements: universities and reputation concerns. I assume that universities' mission is to produce new knowledge and that academic researchers can work as consultants for the private sector. The job of a consultant is to help a firm using the stock of public knowledge. Researchers endogenously sort between the university sector/consultancy and the private sector. The best researchers are hired by universities, and within the university sector researchers are allocated according to PAM: better researchers are assigned to bigger labs. Academic researchers consult for firms that are relatively unproductive, while productive firms carry out in-house R&D by hiring their own researchers. Therefore, the overall allocation of researchers to labs is mixed, with PAM for the best researchers, and NAM for the worst researchers.

The main goal of this extension is to compare the costs and benefits of two different policies: building university labs or subsidizing firms' investment in labs. Building university labs is more expensive than subsidizing firms' investment, because when subsidies are used part of the cost of building labs is covered by firms. However, university labs have a distinctive advantage over subsidies, because within the university research sector the policymaker can implement the knowledge-maximizing allocation of researchers to labs. It follows that if the decentralized allocation of scientific talent is efficient, the only policy that will be used are subsidies. However, when scientific talent is misallocated in equilibrium there is scope for building universities. Hence, in this model, the university research sector exists because the equilibrium allocation of researchers to firms is inefficient.

Finally I assume that producing new knowledge has an additional benefit: it increases a researcher's reputation. Because of a cash constraint, researchers cannot transfer the value of reputation back to firms. I show that, if reputation concerns are strong enough, the equilibrium in the private sector may switch from NAM to PAM. Intuitively, when researchers' reputation concerns are strong enough, they may receive most of their compensation in the form of reputation rather than a monetary payment. Because reputation increases with the amount of knowledge produced, researchers prefer to work for firms with large labs. Hence,

evidence.

reputation may cause productive researchers to work for firms with large labs, therefore increasing the knowledge produced in the economy and total welfare.

1.1 Literature

Several papers have argued that equilibrium matching patterns may be inefficient and that re-matching policies may increase welfare. However, in most of the existing literature, inefficient matching patterns emerge because peer effects and cash constraints prevent agents from efficiently sharing the surplus generated within each match (see, for example, Estevan, Gall, Legros, and Newman, 2014, who consider the problem of students sorting into schools). In my model, instead, surplus can be freely shared among firms and researchers by using wages. Nonetheless, the presence of widespread externalities in the form of knowledge spillovers may lead to an inefficient matching pattern.

Matching markets with widespread externalities have been studied theoretically by Hammond, Kaneko, and Wooders (1989), Hammond (1995), and Kaneko and Wooders (1996). Hammond et al. (1989) and Kaneko and Wooders (1996) show the existence of the competitive equilibrium in large economies with widespread externalities. Hammond (1995) show existence of the equilibrium in economies with widespread externalities and different types of taxes or subsidies. My paper studies the effect of widespread externalities in an applied, policy-relevant matching problem. Also, to the best of my knowledge, it is the first one to study a matching model in which there are both widespread externalities and a pre-matching investment phase similar to Cole, Mailath, and Postlewaite (2001).

There is a large literature examining the determinants and the efficiency consequences of the allocation of talent to different tasks and professions.[6] Closest to my paper are Acemoglu, Akcigit, and Celik (2014) and Vandenbussche, Aghion, and Meghir (2006). In Acemoglu et al. (2014), managers can pursue either radical innovation or incremental innovation. In Vandenbussche et al. (2006), skilled and unskilled workers can be employed either in innovative activities (i.e. pushing the knowledge frontier) or in the adoption of new technology (i.e. reducing the distance to the knowledge frontier). Here, instead, I analyze the allocation of scientific talent within the same sector and the same activity, and not the allocation of scientific talent across sectors and activities. In particular, following the litera-

[6] See for example Murphy et al. (1991); Baumol (1996); Acemoglu, Aghion, and Zilibotti (2006) and, related to knowledge production, Jovanovic and Rob (1989) and Lucas Jr and Moll (2014).

ture on absorptive capacity in my model creating new knowledge and reducing the distance to the knowledge frontier are byproducts of the same activity. Therefore, at a firm level there is no trade-off between innovation and adoption of new knowledge, because they are both increasing in the amount of R&D carried out. Despite this, a trade-off between the two activities emerges endogenously in the assignment of researchers to labs.

Hammerschmidt (2009), Kamien and Zang (2000) and Leahy and Neary (2007) study the theoretical implications of absorptive capacity, and show that it generates a new set of strategic considerations for firms. I also study the theoretical implications of absorptive capacity, but I abstract away from strategic considerations because each firm is assumed to be small relative to the number of firms active on the labor market for researchers. My work is also part of a growing literature recognizing that the inputs in the research process are non-homogeneous, and that the way these inputs are combined is a key determinant of research outcomes. Most papers in this literature are concerned with the determinants of research team size (see, for example, Bikard, Murray, and Gans, 2015). Here I keep the number of inputs in the production of knowledge fixed, and I argue that the sorting of research inputs matters for aggregate research output.

The remainder of the paper proceeds as follows. In the next section, I describe the model. In the third section, I define the equilibrium concept. In the fourth section I derive the equilibrium of the model. In the fifth section I discuss the normative aspects of the equilibrium. In the sixth section I introduce the university research sector, and reputation concerns. In the last section, I conclude. All proofs missing from the text are in appendix.

2 The Model

The economy is populated by a continuum of heterogeneous firms and a continuum of heterogeneous researchers. Firms differ in their productivity p which is continuously distributed over $P = [\underline{p}, \bar{p}]$ (with $\underline{p} < \bar{p}$), and researchers differ in their ability a which is continuously distributed over $A = [0, \bar{a}]$ (with $\bar{a} > 0$). Firms' objective is to maximize profits, while researchers' objective is to maximize their wage. All agents have the same outside option, which is assumed to be zero. The economy runs for three periods. In the first period firms invest in labs; in the second period each firm hires one researcher and produce new knowledge; in the last period profits are realized and wages are paid.

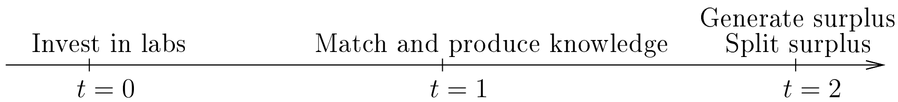

Fig. 1: Timeline

Period $t = 0$: investing in Labs

In period $t = 0$, firms build labs. If a firm p builds a lab of size L, this firm bears a cost $c(p, L)$ continuous, positive, with continuous first and second derivative, increasing in L, decreasing in p, with $\frac{\partial^2 c(p,L)}{\partial L^2} \geq 0$ and $c(p, 0) = 0\, \forall p$.

Therefore, p represents a firm's comparative advantage in investing in labs. For example, although not explicitly considered here, some firms may be active in multiple scientific fields and labor markets for researchers. It is possible that those firms benefit from economies of scale when building labs, and therefore have higher p compared to firms that are active in only one scientific field.[7]

Period $t = 1$: producing new knowledge

In period $t = 1$, each firm hires one researcher to work in the firm's lab. The amount of knowledge produced within each lab is:

$$k = af(L),$$

where $f(L) \geq 0$, $f'(L) > 0$, and $f''(L) \leq 0$. The reader should interpret the lab size L as inclusive of all homogeneous inputs that can increase the chance of a scientific or technological discovery for given researcher's ability, including machines (e.g. a bigger telescope, a more powerful microscope and a state-of-the-art DNA sequencing machine). The fact that some of these inputs do not require an investment ex-ante but can be purchased after hiring the researcher will turn out to be not relevant. I will show in the next section that, in equilibrium, each firm invests taking as given the ability of the researcher working in its lab, which implies

[7] The case $\underline{p} = \overline{p}$ (i.e. identical firms) yields results that are identical to the ones presented here, but the derivations are more convoluted. The reason is that, in this case, identical firms behave differently depending on what researcher they expect to be matched with. Hence, the equilibrium investment in labs is a correspondence and not function. However, after the investment stage (i.e. for given distribution of labs) the model will be identical to the one described here.

that the timing could be reversed with no effect on the equilibrium investment.

Also, researchers typically work in teams. This fact can be incorporated into the model by defining a as the research team's average quality. A previous matching stage determines how researchers form research teams, and how the distribution of a is determined from the distribution of individual ability. To keep the model as simple as possible, I do not pursue this interpretation further. Finally, in the above specification, the amount of knowledge produced does not depend on the stock of prior public knowledge—i.e., there is no "standing on the shoulders of giants" effect. Because the model is static, this modeling choice is without loss of generality as long as all firms have equal access to the stock of prior public knowledge.

Period $t = 2$: wages and profits.

Call V the stock of new public knowledge, available in the economy at the beginning of period 2, and taken as given by firms and researchers. The surplus generated withing a firm depends on the knowledge produced in-house and on the stock of new public knowledge, so that:

$$\Phi(a, L) = \phi(k, V)$$

$$\text{s.t. } k = af(L),$$

where $\phi(,)$ is increasing in both arguments, with $\frac{\partial^2 \phi(k,V)}{\partial k^2} < 0$. For example, $\phi(k, V)$ may represent the revenues earned by inventing new products and bringing them to market. The surplus generated is then split between a wage to the researcher, and profits to the firm:

$$\phi(k, V) = w(a) + \pi(L)$$

where wages for each ability level $w(a) : A \to \mathbb{R}^+$ and profits for each investment in labs $\pi(L) : \mathbb{R}^+ \to \mathbb{R}^+$ are endogenous and determined in equilibrium.

Note that this specification abstracts away from competition in the product market. The reason is that firms may apply the same knowledge to very different products. For example, all biotech firms benefit from the same scientific knowledge. However, some firms develop DNA sequencing machines, some develop drugs and others develop bacteria that can produce biofuel out of garbage. Some firms compete with each other, some complement each other, and some belong to very different product markets. For this reason, I do not consider firms' competition on the product market and focus on competition for researchers.

Endogenous Science

The stock of public knowledge is taken as given by firms and researchers but is determined endogenously, by aggregating all the knowledge produce withing each firm:

$$V = \nu \int m(L) f(L) h(L) dL, \tag{1}$$

where $h(L)$ the *p.d.f.* of the distribution of labs L and the function $m(L) : \mathbb{R}^+ \to \{A, \emptyset\}$ assigns labs to researchers (with $m(L) = \emptyset$ representing an unmatched lab).[8] Both $h(L)$ and $m(L)$ are determined in equilibrium. The parameter $\nu \geq 0$ captures both the size of knowledge spillovers, and the possibility of complementaries between knowledge produced within different firms.

The above specification implies that ability a and lab size L are complements in the knowledge production function. In turn, this implies that the allocation of researchers to labs that maximizes the aggregate production of knowledge is Positive Assortative Matching (PAM), in which the most productive researcher is allocated to the biggest lab. This is a key assumption that I will maintain throughout the paper. It is justified by the observation that, when research funds are allocated by grant-giving institution having the explicit goal of producing new knowledge, researchers who can demonstrate higher ability and better past performance typically have a higher chance of receiving funds and are allocated larger grants.[9]

Finally, the knowledge produced by a given firm may interact with the knowledge produced by other firms and determine the stock of public knowledge in complex ways. As a consequence, a more general functional form such as $V = \int \varrho \left(m(L) f(L) \right) h(L) dL$ may be desirable. As long as $\varrho()$ is not too curved, the two inputs in the knowledge-production

[8] No labs will be unmatched in equilibrium. If a firm anticipates that it will not be able to hire a researcher, this firm will not build a lab.

[9] For example, Arora and Gambardella (2005) analyze the funding allocation decisions of NSF and show that the reputation (past publication record) is positively correlated with the probability of being awarded the grant and with the size of the grant. Arora, David, and Gambardella (1998) analyze the funding allocation decisions of the Italian CNR (equivalent to NSF) and find similar results. They also show that the elasticity of research output with respect of research budget is higher then average for prestigious scientists, which implies complementarity between ability and funds.

process remain complements. However, because the main results of the paper do not depend on the specific shape of $\varrho()$, for ease of exposition I only consider the linear case.[10]

Total welfare

The total welfare of the economy is equal to the sum of all profits and all wages earned, minus the cost of setting up the labs. Therefore, for given matching function $m(L)$ and given investment in labs by each firm $i(p)$ (which will be determined in equilibrium), total welfare is given by:

$$SW = \int \left[\Phi(m(i(p)), i(p)) - c(p, i(p))\right] \gamma(p) dP, \tag{2}$$

where $\gamma(p)$ is the p.d.f. of p. Therefore, whereas firms and researchers take the total stock of knowledge V as given, from the social point of view V is determined endogenously and depends both on firms' investment decisions and on the equilibrium matching pattern.

3 Definition of the Equilibrium

Ex-post, partial equilibrium.

To start, I define the period-1 equilibrium assignment of researchers to firms and their respective equilibrium payoffs taking the stock of public knowledge and the distribution of labs as given. Remember that, in period $t = 1$, firms differ only in the size of the labs that they own.

Definition 1. For a given V, the functions $m(L)$, $\pi(L)$ and $w(a)$ constitute an ex-post equilibrium if the following two conditions are satisfied:

- Feasibility: $\pi(L) + w(m(L)) \leq \Phi(m(L), L)\ \forall L$.
- Stability: $\pi(L) + w(m(L)) \geq \Phi(m(L'), L)\ \forall L, L'$.

[10] The results do not depend on $\varrho()$ in the sense that, for any $\varrho()$, if the marginal private-return on knowledge production decreases fast enough, a and L are substitutes in the private surplus function (see Lemma 1), and the equilibrium allocation may be inefficient (see Proposition 1). However, the exact curvature determining whether a and L are substitutes in the private surplus function depends on $\varrho()$.

To understand the definition of equilibrium, note that feasibility and stability imply that

$$\Phi(m(L), L) - w(m(L)) \geq \Phi(m(L'), L) - w(m(L')) \; \forall L, L'.$$

In other words, given the equilibrium wage function $w()$, each firm "shops for researchers" in a competitive market.[11] Profits are equal to the difference between the total surplus and the wage paid. A matching function $m()$ is part of the equilibrium if each firm prefers to hire the researcher specified by $m()$ rather than any other researcher. The existence of a unique equilibrium for a given V and given distribution of labs is a standard result in matching theory. Also, because L and a have continuous distributions, the equilibrium matching function $m(L)$ is a bijection: each L is matched with one and only one a.[12]

Ex-ante, general equilibrium.

By working backward, I can define an ex-ante equilibrium, in which firms' investment levels and the aggregate amount of knowledge produced V are endogenous. I introduce the following notation:

- $i(p) : P \rightrightarrows \mathbb{R}^+$, the investment made by firm p, determining the size of this firm's lab.
- $\tilde{m}(p) \equiv m(i(p)) : P \rightrightarrows A$, the matching rule on the equilibrium path (for labs built by some firms) mapping firms to researchers.[13]
- $\tilde{\pi}(p) \equiv \pi(i(p)) : P \to \mathbb{R}^+$, firms' profits on the equilibrium path.
- $l(a) \equiv i(\tilde{m}^{-1}(a))$ the lab a researcher of ability a receives in equilibrium.

[11] Note that stability and feasibility also imply that $\Phi(m(L), L) - \pi(L) \geq \Phi(m(L), L') - \pi(L') \; \forall L, L'$. In other words, we could equivalently imagine that researchers shop for firms.

[12] See, for example, Kamecke (1992). Note that the equilibrium allocation is always unique, but the equilibrium payoff functions $\pi(L)$ and $w(a)$ are unique only if there is a continuous of types on each side of the market. The existence of an equilibrium for given distribution of labs but endogenous V is shown in Kaneko and Wooders (1996), who consider a two-sided matching model with a continuum of players and externalities. The existence of an equilibrium in which both the distribution of labs and V are determined endogenously is shown below (Proposition 2).

[13] Both $i(p)$ and $\tilde{m}(p)$ are functions because firms are heterogeneus in p. If all firms are identical, $i(p)$ and $\tilde{m}(p)$ are correspondences.

The definition of equilibrium that I use is similar to the one in Cole, Mailath, and Postlewaite (2001) with the difference that, here, the investment in labs generates a positive externality on the other firms.

Definition 2. The quadruple $\{i(.), m(.), \pi(.), w(.)\}$ constitutes an equilibrium if the following conditions are fulfilled:

1. The investment is optimal:

$$i(p) = \arg\max_{L \geq 0} \{\pi(L) - c(p, L)\}$$

2. Ex post, the matching $\{i(.), \tilde{m}(.), \tilde{\pi}(.), w(.)\}$ is feasible and stable, where:

   - Feasibility: $\tilde{\pi}(p) + w(\tilde{m}(p)) \leq \Phi(\tilde{m}(p), i(p))\ \forall p \in P$.[14]
   - Stability: $\tilde{\pi}(p) + w(\tilde{m}(p')) \geq \Phi(\tilde{m}(p'), i(p))\ \forall p, p' \in P$.

3. For $L \notin \{L : L = i(p) \text{ for some } p \in P\}$ (investments off the equilibrium path):

$$\pi(L) = \max_a \{\Phi(a, L) - w(a)\}$$

4. $\Phi(a, L) = \phi(k, V)$, with $k = af(l(a))$ and $V = \nu \int af(l(a))z(a)da$ (where $z(a)$ is the p.d.f. of a).

To understand the definition, assume that there is an equilibrium, and consider a deviation made by a single firm. Because there is a continuum of firms, any change in the investment level of a single firm does not affect the distribution of labs, and has no impact on the equilibrium $w(a)$. In other words, firms are price takers in the market for researchers, both for on-equilibrium and off-equilibrium investment levels. Therefore, when choosing the investment level, each firm takes $w(a)$ as given. In addition, regardless of the size of the lab built, each firm can match with any researcher a provided that it pays the market wage $w(a)$.

[14] The general definition of feasibility is more complicated (see Cole et al., 2001). However, here this simpler version can be used because all distributions are continuous.

4 Solution

4.1 Partial equilibrium.

In this section, I derive the equilibrium for given distribution of labs and for given V. I establish that the shape of the equilibrium matching function $m(L)$ and the welfare properties of the competitive equilibrium depend on the curvature of the surplus function $\phi(k, V)$. In the next subsection, I will argue that the curvature of the surplus function $\phi(k, V)$ can be linked to the two motives for knowledge production discussed in the introduction (i.e. new knowledge and absorptive capacity).

Lemma 1. *Consider the equilibrium matching function $m(L)$ and a given $\hat{L}$ such that $m(L)$ is differentiable at $\hat{L}$. Define $\hat{k} = m(\hat{L})f(\hat{L})$.*

- *the equilibrium is local NAM (i.e. $m'(\hat{L}) < 0$) if and only if $\phi(k, V)$ is* ***more curved than a logarithmic function*** *in k at $\hat{k}$: for k in the neighborhood of $\hat{k}$, $\phi(k, V) = \chi(log(k), V)$ where $\chi(.,.) : \mathbb{R}^2 \to \mathbb{R}$ is strictly increasing and concave in $log(k)$.*
- *the equilibrium is local PAM (i.e. $m'(\hat{L}) > 0$) if an only if $\phi(k, V)$ is* ***less curved than a logarithmic function*** *in k at $\hat{k}$: for k in the neighborhood of $\hat{k}$, $\phi(k, V) = \chi(log(k), V)$ where $\chi(.,.) : \mathbb{R}^2 \to \mathbb{R}$ is strictly increasing and convex in $log(k)$.*

Whenever the function $\phi(k, V)$ is more curved than a logarithmic function in k, the marginal benefit of conducting R&D decreases rapidly with the level of knowledge produced in-house, and the allocation of researchers to labs is NAM. The reason is that researchers and labs are substitutes from the firms point of view. As an illustration, consider two firms that have two labs of equal size but hire two researchers of different productivity. If both firms increase the size of their labs by one dollar, the firm with the more productive researcher sees a larger increase in its knowledge output than the other firm. However, the firm with the less productive researcher was producing less knowledge than the other firm. If the marginal benefit from producing knowledge decreases fast enough, the increase in total surplus will be greater for the firm with the less productive researcher. It follows that, in a neighborhood of $\hat{L}$, the firm with the smallest lab benefits the most from matching with a productive researcher, and therefore will hire him in equilibrium. On the other hand, whenever the function $\phi(k, V)$ is less curved than a logarithmic function in k, the marginal benefit of conducting R&D decreases slowly with the level of R&D, and the allocation of researchers to labs is PAM.

Corollary 1. *Define $\overline{L}$ as the largest lab in the economy, and $\overline{k} = \overline{a}f(\overline{L})$ as the maximum amount of knowledge that can be produced by any firm.*

- *If $\phi(k,V) = \chi(log(k),V)\,\forall k \leq \overline{k}$ with $\chi(.,.) : \mathbb{R}^2 \to \mathbb{R}$ strictly increasing and concave in $\log(k)$, then $m'(L) < 0$ for every L that is matched (i.e. global NAM.)*
- *If $\phi(k,V) = \chi(log(k),V)\,\forall k \leq \overline{k}$ with $\chi(.,.) : \mathbb{R}^2 \to \mathbb{R}$ strictly increasing and convex in $\log(k)$, then $m'(L) > 0$ for every L that is matched (i.e. global PAM.)*

Hence, if the function $\phi(k,V)$ is everywhere more curved than a logarithmic function in k, the equilibrium matching pattern is global NAM, in which the least productive researcher among the ones who are hired is allocated to the largest lab. Conversely, if the function $\phi(k,V)$ is everywhere less curved than a logarithmic function in k, the equilibrium matching pattern is global PAM: the best researcher is allocated to the largest lab. Note that, in general, the function $\phi(k,V)$ may be somewhere more curved than a logarithm, and somewhere less curved than a logarithm. When this happens, a and L are somewhere local complements, and somewhere local substitutes in $\phi(af(L),V)$. Depending on the distribution of labs and researchers' ability, the equilibrium allocation of researchers to firms can be PAM, NAM, or PAM over some range and NAM over some other range.

Proposition 1. *Suppose that $m'(\hat{L}) < 0$ at some $\hat{L}$ (i.e. local NAM). For ν sufficiently large the equilibrium allocation is inefficient: social welfare can be increased by implementing PAM in the neighborhood of $\hat{L}$.*

Because labs and researchers' abilities are complements in the knowledge production function, the matching rule that maximizes the stock of public knowledge is PAM: the best researcher should work in the biggest lab. On the other hand, lemma 1 shows that when $\phi(k,V)$ is sufficiently curved, for a given distribution of labs the equilibrium allocation is NAM, which is the allocation that *minimizes* the stock public knowledge V. If the value of public knowledge ν is sufficiently large, the equilibrium allocation of researchers to labs is inefficient. It is possible to achieve a Pareto improvement by reallocating researchers to firms and implementing PAM over some range.

Corollary 2. *Suppose that $m'(L) > 0$ for all L that are matched (i.e. global PAM). The allocation of researchers to labs is efficient: it is not possible to re-allocate researchers to labs and increase welfare.*

The above corollary follows directly from the fact that the amount of knowledge produced in the economy is maximized under a PAM allocation, which is also the equilibrium allocation. Hence, firms and researchers are simultaneously maximizing their individual payoffs, and maximizing the positive externality they generate.[15]

Overall, the fact that knowledge is a public good may or may not imply that the allocation of researchers to labs is inefficient.[16] The key determinant of whether welfare can be improved by re-allocating researchers to firms is the curvature of $\phi(k, V)$. When the surplus function is very curved in the amount of knowledge produced in-house, then inefficiencies are possible because the equilibrium allocation may be NAM.

4.1.1 The two faces of R&D: direct benefit and absorptive capacity.

In the introduction I argued that conducting in-house R&D activities produces two distinct outputs: new knowledge and absorptive capacity ("the two faces of R&D," according to Cohen and Levinthal, 1989). New knowledge is beneficial to the firm because it can lead, for example, to new patents and new products. Absorptive capacity is beneficial to the firm because it reduces the cost of using outside science. Rosenberg (1990, p. 170) explains:

> *Knowledge is regarded by economists as being "on the shelf" and costlessly available to all comers once it has been produced. But this model is seriously flawed because it frequently requires a substantial research capability to understand, interpret and to appraise knowledge that has been* placed *upon the shelf.*

In this section, I introduce absorptive capacity into the model by assuming that the stock of new public knowledge V can be used by firms only at a cost, and this cost is decreasing in the knowledge produced in-house. Therefore, in-house R&D activities generate two separate benefits:

[15] In equilibrium, a firm that expects to remain unmatched will not build in a lab. Hence, after the investment phase, a social planner cannot increase welfare by increasing the number of researchers matched. However, when considering also the investment choice, a PAM matching function may still be inefficient in the sense that not enough labs are built and not enough researchers are matched.

[16] It will, however, imply that the investment in labs is always inefficient. See lemma 2.

- A direct benefit coming from the application of the knowledge produced in-house toward the production of patents and new products, or toward improving the production process. I call this direct benefit $d(k) > 0$, with $d'(k) > 0$.

- A reduction in the cost of using the stock of new public knowledge. I call this cost $q(k) < 0$ with $q'(k) < 0$.

Hence, I impose the following functional form on the surplus function.

$$\Phi(a, L) = d(af(L)) + \alpha \left(V + q(af(L))\right) \equiv g(af(L)) + \alpha V, \tag{3}$$

where the parameter $\alpha > 0$ measures the importance of outside knowledge for the firm.[17]

The two benefits of knowledge production can be linked to the conditions on the curvature of the surplus function discussed in the previous section. When the surplus function is very curved, the marginal benefit of producing one additional unit of knowledge decreases very rapidly, which can be due to absorptive capacity. Intuitively, by producing the first unit of knowledge, the surplus generated within a firm increases significantly because the firm is now able to tap into the knowledge produced by other firms. However, once the amount of knowledge produced in-house is sufficiently high, the benefit of every additional unit of knowledge produced is low, as the firm is already able to use most of the public stock of knowledge. As a consequence, the total benefit derived from increasing absorptive capacity $V + q(k)$ is bounded above by the stock of knowledge in the economy V (remember that $q(k)$ is negative and increasing). On the other hand, there is no reason to believe that the benefit derived from pushing the knowledge frontier $d(k)$ should have an upper bound.

To see more clearly the link between the two motives for knowledge production and the curvature of the surplus function, assume that the functions $d(k)$ and $q(k)$ are both isoelastic functions:

$$d(k) = \frac{k^{1-\sigma_d}}{1-\sigma_d}$$

$$q(k) = \frac{k^{1-\sigma_q}}{1-\sigma_q},$$

with $\sigma_d < 1$ (so that $d(k)$ is positive and unbounded above) and $\sigma_q > 1$ (so that $q(k)$ is

[17] The parameter α is assumed constant but could, in principle, also depend on the amount of research produced by a firm. I discuss this possibility at the end of this section.

negative and bounded above by zero). The matching pattern is NAM at a given knowledge level k if:

$$k^{\sigma_q-\sigma_d} < \frac{\alpha(\sigma_q-1)}{1-\sigma_d},$$

and is global NAM if:

$$\overline{k}^{\sigma_q-\sigma_d} < \frac{\alpha(\sigma_q-1)}{1-\sigma_d}, \tag{4}$$

hence, the equilibrium matching pattern is NAM if $q(k)$ is sufficiently curved, if $d(k)$ is not very curved, and if the cost-reduction component is particularly important in the firms' surplus function (high α). In this sense, the equilibrium in the economy is NAM whenever absorptive capacity is the dominant motive for knowledge production as by (4).

Finally, the only case in which the equilibrium is either global PAM or global NAM for any distribution of a and L is when $g(k)$ is an isoelastic function:

$$g(k) = \frac{k^{1-\sigma}}{1-\sigma} + C$$

for some constant C. Under this assumption, the two inputs in the knowledge production process are substitutes in $\Phi(a, L)$ (and the equilibrium is NAM) if the overall benefit from producing science is bounded above (i.e. $\sigma > 1$). Conversely, the two inputs in the knowledge production process are complements in $\Phi(a, L)$ (and the equilibrium is PAM) if the overall benefit from producing science is unbounded above (i.e. $\sigma < 1$).

Whether in a given sector absorptive capacity or knowledge production is the dominant motive behind firms' R&D activities depends on several factors, which are not explicitly modeled here. For example, each firm's market share, the IP regimes, the market structure, all determine the benefit of research activities and the shape of the function $g(k)$. However, in what follows I assume that the dominant motive for knowledge production is absorptive capacity, in the sense that $g(k)$ is more curved than a logarithmic function.

Assumption 1. *$g(k) = \chi(log(k))\,\forall k \leq \overline{k}$ with $\chi() : \mathbb{R} \to \mathbb{R}$ strictly increasing and concave.*

Under the opposite assumption (i.e. $g(k)$ is *less* curved than a logarithmic function) the equilibrium allocation of researchers to labs is global PAM. In this case, there is no inefficiency at the matching stage but there is under-investment in labs because firms do not capture the full benefit of their R&D effort (see Lemma 2 in the next section). Hence, if $g(k)$

is *less* curved than a logarithmic function the model collapses back to a standard model of knowledge production where the only source of inefficiency is the firms' underinvesment. If instead $g(k)$ is *more* curved than a logarithmic function there is a novel form of inefficiency, that is worth exploring in greater details.[18]

To conclude this section, note that, in principle, absorptive capacity could affect not only the cost of using the stock of public knowledge, but also the fraction of public knowledge that is "absorbed" by firms. In this case, when the stock of public knowledge is expected to be large, absorptive capacity becomes more relevant in the surplus function, and NAM is more likely to emerge. Conversely, when the stock of public knowledge is expected to be small, absorptive capacity becomes less relevant in the surplus function, and PAM is more likely to emerge. Because NAM minimizes the amount of knowledge produced while PAM maximizes it, the equilibrium for given distribution of labs is always unique. Furthermore, the equilibrium matching depends on the curvature of $q()$ and $d()$ in a way that is similar to what discussed above. Hence, for given distribution of labs, the functional form assumed in equation (3) can be weakened without significant changes to the results.

Note, however, that equation 3 implies that the marginal incentive to invest in labs does not depend on V. This fact will be relevant in the next section, because it implies that V does not affect firms' equilibrium investment in labs but only the mass of firms investing. I will show that, as V increases, more and more firms enter the market and produce a positive amount of knowledge. Hence, there is a complementarity between aggregate knowledge produced and firms investment, but this complementarity operates exclusively through the extensive margin. Restricting the attention to the extensive margin simplifies the characterization of the general equilibrium of the economy.

4.2 General equilibrium.

I can now solve for the equilibrium investment in labs and equilibrium stock of public knowledge.

[18] In case $g(x)$ is somewhere more curved and somewhere less curved than a logarithmic function, for some distribution of a and L the equilibrium is local NAM somewhere, and by proposition 1 the equilibrium matching function may be inefficient. However, in this case it is not possible to derive the shape of the function $m(L)$ analytically.

Lemma 2. *In equilibrium, for $L \geq 0$:*

$$\frac{\partial \pi(L)}{\partial L} = \frac{\partial \Phi(a, L)}{\partial L}|_{a=m(L)}.$$

Proof. By applying the envelope theorem to point 3 of the definition of the equilibrium. □

Lemma 2 implies that a firm's investment solves:

$$\frac{\partial c(p, L)}{\partial L} = \frac{\partial \Phi(a, L)}{\partial L}|_{a=m(L)}, \tag{5}$$

which implies that firms maximize surplus taking as given V and the researchers they will hire in equilibrium. Because the social planner would take into account the impact of the individual investment in L on the stock of public knowledge, Lemma 2 implies that, for given researcher's ability a, the investment is inefficient.

In addition, because the matching pattern expected to emerge in the following period affects the investment decisions, there is an interesting interaction between the inefficiency in the matching stage and the inefficiency in the investment phase. Firms with a low cost of building labs underinvest for two reasons: because they do not capture the full social benefit of knowledge production, and because they expect to be matched with an unproductive researcher. Firms with a high cost of building labs may actually overinvest. On the one hand, they do not capture the full social benefit of knowledge production. On the other hand, they expect to be matched with a productive researcher. Therefore, the inefficiency in the matching phase causes firms with a comparative advantage at building labs to further drop their investment level, and those with a comparative disadvantage at building labs to increase their investment levels.

To simplify the derivation of the general equilibrium, I make the following assumption.

Assumption 2. $\lim_{k \to 0} g(k) = -\infty$.

The above assumption implies that for any level of public knowledge V there are always researchers who are not productive enough and will not be hired. Knowing this, it is possible to derive the equilibrium payoffs for researchers and firms.

Proposition 2. *If the value of research ν is high enough, equilibria where a positive amount of knowledge is produced exist. In these equilibria, researchers belonging to the set $[\underline{a}, \overline{a}]$*

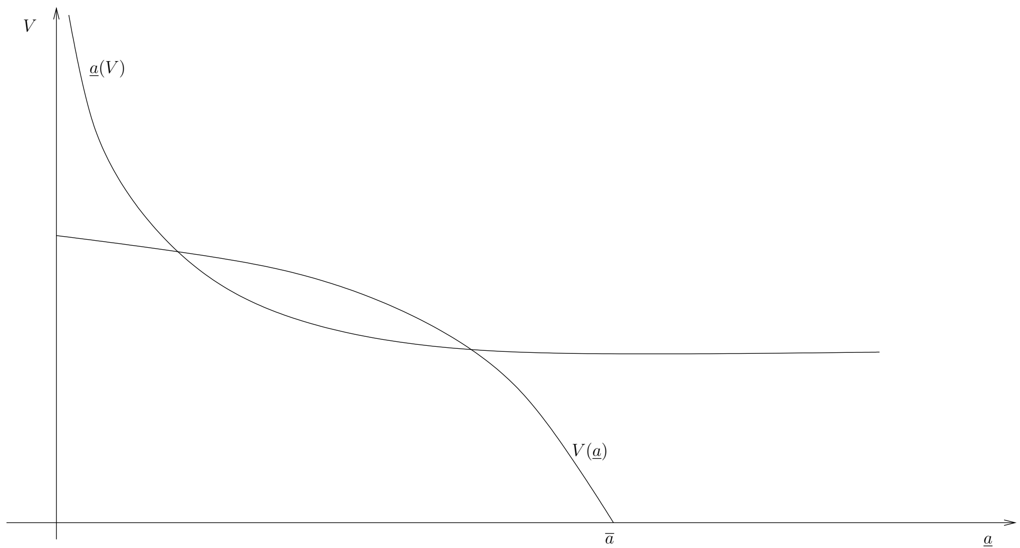

Fig. 2: Equilibrium $\underline{a}$ and V.

match with firms investing $l(a)$*, where:*

$$l(a) = max\left\{\left\{L \in \mathbb{R}^+ : \frac{\partial \Phi}{\partial L} = \frac{\partial c}{\partial L}\right\}, 0\right\} \tag{6}$$

$$\underline{a} : \alpha\nu \int_{\underline{a}}^{\overline{a}} af(l(a))z(a)da = P(\underline{a}) - g(\underline{a}f(L(\underline{a}))) \tag{7}$$

$$P(\underline{a}) = \int_{\tilde{m}^{-1}(\overline{a})}^{\tilde{m}^{-1}(\underline{a})} \frac{\partial c(s, i(p))}{\partial L} \gamma(p)dp, \tag{8}$$

$z(a)$ *is the p.d.f. of* a*, and* $\gamma(p)$ *is the p.d.f of* p*.*

Figure 2 illustrates the case of two positive investment equilibria, given by the intersection of $V(\underline{a})$ and $\underline{a}(V)$, where $V(\underline{a})$ represents the aggregate knowledge produced as a function of the least productive researcher employed, and $\underline{a}(V)$ represents the least productive researcher employed in the economy as a function of the stock of public knowledge V. The function $V(\underline{a})$ is increasing in $\underline{a}$, while $\underline{a}(V)$ is decreasing in V. That is, if the stock of public knowledge is expected to be high, researchers with low productivity will be employed. At the same time, when researchers of low productivity are employed, the stock of public knowledge produced is high because a large fraction of the researchers in the economy are employed. It follows that multiple equilibria are possible, a case represented in Figure 2. Of the two equilibria depicted in Figure 2, one can be considered stable (the high V, low $\underline{a}$ one) and the other unstable. As ν increases, $V(\underline{a})$ increases while $\underline{a}(V)$ remains constant, implying that if ν is sufficiently high $V(\underline{a})$ and $\underline{a}(V)$ will cross.

By focusing on the stable equilibrium, I can make a few comparative static exercises. If the value of public knowledge ν increases, $V(\underline{a})$ moves upward: more researchers are matched and more research is produced. It is also possible to introduce an exogenous stock of knowledge V^f, which is the knowledge produced, for example, by a foreign country. The graph should be modified by writing on the vertical axes V^h (research produced at home) instead of V. To the extent that the knowledge available in the home country increases in V^f, then $\underline{a}(V^h)$ shifts downward as V^f increases. As a consequence, as one country increases its research output, the other country also increases its research output. Obviously, all of the comparative statics are reversed if we consider the unstable equilibrium.

5 The First Best

Because of equation 3, the expression for aggregate welfare in the economy (Equation 2) becomes:

$$SW = \int \left(\alpha\nu m(i(p))f(i(p)) + g(m(i(p))f(i(p))) - c(p, i(p))\right)\gamma(p)dP.$$

Therefore, the welfare generated within each match between a researcher a and a firm $\{L, p\}$ is:

$$sw(a, L, p) = \alpha\nu a f(L) + g(af(L)) - c(p, L),$$

which is neither supermodular nor submodular in $\{a, L\}$.[19] It follows that the optimal allocation of researchers to labs can only be derived numerically, and may involve implementing PAM over some range, and NAM over some other range. Intuitively, the social planner may give priority to the production of knowledge by implementing PAM across some firms, and to the absorption of knowledge by implementing NAM across some other firms.

[19] Note that, for given L, p does not play any role in determining the optimal allocation of researchers to firms. The reason is that $\frac{\partial^2 sw(a,L,p)}{\partial a \partial p} = 0$.

However, the social planner problem has a unique solution. Hence, the first best can be achieved whenever the social planner can mandate a given investment in labs and a given allocation of researchers to labs. Similarly, the first best can be achieved with a mix of subsidies to the investment in labs and the direct allocation of researchers to labs. Finally, absent the possibility of allocating researchers to labs, the first best can be implemented if transfers based on the amount of knowledge produced by each firm are feasible.

Proposition 3. *The first best can be achieved if the following rule is announced: every firm producing knowledge receives a transfer equal to* $\alpha\,(\nu a f(L) - V)$.[20]

Because there is a mass 1 of firms, V is equal to the average amount of knowledge produced. Therefore, implementing the first best implies subsidizing firms that produce more than the average, while taxing the others.

6 Extensions

The model just developed abstracts away from several elements that may shape the aggregate distribution of scientific talent to labs. Here, I consider two such elements: the university research sector and reputation concern. In both cases, the model's predictions become more nuanced, with the equilibrium allocation of researchers to lab depending on the research sector and on the strength of researchers' reputation concerns.

6.1 The University Research Sector

So far, I have assumed that only profit-maximizing firms are active in the labor market for researchers. However, in most countries and in many scientific fields non-for-profits research institutions, and in particular universities, produce the vast majority of knowledge and are therefore an important player on the labor market for researchers.

In this section, I extend the model by incorporating the university research sector. To simplify the analysis, I make the following assumptions:

- Firms benefit from producing knowledge exclusively because it reduces the cost of using outside knowledge (i.e. $d(af(L)) \equiv 0$ and $\alpha = 1$).

[20] This result is closely related to Hammond (1995), Section 6.

- Each university owns a single lab and hires a single researcher. The amount of knowledge produced within a university is $K^u = f(L^u)\,a$, where L^u is the size of the university lab, and a is the ability of the researcher hired by the university. The universities' objective (their mission) is to produce new knowledge.

- In period 1 researchers can work in universities, and in period 2 academic researchers can work as consultant for firms.

The last assumption is motivated by the literature on *star scientists.* Zucker, Darby, and Brewer (1998) show that the birth of the biotechnology industry during the 1970s in a particular region can be explained by the presence of star scientists: researchers with an outstanding research track in genetics. These researchers worked in academia, and, at the same time, were active as consultants, were part of the board of companies, and sometimes even created their own start-ups. Doing so, they brought into these private labs the public science they contributed to create. More in general, it is commonly observed that academic researchers collaborate with private firms in various ways. For simplicity, I will refer to all these activities as consulting.

Consider a given distribution of university labs and of firms' labs (these distributions will be derived endogenously). I assume that the surplus created by a match between a firm and an academic researcher working as consultant is given by:

$$\Phi(a, L) = V + q(af(l^u(a)),$$

where $l^u(a)$ is the university lab researcher a worked in. Intuitively, previous research output affects the ability of a researcher to find and explain the bits of public knowledge that are relevant to the firm. Therefore, from the point of view of a firm, the benefit of hiring an academic researcher as consultant is increasing in the amount of knowledge produced by this academic researcher. Note also that the lab owned by the firm L plays no role in determining total surplus. Instead, if a researcher joins a firm in period 1, the surplus generated in period 2 is:

$$\Phi(a, L) = V + q(af(L)),$$

where L is the lab owned by the firm. It follows quite immediately that whenever $l^u(a) \geq L$ surplus is maximized when the researcher joins the university research sector and then acts as consultant.

Finally, note that academic researchers are not paid to do research within universities. They earn their payoff by consulting for firms. This is a useful simplification because it implies that the sorting of researchers between universities and firms is independent on the objective function of universities, and is fully determined by whether $l^u(a)$ or L is larger. Despite this, in the next section I show that, in equilibrium, the most productive researchers in the economy join the university sector (see Lemma 3). Hence, universities cannot improve on the quality of the researchers hired or on the amount of knowledge produced by paying wages to their researchers.

Endogenous university research sector. In order to derive the size of the university sector endogenously, I introduce into the model a government, and I assume that its objective is to maximize the stock of public knowledge produced in the economy, under an exogenous resource constraint G.[21] The instruments available to the government are two: a subsidy to the investment in labs, and the possibility of creating universities. Subsidies are cheaper than building universities because part of the cost of building labs is covered by firms. However, subsidies to the investment in labs have no impact on the equilibrium matching pattern. Building universities, although more expensive, allows the government to choose the optimal allocation of researchers to labs. The main goal of this section is to show that, depending on the parameters of the model, the government may use one, the other or both policy instruments. This is relevant because, if there are no inefficiencies in the equilibrium matching pattern, the government should only use subsidies to increase the production of knowledge. Hence, here universities exist to reallocate researchers to labs, which is justified by the fact that the private sector allocation of researchers to firms is inefficient.

Before the investment phase begins, the government announces $l^u(a)$, the lab a given researcher will receive if he joins the university sector. Furthermore, the government announces a transfer to firms $\tau(L)$, continuous and differentiable. The private surplus function is now $\Phi(a, L) + \tau(L)$. By lemma 2, in equilibrium $\frac{\partial \Phi(a,L,s)}{\partial L} + \frac{\partial \tau}{\partial L} = \frac{\partial c}{\partial L}$ and the worst researcher matched is given by $\Phi(\underline{a}, l(\underline{a})) + \tau(l(\underline{a})) - P(\underline{a}) = c(l(\underline{a}))$. Assuming that $\tau(l(\underline{a})) = 0$ (so that not all firms receive a subsidy), proving that a competitive equilibrium exists is analogous

[21] In the model, the government is uniquely characterized by its objective function. Readers may safely substitute the word "government" with, for example, "foundations."

to the problem solved in the previous section.

The government problem can be formalized in the following way:

$$\max_{L^u(a),\tau(l(a))} \{\nu \int_{\underline{a}}^{\overline{a}} af(\hat{l}(a))z(a)da\} \tag{9}$$

$$\text{s.t.} \begin{cases} \hat{l}(a) = \max\{l(a), l^u(a)\} & \text{(I)} \\ G = \int_{\underline{a}}^{\overline{a}} \left(\tau(l(a)) + l^u(a)\right) z(a)da & \text{(II)} \\ l(a) = \left\{L : \frac{\partial \Phi(a,L)}{\partial L}|_{a=m(L)} + \frac{\partial \tau}{\partial L} = \frac{\partial c}{\partial L}\right\} & \text{(III)} \\ \frac{\partial l(a)}{\partial a} \leq 0 & \text{(IV)} \\ \underline{a} : \nu \int_{\underline{a}}^{\overline{a}} af(l(a))z(a)da = c(l(\underline{a})) - q(\underline{a}f(l(\underline{a}))) + P(\underline{a}) & \text{(V)} \\ \tau(L) \geq 0 & \text{(VI)} \end{cases}$$

where $\hat{l}(a)$ are the labs in use in the economy, some of which are private $l(a)$ and some of which belong to universities $l^u(a)$. The first constraint says that a given researcher will work in the sector where he/she receives the largest lab. The second constraint is the government budget constraint. The following three constraints require that the economy is in a competitive equilibrium. The last line restricts $\tau(L)$ to be a subsidy rather than a tax.

It is possible to characterize the solution to the government problem.

Lemma 3. *The best researchers join the university sector and, within the university research sector, better researchers work in bigger labs. Subsidies are such that all firms receiving subsidies invest the same amount. University labs are larger than subsidized private labs.*

Proof. In appendix. □

Figure 3 provides an illustration of the costs and benefits of different types of government interventions. In the top graph, the shaded area represents the cost borne by the government. In the bottom graph, the shaded area represents the increase in V due to government intervention. To determine when the government should subsidize firms' investment, build universities or do both I resort to numerical methods (the details of the simulation are in the appendix). The results are reported in Figures 4 and 5.

In Figure 4 different quadrants report the distribution of labs for different values of $\overline{a}$ and G (G increases going from left to right, and $\overline{a}$ increases going from the top down). Figure

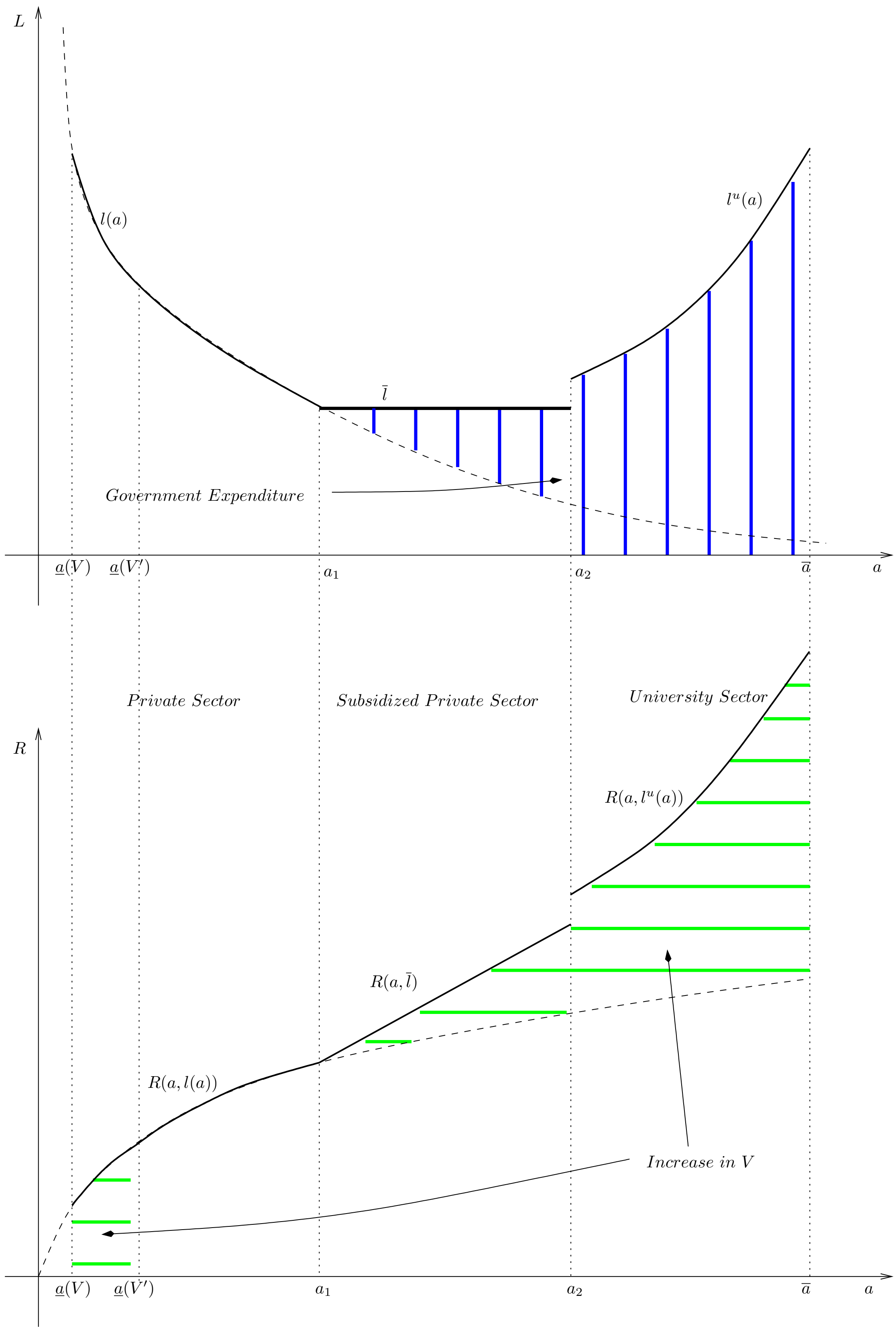

Fig. 3: Cost and benefit of government intervention.

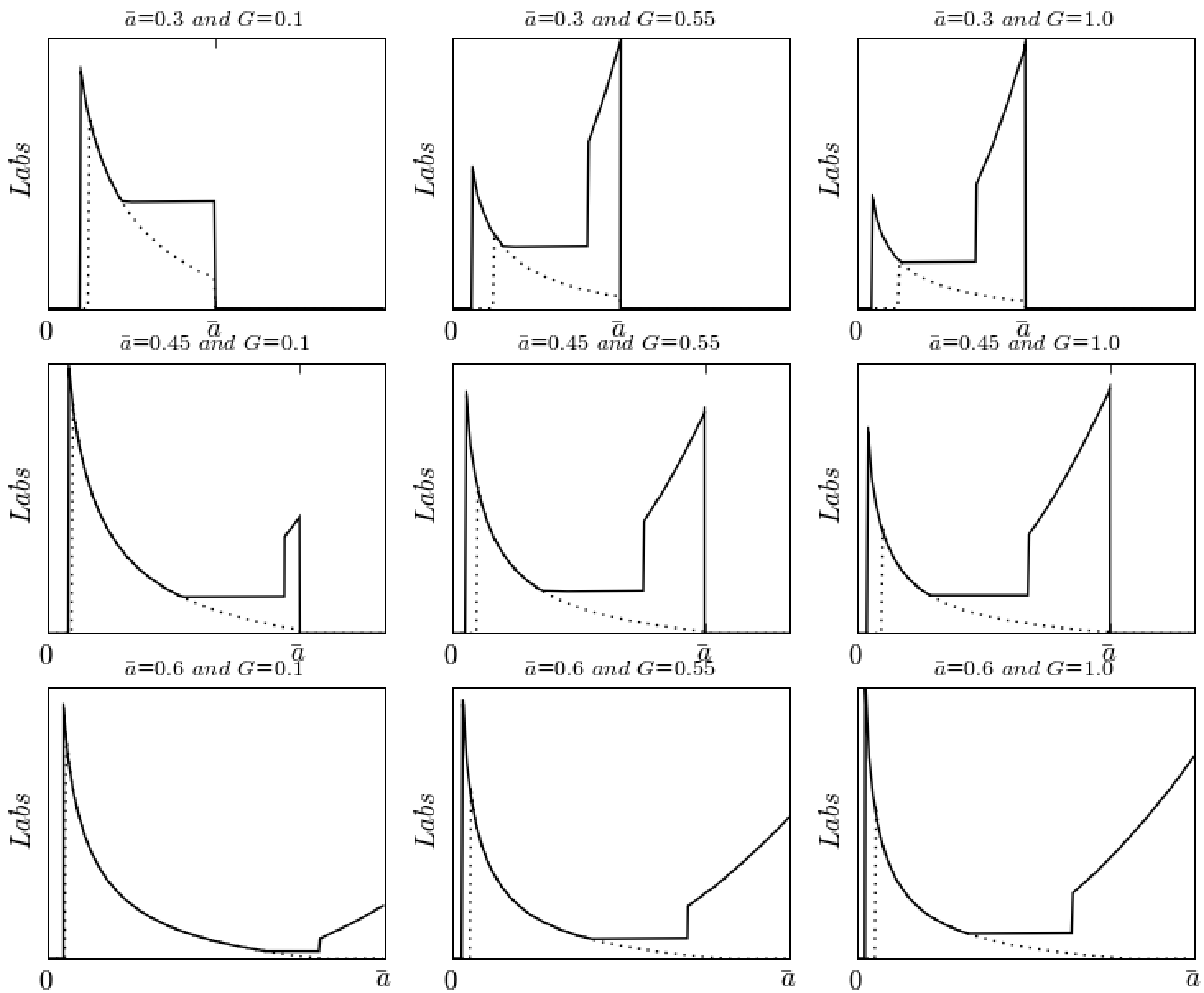

Fig. 4: Equilibrium distribution of labs (dotted line, no government; solid line, with government).

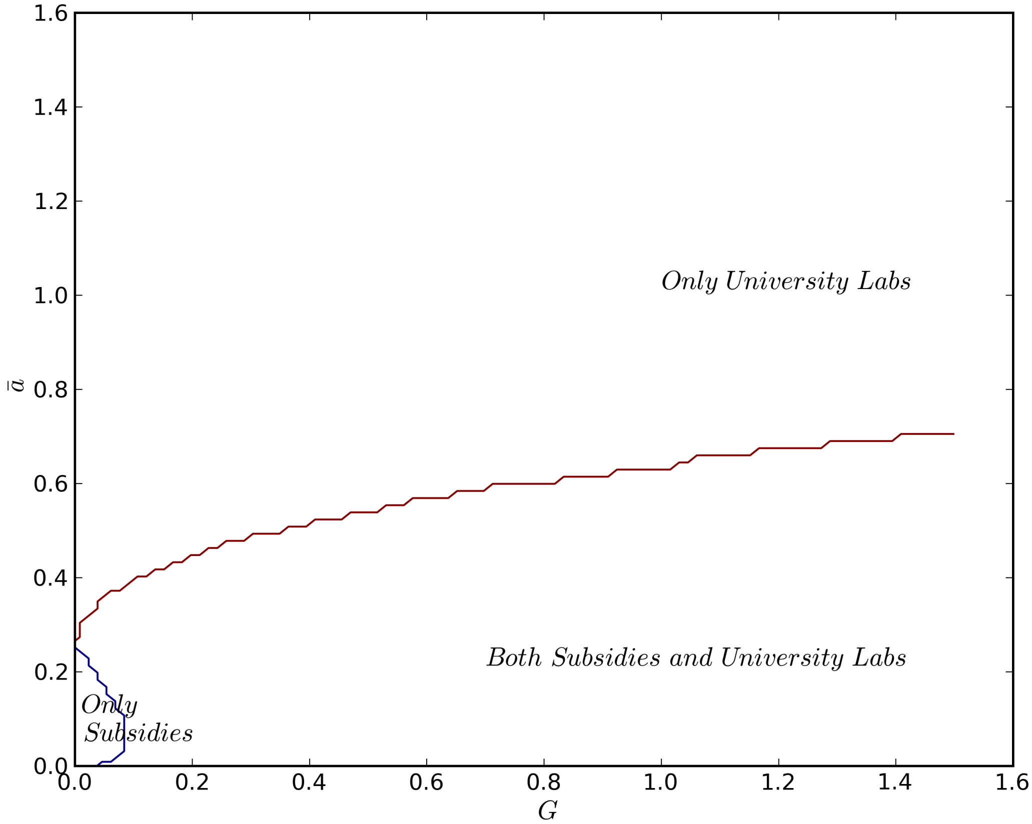

Fig. 5: Optimal government policy, as a function of $\bar{a}$ and G.

5 summarizes the results of the same exercise for a wider range of $\bar{a}$ and G. In both figures it is evident that, if the quality of the best researcher increases, the government is more likely to build university labs. When a researcher is very productive, the lab that he would work with in the private sector is very small: building universities allows the government to allocate more resources on the most productive researchers. Finally, Figure 5 shows that when the government has more resources, it is more likely to use a mix of university labs and subsidies, rather than only one of the two policies.

The government's intervention increases the equilibrium V. Compared with the economy without a government, now more firms invest and more researchers are matched. This is represented in the bottom graph of Figure 3 by a decrease in $\underline{a}$ from $\underline{a}(V')$ (where V' is the stock of knowledge before the government intervention) to $\underline{a}(V)$. Public expenditure in research is a complement or a substitute to private expenditure in research depending on the number of new firms investing in research compared to the number of firms that stop investing because of the creation of university labs. For example, in Figure 4, the researchers joining the university sector would work with small labs in the private sector, so there is little decrease in private investment if the government increases its expenditure. David, Hall, and Toole (2000) review the existing econometric evidence looking at whether publicly-funded R&D and privately-funded R&D are substitutes or complements. They report that most of the papers looking at aggregate measures find a complementarity effect, while, at the single firm level, there is evidence of a substitution effect. These results are consistent with the model, because government subsidies and the creation of universities decrease the investment of firms, but increase the number of firms investing.

6.2 Extension: Reputation Concerns.

In the benchmark model, I implicitly assumed that the creation of new knowledge generates only monetary benefits, which can be freely split between firms and researchers. However, since the work of Merton (1957), it is well known that researchers care about reputation. Merton calls scientists' concern for reputation the *race for priority*: researchers want to be recognized as the first to discover something. The role of reputation in the production of knowledge was explored in the economic literature by Dasgupta and David (1985). They argue that, on the one hand, reputation motivates researchers, which is important because an incentive scheme based exclusively on the quality of scientific output would be hard to implement. On the other hand, reputation fosters openness, which guarantees the circulation of ideas and generates a faster pace of scientific progress.

In this section, I introduce reputation into the model as an additional output of R&D, and I show that reputation may affect the production of knowledge in a new way: by changing the equilibrium allocation of researchers to firms. The reason is that reputation accrues to the researcher but cannot be transferred back to the firm because of liquidity constraints, and therefore transforms the matching problem in a Non-Transferrable Utility (NTU) problem (see Legros and Newman, 2007).[22] I show that NTU has the usual effect on the equilibrium matching: if it is strong enough it will generate a PAM allocation. More interestingly, here NTU may actually improve welfare because PAM is the knowledge-maximizing allocation of researchers to labs.

Formally, I assume that the researchers' utility is:

$$U(a) = w(a) + \rho\Phi(a, L)$$

where $w(a) \geq 0$ is the monetary payment received and $\rho \in (0, 1)$ is the fraction of surplus that accrue to the researcher as *reputation*: the utility derived from producing new knowledge. Researchers may care about knowledge because their future earnings depend on it, or simply because they enjoy producing knowledge. The remaining fraction of surplus $(1 - \rho)\Phi(a, L)$ is in monetary form and can be split among researchers and firms.

Lemma 4. *If ρ is sufficiently large, the equilibrium matching is PAM.*

Proof. See the mathematical appendix. □

Intuitively, here researchers are willing to give up their cash payments to produce new knowledge, the more so the larger is ρ.[23] Because of the complementarity between labs and researchers, a productive researcher is always willing to give up more than an unproductive researcher for the right to work in a firm with a given lab. Hence the payoff for both sides (firms and researchers) increases with the other side's type. It follows that the equilibrium matching is PAM whenever ρ is large enough.

If reputation concerns change the equilibrium matching from NAM to PAM, the effect on the social welfare is threefold. For given distribution of labs, firms' cost of using the aggregate stock of public knowledge will increase (i.e., low $g(af(L))$), but the stock of public knowledge will also increase. The net effect will depend on the parameter ν (measuring the value of a unit of new knowledge), and is positive as long as ν is sufficiently large.

[22] NTU can arise in many other cases. For example, if researchers' effort impact the value of the science produced and firms cannot implement incentive schemes depending on the value of output, the particular surplus split agreed ex ante determines total surplus.

[23] This finding is consistent with Stern (2004). In his paper "Do Scientists Pay to be Scientists?" the author collects data on job offers received by a sample of biology Ph.D. job market candidates. He finds that firms engaged in scientific research offer wages 25% lower than firms that are not engaged in scientific research.

Finally, reputation has an effect on the investment made by firms. As ρ increases, both the benefit and marginal benefit from investing in labs may decrease, because the fraction of surplus that is in monetary form and can be transferred to firms decreases. However, when a firm's investment is sufficiently inelastic, the overall welfare effect of reputation concerns is positive. For example, setting $c(p, L) = \left(\frac{L}{p}\right)^{\Delta}$ for Δ large enough guarantees that firms will invest $L \approx p$ regardless of the value of ρ. Overall, if firms investment is sufficiently inelastic and ν is sufficiently large the introduction of reputation concerns increases total welfare.

7 Conclusions

In this paper, I build a model of knowledge production in which firms invest in labs and hire researchers on a competitive market, and I analyze the competitive allocation of researchers under different motives for knowledge production. I show that whenever firms' return on R&D is sufficiently curved, scientific talent may be misallocated in equilibrium. I also argue that talent misallocation is more likely to occur when firms' R&D effort is motivated by absorptive capacity. In case of misallocation, subsidies to the investment in labs cannot restore full efficiency as they do not affect the equilibrium matching pattern. There is scope for policies that distort the market allocation of researchers, because reallocating researchers from one firm to another firm may generate an aggregate welfare gain (in addition to a local gain and a local loss).

I extend the model in two directions, by separately introducing the university research sector and reputation concerns. I show that the presence of reputation concerns may change the equilibrium matching pattern to PAM: good researchers work in big labs because they benefit directly from their research output. It follows that reputation concerns may increase the aggregated knowledge produced and total welfare. It also follows that the empirical matching patterns between researchers and labs should depend on the strength of researchers' reputation concerns. For example, assuming that reputation concerns are stronger for young researchers, we should observe productive young researchers working for firms with large labs, but productive old researchers working for firms with small labs.

I also analyze the role of universities, which are assumed to value knowledge production and not absorptive capacity. In equilibrium, the best researchers are hired by universities and, within the university sector, the allocation of researchers to labs is PAM. Hence, from the social point of view, building universities has a unique advantage relative to subsidizing the firm's investment in R&D, as universities can affect the way researchers are matched with labs. Depending on the parameters of the model, a policymaker aiming to increase the total knowledge produced in the economy may build university, subsidize the private investment in labs, or do both.

I focus on absorptive capacity due to the vast empirical literature showing its importance for firms. However, other mechanisms may generate a return on knowledge production that is sufficiently curved, and as a consequence cause a misallocation of scientific talent in equilibrium. For example, assume that conducting R&D activities is equivalent to making draws, where each realization of a draw represents the expected commercial value of an invention (where the parameter $k = af(L)$ in the model can be interpreted as the number of draws). Furthermore, assume that the outcome of a draw is always valuable from the social point of view (i.e. even bad draws are informative) but firms' revenues depend exclusively on the highest-valued draw. In this case, the social value of making draws is always greater than the private value of making draws. More interestingly, the social value of making draws grows linearly in the number of draws, whereas the private value grows much slower than linearly. Also in this case, the expected marginal private benefit from carrying out R&D activities decreases rapidly with the number of draws, meaning that the allocation of researchers to labs may be inefficient. Therefore, scientific talent misallocation may be present also in industries or scientific fields in which absorptive capacity is not the dominant motive for science production.

Finally, I characterize the optimal allocation of researchers to labs only partially. I show that whenever the allocation of researchers to firms is inefficient, it is possible to increase total welfare by re-allocating productive researchers from small labs to larger labs. However, the first-best allocation of researchers to firms depends on the exact distribution of labs and researchers' ability, and may involve PAM over some range and NAM over some other range. Deriving the first-best re-allocation policy and describing which researcher should move to what lab is left for future work.

Appendix: mathematical derivations.

Proof of Lemma 1

Proof. By standard matching theory, local substitutability implies local NAM, and local complementarity implies local PAM. Substitutability or complementarity at a given $\hat{a} = m(\hat{L}), \hat{L}$ depends on the sign of:

$$\frac{\partial \phi(af(L), V)}{\partial a \partial L}|_{a=m(\hat{L}), L=\hat{L}}$$

that itself depends on the sign of

$$\frac{\partial \chi(\log(af(L)), V)}{\partial a \partial L}|_{a=m(\hat{L}), L=\hat{L}}$$

Using the chain rule, it is possible to show that the above expression is negative if $\frac{\partial^2 \chi(x,V)}{\partial x^2}|_{x=\log(m(\hat{L})\hat{L})} < 0$ (i.e. $\chi(,)$ is concave in its first argument) and positive otherwise. □

Proof of Proposition 1

Proof. Consider an equilibrium matching function $m(L)$ and assume that $m'(\hat{L}) < 0$ at some $\hat{L}$. Consider an alternative matching function $m_\epsilon(L)$ with $m_\epsilon(L) = m(L)$ for $L \notin [\hat{L} - \epsilon, \hat{L} + \epsilon]$, and $m'_\epsilon(L) > 0$ for all $L \in [\hat{L} - \epsilon, \hat{L} + \epsilon]$ for some $\epsilon > 0$ small. In other words, the alternative matching function is a perturbation of the the equilibrium matching function, and implements PAM instead of NAM in the interval $[\hat{L} - \epsilon, \hat{L} + \epsilon]$. Call V the stock of aggregate knowledge produced under the equilibrium matching function $m(L)$, and V_ϵ stock of aggregate knowledge produced under the matching function $m_\epsilon(L)$.

When the matching function is perturbed in the way described above, the change in the surplus generated by a firm with lab L is[24]

$$\frac{d\phi(m(L)L, \nu \int m(L)Lh(L)dL)}{d\epsilon} = \underbrace{\frac{\partial \phi(m(L)L, \nu \int m(L)Lh(L)dL)}{\partial k}}_{\geq 0} \underbrace{\frac{\partial [m(L)L]}{\partial \epsilon}}_{<0 \text{ for some firms}} + \underbrace{\frac{\partial \phi(m(L)L, \nu \int m(L)Lh(L)dL)}{\partial V} \cdot \frac{\partial [\int m(L)Lh(L)dL]}{\partial \epsilon} \cdot \nu}_{>0}$$

Note that for some firms $\frac{\partial [m(L)L]}{\partial \epsilon} < 0$: under the alternative matching function some firms end up

[24] Remember that, by definition $k = m(L)L$ and $V = \nu \int m(L)Lh(L)dL$.

matched with a worse researcher compared to the equilibrium. On the other hand, $\frac{\partial[\int m(L)Lh(L)dL]}{\partial\epsilon} > 0$ for all $\epsilon > 0$: the total stock of public knowledge is greater when local NAM is transformed into local PAM. The parameter ν measures the value of the new knowledge produced. When it is sufficiently large, the effect of a perturbation in the matching function on the surplus generated by a given firm is positive, even for firms that are matched with a worse researcher.

Hence, for ν is sufficiently large, implementing the alternative matching function increases the surplus generates by each firm in the economy. It follows that total surplus is greater under the alternative matching function rather than the equilibrium matching function.

□

Proof of Proposition 2

Proof. In general, if all of the researchers and firms in the economy were matched, the worst member of each group could enjoy a strictly positive payoff in equilibrium. In our case, because of Assumption 2, in equilibrium there are always researchers and firms that remain unmatched. Consider the match between the firm that invested the most and the worst researcher matched. The researcher receives a payoff equal to zero, while the firm receives

$$\tilde{\pi}(\overline{p}) = \int_{\tilde{m}^{-1}(\overline{a})}^{\tilde{m}^{-1}(\underline{a})} \frac{\partial\Phi(\tilde{m}(p), i(p))}{\partial L}\gamma(p)dp = \int_{\tilde{m}^{-1}(\overline{a})}^{\tilde{m}^{-1}(\underline{a})} \frac{\partial c(p, i(p))}{\partial L}\gamma(p)dp \equiv P(\underline{a})$$

where $\gamma(p)$ is the p.d.f. of p, and the second equality follows from Lemma 2 and Equation 5. In other words, the payoff received by the most productive firm depends on the productivity of the worst researcher matched. The equilibrium $\underline{a}$ and V are the solutions to:

$$\underline{a} = \{a : \Phi(a, l(a)) = P(a)\} \tag{10}$$

and:

$$V = \nu \int_{\underline{a}}^{\overline{a}} af(L(a))z(a)da \tag{11}$$

The equilibrium with positive investment exists if there is a $\{\underline{a}, V\}$ solution to Equations 10 and 11.

Note that Equation 11 has a finite value at $\underline{a} = 0$, is equal to zero at $\underline{a} = \overline{a}$, and is strictly decreasing. Finally, Equation 10 can be rewritten as:

$$V = P(\underline{a}) - g(\underline{a}f(L(\underline{a}))) \tag{12}$$

Because $\lim_{a \to 0} q(af(L)) = -\infty$ the solution to 12 diverges to infinity for $\underline{a} \to 0$, has finite values for $\underline{a} \in (o, \overline{a}]$, and is continuous. Therefore, if ν is high enough, Equations 10 and 11 will cross.

$\square$

Proof of Proposition 3

Proof. To start, note that the competitive equilibrium maximizes private surplus for given V. For given distribution of labs, this result is due to Kamecke (1992). The fact that the investment in labs maximizes private surplus for given V follows from Equation 5 (see also Cole et al., 2001 for the same result in a more general setup).

If transfers conditional on the amount of knowledge generated are used, the private surplus in each match is:

$$\alpha V + g(af(L)) + T(af(L)) - c(p, L)$$

where $T(af(L))$ are the transfers. Clearly, the transfer described described in the proposition transforms the private surplus into the social surplus. Finally, note that a similar result is derived in Hammond (1995), who shows that Pigouvian taxes can restore efficiency in continuum economies with widespread externalities.

$\square$

Proof of Lemma 3

Proof. Consider two researchers and assume that they work in the university sector. The only constraint that matters in the construction of labs is constraint (II) of Problem 9. Therefore, the government will set:

$$f'(l^u(a)) = \left(\frac{a'}{a}\right) f'(l^u(a'))$$

for all a and a' working in the university sector.

PAM is not achievable using subsidies because of constraint (IV). Therefore, if the government uses subsidies, constraint (IV) is binding:

$$l(a) = \bar{l}$$

for all $l(a)$ receiving a positive subsidy.

This implies that, if some university labs are smaller than the subsidized private labs, total knowledge produced increases by allocating all a such that $l^u(a) < \bar{l}$ to the subsidized private sector. In addition, doing so would reduce government expenditures. Hence, $l^u(a) \geq \bar{l}$ for all researchers in the university sector. This also implies that the best researchers are hired by the university sector.

$\square$

Details of the Simulation.

I choose the following functional forms:

- $c(p, L) = (1+r)L$ so that all firms are identical
- $f(L) = (1+L)^{\frac{1}{2}}$
- $q(k) = -\frac{1}{k}$

and I assume that $\tau(l(\underline{a})) = 0$: the firm matched with the worst researcher receives no subsidy. This can be seen as a restriction on the amount of resources the government has.

The government problem can be written as:

$$\max_{l^u(\overline{a}),\bar{l},a_1,a_2} \{\int_{a_1}^{a_2} a(1+\bar{l})^{\frac{1}{2}} da + \int_{a_2}^{\overline{a}} a^2(1+l^u(\overline{a}))^{\frac{1}{2}} da - \int_{a_1}^{a_2} a(1+l(a))^{\frac{1}{2}} da\} \tag{13}$$

$$\text{s.t.} \begin{cases} \bar{l} = \left(\frac{1}{2(1+r)a_1}\right)^{\frac{2}{3}} - 1 & (1) \\ a_1 \leq a_2 \leq \overline{a} & (2) \\ \left(\frac{a_2}{\overline{a}}\right)(1+l^u(\overline{a})) - 1 \geq \bar{l} & (3) \\ \int_{a_1}^{a_2} \left(\bar{l} - \max\left\{\left(\frac{1}{2(1+r)a}\right)^{\frac{2}{3}} - 1, 0\right\}\right) da + \int_{a_2}^{\overline{a}} \left[\left(\frac{a}{\overline{a}}\right)^2 (1+l^g(\overline{a})) - 1\right] da = G & (4) \end{cases}$$

Figure 3 on page 27 represents it graphically. The objective function is the extra research produced thanks to the policy in place (the shaded area in the lower axes) at a cost summarized by constraint (4) and represented by the shaded area in the upper axis. Note that the increase in research at the bottom of the distribution of labs (between $\underline{a}(V)$ and $\underline{a}(V'))$ can be safely ignored since it is an increasing function of the extra research V produced by the rest of the economy.

The simulation simply compares values of the objective function at different a_2 and $\bar{l}$. The aim is not to determine the exact optimal policy, but to check whether there is an interior solution (both subsidies and university labs) or one of the two corner solutions (only subsidies, only university labs).

I construct a grid $\{0, ..., \overline{a}\}$ containing all possible values of a_2. For every value of a_2, I construct a grid of possible value of $\bar{l} \in \left\{ \left(\frac{1}{2(1+r)a_2} \right)^{\frac{2}{3}} - 1, ..., \tilde{l} \right\}$ where $\tilde{l}$ is an appropriate large number. For every a_2 and $\bar{l}$ I compute $l^u(\overline{a})$ using constraint (4) of 13. I consider the pair a_2 and $\bar{l}$ admissible if $l^u(a_2) = \left(\frac{a_2}{\overline{a}}\right)(1 + l^u(\overline{a})) - 1 \geq \bar{l}$. Finally, I compute the value of the objective function. The final solution is the admissible pair $\{a_2, \bar{l}\}$ returning the highest value. Finally, the value for r is 0.01 and for ν is 100.

Proof of Lemma 4

Proof. Consider a given V and a given distribution of labs. Suppose that, in equilibrium, the equilibrium is PAM and researchers receive no cash payments: their full payoff is in form of reputation. In this equilibrium, the only possible deviation for a research is to move to a firm with a smaller lab and capture a positive monetary payoff. Consider the best researcher $\overline{a}$, that is matched with the firm with the largest lab $\overline{L}$. This researcher could work for the firm with the smallest lab and receive a cash payment equal to the entire surplus generated within this match. This deviation is not profitable as long as:

$$\rho \Phi(\overline{a}, \overline{L}) > \Phi(\overline{a}, \underline{L})$$

If this deviation is not profitable for the most productive researcher it is not profitable for any researcher. Furthermore, for any V and any distribution of labs, we always have $\Phi(\overline{a}, \overline{L}) > \Phi(\overline{a}, \underline{L})$. Hence, if ρ is sufficiently close to 1, the equilibrium is PAM for any V and any distribution of labs. □

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