

## Common Models and Modelling Assumptions

**Particle** - An object which is small in comparison with other sizes or lengths can be modelled as a particle. This means that the mass of the object can be considered to be concentrated at a single point (a particle is often referred to as a point-mass). The fact that a particle has no dimensions means that we can ignore the rotational effect of any forces that are acting on it as well as any effects due to air resistance.

**Rod** - An object with one dimension small in comparison with another (such as a metre ruler or beam) can be modelled as a rod. This means that the mass of the object can be considered to be distributed along a straight line. A rod has no thickness (it is one-dimensional) and is rigid (it does not bend or buckle).

**Lamina** - An object with one dimension (its thickness) very small in comparison with the other two (its length and width) can be modelled as a lamina. This means that the mass of the object can be considered to be distributed across a flat surface. A lamina has no thickness (it is two-dimensional). For example, a sheet of paper or metal could be modelled as a lamina.

**Uniform Body** - If an object is uniform then its mass is evenly distributed over its entire volume. This means that the mass of the body can be considered to be concentrated at a single point (known as the centre of mass), at the 'geometrical centre' of the body. For example, an unsharpened pencil could be modelled as a uniform rod. However, once it is sharpened then its centre of mass would not be at its mid-point and we would model it as a non-uniform rod.

**Light Object** - If the mass of an object is very small in comparison with the masses of other objects, we can model it as being light. This means that we can ignore its mass altogether and treat it as having zero mass. Strings and pulleys are often modelled as being light.

**Inextensible** - If a string does not stretch under a load it is inextensible or inelastic.

**Smooth** - If we want to ignore the effects of friction, we can model a surface as being smooth. This means that we assume there is no friction between the surface and any object which is moving or tending to move along it.

**Rough Surface** - If a surface is not smooth it is said to be rough. We need to consider the friction between the surface and an object moving or tending to move along it. For example, a ski slope might be modelled as a smooth or a rough surface depending on the problem to be solved.

**Wire** - A rigid thin length of metal, which is treated as being one-dimensional, is referred to as a wire. A wire can be smooth or rough. We often consider beads which are threaded on a wire.

**Bead** - A particle which can be threaded onto, and move freely along, a wire or string is called a bead.

**Peg** - A support from which an object can be suspended or on which an object can rest is called a peg. A peg is treated as being dimensionless (it is treated as a point) as is usually fixed. A peg can be rough or smooth.

**Air Resistance** - When an object moves through the air it experiences a resistance due to friction.

**Wind** - Unless it is specifically mentioned, you can usually ignore any effects due to the wind in your models.

**Gravity** - The force of attraction between all objects with mass is called gravity. Because the mass of the earth is very large, we can usually assume that all objects are attracted towards the Earth (ignoring any force of attraction between the objects themselves). We usually model the force of the Earth's gravity as uniform, and acting vertically downwards. The acceleration due to gravity is denoted by  $g$  and is always assumed to be constant at  $9.8\text{ms}^{-2}$ . This value is given on the front of the exam paper.

## Kinematics of a Particle Moving in a Straight Line

- You need to know what these symbols represent

s	displacement (distance)
u	starting (initial) velocity
v	final velocity
a	acceleration
t	time

- If a particle is slowing down it has a negative acceleration. This is called deceleration or retardation.
- All measurements need to be converted into base SI units before substituting their values into the formulae

Measurement	SI Unit
<b>Time (t)</b>	Seconds (s)
<b>Displacement (s)</b>	Metres (m)
<b>Velocity (v or u)</b>	Metres per Second ( $ms^{-1}$ )
<b>Acceleration (a)</b>	Metres per Second per Second ( $ms^{-2}$ )

- The five formulae for solving problems about particles moving in a straight line with constant acceleration are
  - $v = u + at$
  - $s = \frac{1}{2}(u + v)t$
  - $v^2 = u^2 + 2as$
  - $s = ut + \frac{1}{2}at^2$
  - $s = vt - \frac{1}{2}at^2$
- An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of  $g = 9.8ms^{-2}$
- The gradient of a speed-time graph illustrating the motion of a particle represents the acceleration of the particle
- The area under a speed-time graph illustrating the motion of a particle represents the distance moved by the particle
- Area of trapezium = *average of the parallel sides x height*  
 $= \frac{1}{2}(a + b) \times h$
- At constant speed, *distance = speed x time*

## Dynamics of a Particle Moving in a Straight Line

- The unit of force is the Newton (N). It is defined as the force that will cause a mass of 1kg to accelerate at a rate of  $1\text{ms}^{-2}$

$$F = ma$$

- The force due to gravity is called the weight of an object, and it acts vertically downwards. A particle falling freely experiences acceleration of  $g = 9.8\text{ms}^{-2}$

$$W = mg$$

- The component of a force of magnitude  $F$  acting in a certain direction is  $F \cos(\theta)$ , where  $\theta$  is the size of the angle between the force and the direction
- The maximum or limiting value of the friction  $F_{MAX}$  between two surfaces is given by

$$F_{MAX} = \mu R$$

where  $\mu$  is the coefficient of friction and  $R$  is the normal reaction between the two surfaces

- If a force  $P$  is applied to a block of mass  $m$  which is at rest on a rough horizontal surface and  $P$  acts at an angle to the horizontal:
  - The normal reaction  $R$  is not equal to  $mg$
  - The force tending to pull or push the block along the plane is not equal to  $P$
- A particle placed on a rough inclined plane will remain at rest if  $\tan(\theta) \leq \mu$  where  $\theta$  is the angle the plane makes with the horizontal and  $\mu$  is the coefficient of friction between the particle and the plane
- Provided all parts of a connected system are moving in the same straight line you can treat the whole system as a single particle
- In problems involving particles which are connected by string(s) which pass over pulley(s) you cannot treat the whole system as a single particle. This is because the particles are moving in different directions.
- The momentum of a body of mass  $m$  which is moving with velocity  $v$  is  $mv$

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

$$p = mv$$

- If a constant force  $F$  acts for time  $t$  then we define the impulse of the force to be  $Ft$

$$\text{Impulse} = \text{Force} \times \text{Time}$$

$$I = Ft$$

- The Impulse-Momentum Principle states that
  - $\text{Impulse} = \text{Final Momentum} - \text{Initial Momentum} = \text{Change in Momentum}$
  - $I = mv - mu$
- The principle of Conservation of Momentum states that

*Total Momentum Before Impact = Total Momentum After Impact*

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

## Statics of a Particle

- A particle is said to be in equilibrium when it is acted upon by two or more forces and motion does not take place. This means that the resultant of the forces is zero and the particle will remain at rest or stationary, as it is not subject to acceleration.
- To solve problems in statics you should:
  - draw a diagram showing clearly the force acting on the particle(s)
  - resolve the forces into horizontal and vertical components or, if the particle is on an inclined plane, into components parallel and perpendicular to the plane
  - set the sum of the components in each direction equal to zero
  - solve the resulting equations to find the unknown force(s) and angle(s)
- You can add forces of weight, tension, thrust, normal reaction and friction to a force diagram as appropriate
- The maximum value of the frictional force  $F_{MAX} = \mu R$  is reached when the body you are considering is on the point of moving. The body is then said to be in limiting equilibrium.
- In general the force of friction  $F$  is such that  $F \leq \mu R$ , where  $R$  is the normal reaction.
- The direction of the friction force is opposite to the direction in which the body would move if the friction force were absent.

## Moments

- A force applied to a rigid body can cause the body to rotate
- The moment of a force measures the turning effect of the force on the body on which it acts.
- The moment of the force  $F$  about a point  $P$  is the product of the magnitude of the force  $F$  and the perpendicular distance from the point  $P$  to the line of action of  $F$ .  
Unit: Nm.

$$\mathbf{Moment = F \times d}$$

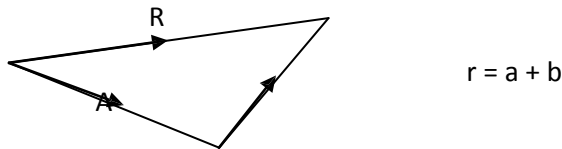
- When you have several coplanar forces acting on a body, you can add the moments about a point. You need to choose a positive direction (clockwise) and consider the sense of rotation of each moment.
- When the distance given is not the perpendicular distance of the line of action from the pivot you can find the moment by resolving the force into components.
- If a body is resting in equilibrium the resultant force in any direction is zero, and the sum of the moments at any point is zero.
- The mass of a non-uniform rigid body can be modelled as acting at its centre of mass.

## Vectors

- A vector is a quantity which has both magnitude and direction.
- A vector can be represented as a directed line segment.



- Two vectors are equal if and only if they have the same magnitude and the same direction.
- Two vectors are parallel if and only if they have the same direction.
- You can add vectors using the triangle of law of addition



- The unit vectors along the Cartesian axes are usually denoted by **i** and **j** respectively. You can write any two dimensional vector in the form  $a\mathbf{i} + b\mathbf{j}$ .
- When vectors are written in terms of the unit vectors **i** and **j** you can add them together by adding the terms in **i** and the terms in **j** separately. You can subtract vectors in a similar way.
- When a vector is given in terms of the unit vectors **i** and **j** you can find its magnitude using Pythagoras' Theorem. The magnitude of a vector **a** is written  $|\mathbf{a}|$ .
- The velocity of a particle is a vector in the direction of motion. The magnitude of the velocity is the speed of the particle. The velocity is usually denoted by **v**.
- If a particle starts from the point with position vector  $\mathbf{r}_0$  and moves with constant velocity **v**, then its displacement from its initial position at time t is  $\mathbf{vt}$  and its position vector **r** is given by

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{vt}$$

- The acceleration of a particle tells you how the velocity changes with time. Acceleration is a vector, usually denoted by **a**. If a particle with initial velocity **u** moves with constant acceleration **a** then its velocity, **v**, at time t is given by
- $$\mathbf{v} = \mathbf{u} + \mathbf{at}$$
- A force applied to a particle has both a magnitude and a direction, so force is also a vector. The force causes the particle to accelerate
- $$\mathbf{F} = m\mathbf{a}, \text{ where } m \text{ is the mass of the particle}$$
- If a particle is resting in equilibrium then the resultant of all the forces acting on it is zero. This means that the sum of the vectors of the forces is the zero vector.