Microeconomic Theory I

4. Technology and Production

Andrea Canidio

CEU

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Production and Technology

Basic concepts

- **Production plan**: a vector containing the inputs and outputs of a production process, with $-$ and $+$ signs, respectively

Example

$$y = \begin{bmatrix} +1 \text{ apple pie} \\ -500\text{g flour} \\ -5 \text{ apples} \\ -100\text{g butter} \\ -100\text{g sugar} \end{bmatrix}$$

Basic concepts

Production possibility set Y: all production plans that are **technologically feasible**

- *Notation*: we sometimes break the production plan in input and output in the following way

$$y = \begin{bmatrix} y \\ -x \end{bmatrix}$$

where y is the vector of output and x is the vector of inputs.

Basic concepts

Definitions

If there is only one output, for every $y \in Y$ we can write

$$y = \left[\begin{array}{c} f(x) \\ -x \end{array} \right]$$

$f(x)$ is called a **production function** and it is the highest level of output achievable with a given input vector.

Property of Technology

Assumption

The technology is **continuous** if Y is closed.

Assumption

$$Y \cap \mathbb{R}_+^L = \{\underline{0}\}$$

Assumption

There is free disposability if

$$z \in Y \;\&\; y \leq z \implies y \in Y \;.$$

Property of Technology

Definitions

A production set Y exhibits

- **increasing returns to scale** (IRS) if for any $y \in Y$ and $\alpha > 1$, $\exists y' > \alpha y$ s.t. $y' \in Y$
- **decreasing returns to scale** (DRS) if for any $y \in Y$
- **constant returns to scale** (CRS) if for any $y \in Y$ and

Property of Technology

Definition

A technology is **convex** if Y is convex, i.e.

$$\forall y, y' \in Y, \ \forall \alpha \in [0,1]$$
$$\alpha y + (1-\alpha)\, y' \in Y \ .$$

- Convexity $\Longrightarrow$ CRS or DRS

Definition

A technology is **additive** if

$$x, y \in Y \implies x + y \in Y \ .$$

- Additivity $\Longrightarrow$ CRS or IRS

Property of Technology

Proposition

A production function f is

$$\begin{aligned} \text{CRS} &\iff f(tx) = tf(x) \quad (\forall t), \text{ i.e. } f \text{ is } O^1 \\ \text{IRS} &\iff f(tx) > tf(x) \quad (t > 1) \\ \text{DRS} &\iff f(tx) < tf(x) \quad (t > 1)\ . \\ &(\text{alt.: } f(tx) > tf(x) \quad (0 < t < 1)\ ) \end{aligned}$$

Can we measure the returns to scale?

- Suppose that we measure the returns to scale of a technology, and we find that it is DRS.
- **Problem:** the technology is ether really DRS, or we failed to measure properly one of the inputs.

Definition

Hidden input (z): the true production function is:

$$F(z, \underbrace{x}_{\text{measurable}}) \equiv z\, f\left(\frac{x}{z}\right)\ ,\ f \text{ is concave}$$

Claim

the true production function $F(z, x)$ is CRS.

Firm's optimal behavior: profit maximization

Firm's optimal behavior

- Technology determines what is **possible** for a firm to do. But what is the optimal thing to do?
- We will analyze the problem from two point of views:
  - profit maximization
  - cost minimization.
- In both cases firms choose the optimal *production plan* for *given technology* as a *function of prices*, and ***prices are taken as given.***

Profit maximization

Definition

$$\pi(p) = \max_{y} py$$
$$\text{s.t. } y \in Y$$

$\pi(p)$ is the **Profit Function**

Definition

$$y(p) = \underset{y \in Y}{\text{argmax}}\, py$$

where if $y_i(p)$ is positive we call it a *supply function*, if $y_i(p)$ is negative we call it a *factor demand function.*

Profit maximization

- If we can use the production function:

$$\pi(p) = \max_x \{p_y f(x) - \boldsymbol{w}x\}$$

where $p_y f(x)$ is **revenue** and wx is **cost**

- FOC (do not use automatically!):

$$p_y \frac{\partial f(x^*)}{\partial x_i} = w_i \quad \forall i$$

i.e. equalize marginal cost and marginal benefit of each input.

Profit maximization

Proposition

If Y has non-decreasing returns to scale, then $\pi(p)$ is not defined for some $p \gg 0$.

Proof.

- Take $y' \in Y$ and $p \gg 0$ s.t. $py' > 0$
- Suppose $y(p) = \arg\max(py)$ exists
- We have $py(p) \geq py' > 0$
- We know that $\alpha y(p) \in Y$ for any $\alpha > 1$ but $p\alpha y(p) > py(p)$ $\implies y(p)$ is not profit max

□

Comparative statics

Proposition

$$\left(p - p'\right)\left(y(p) - y(p')\right) \geq 0$$

Proof.

$$py(p) \geq py(p') \quad \text{since } y(p) = \arg\max yp \qquad (1)$$
$$p'y(p') \geq p'y(p) \qquad (2)$$

$$(1) \implies p\left(y(p) - y(p')\right) \geq 0$$
$$(2) \implies -p'\left(y(p) - y(p')\right) \geq 0$$

$$\left(p - p'\right)\left(y(p) - y(p')\right) \geq 0$$

□

Comparative statics

Hotelling's lemma

$$\frac{\partial \pi(p)}{\partial p_i} = y_i(p)$$

Proof.

By envelope theorem... □

Firm's optimal behavior: cost minimization

Cost minimization

Definitions

Cost function:

$$C(w, \bar{y}) = \min wx$$
$$\text{s.t. } f(x) = \bar{y}$$

Conditional factor demand function:

$$z(w, \bar{y}) = \arg\min wx$$
$$\text{s.t. } f(x) = \bar{y}$$

- Choose the inputs vector so to minimize the cost of producing a given output.
- Advantage: the solution to the cost minimization problem always exists.

Cost minimization

- FOC:

$$\lambda \frac{\partial f(x^*)}{\partial x_i} = w_i \quad \forall i$$

$$\frac{\frac{\partial f(x^*)}{\partial x_i}}{\frac{\partial f(x^*)}{\partial x_j}} = \frac{w_i}{w_j}$$

- The LHS is known as the **marginal rate of technical substitution** (MRTS)

Exercise

$$f(K, L) = AK^{\alpha}L^{1-\alpha} \quad 0 < \alpha < 1$$

1. Find the conditional factor demands.
2. Find the cost function.
3. Find the supply function.
4. Find the profit function.