A Theory of Incomplete Agreements in Negotiations*

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Abstract

We study negotiations between players having context-dependent preferences. We show that the players may negotiate in stages, by first signing an incomplete agreement and then finalizing the outcome of the negotiation, which never happens when players are fully rational. Furthermore, if preferences are context dependent because of the focusing effect, incomplete agreements are used to eliminate extreme, off-equilibrium outcomes from the set of possible bargaining solutions. This remains true when previous incomplete agreements can be renegotiated or ignored.

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1 Introduction

The literature on decision theory and behavioral economics has long argued that when preferences are *context-dependent*—i.e., the preference ranking over two consumption bundles depends on the set of available consumption bundles—a person may value

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eliminating certain options from her choice set before making her final choice.[1] In this paper, we introduce context-dependent preferences in a bargaining game. We argue that the negotiating parties may prefer to negotiate in stages, by first restricting the set of possible bargaining options via an *incomplete agreement*, and then finalizing the outcome of the negotiation.

Similarly to a decision problem, a person with context-dependent preferences may anticipate that upon entering a negotiation her preferences will be distorted by the set of possible bargaining outcomes. For example, this person may anticipate that one of the issues to be discussed will be very salient, and because of this she is likely to overvalue achieving a good result on that specific issue. As a consequence, the outcome of the negotiation may not be optimal from the ex-ante (that is, before the negotiation begins) viewpoint. It is possible that the negotiating parties will fail to find an agreement over all issues on the table despite the fact that such comprehensive agreement would be ex-ante optimal. When this is the case, this person may benefit from restricting the set of future bargaining options.

Our main result is that even if this individual is unable to unilaterally manipulate the future bargaining set, the two parties may jointly impose a restriction on the future bargaining set via an incomplete agreement. Such an incomplete agreement will be signed whenever restricting the set of bargaining options is beneficial to both players. For example, eliminating outcomes that are particularly bad for a player and will not be achieved in equilibrium may increase the willingness to compromise of that player, the number of issues included in the agreement and, as a consequence, the value of reaching an agreement for both players. Therefore, the novelty of our paper relative to the existing literature is that restricting the set of future options is a *joint* decision made by players who have, otherwise, opposed interests.

Note that the above reasoning relies on the fact that a person's preferences while bargaining over an incomplete agreement may be different from this same person's preferences during the final negotiation round. But why should this be the case? We consider two possibilities. First, preferences may be context dependent during the last stage of the negotiation but rational if far away from that moment. For example, the last round of negotiation is the last moment when a player can walk away from the negotiation, which may create additional pressure and distort preferences. While negotiating the incomplete agreement instead, the players anticipate that they have

[1] See, for example, Strotz (1955), Gul and Pesendorfer (2001), Dekel, Lipman and Rustichini (2001), Sarver (2008), Noor (2011), Kőszegi and Szeidl (2013), Bordalo, Gennaioli and Shleifer (2013), Bushong, Rabin and Schwartzstein (2015).

an additional chance to trigger the disagreement outcome, easing off the pressure and making them behave as rational. In this case, the players will use incomplete agreements to move the final outcome of the negotiation closer to the outcome that would be achieved if players were always rational.

A second possibility is that players' preferences are context dependent also ex-ante, that is, when bargaining over incomplete agreements. We address this case by assuming that the ex-ante context is defined by the set of possible outcomes that are achievable via an incomplete agreement. We show that players may value using incomplete agreements also in this case. However, there is no presumption that the outcome achieved by the parties via an incomplete agreement is always better (according to some rational benchmark) than the outcome achieved without the use of incomplete agreements.

Our theory therefore provides an explanation to why, in the context of negotiations, many agreements are incomplete, in the sense that they specify only some aspects of the final outcome and rely on future bargaining rounds to define the missing provisions. A case in point is the use of framework agreements in international negotiations and in procurement. In procurement, a framework agreement may define, for example, a set of prices and quality levels of a possible future transaction, with the understanding that the details of this transaction will be established in a future agreement. Similarly, international negotiations are often structured as a sequence of negotiating rounds. Each round is concluded by an agreement, which is neither final nor binding but provides the framework for a later round of negotiations.[2] Importantly, practitioners believe that incomplete agreements can be used by the negotiating parties to affect the final outcome of the negotiation (see, for example, the influential textbook by Raiffa, Richardson and Metcalfe, 2002, pp. 206-208), which is in line with our results.

For most of the paper, we consider a specific source of context dependence, namely the *focusing effect*. The focusing effect (or focusing illusion) occurs whenever a person places too much importance on certain aspects of her choice set (i.e., when certain elements are more *salient* than others). Intuitively, an agent's attention is unconsciously and automatically drawn toward certain attributes, which are therefore overvalued when making a choice. Kőszegi and Szeidl (2013) formalize this concept by assuming

[2] This principle is sometimes explicitly stated as "*Nothing is agreed until everything is agreed*." See, for example, the rules governing the Doha round of trade negotiations. http://www.wto.org/english/tratop_e/dda_e/work_organi_e.htm (accessed on the 28th of December 2015).

that agents maximize a *focus-weighted utility*

$$\tilde{U}(x_1, x_2, .., x_n) = \sum_{s=1}^{n} h_s u_s(x_s)$$

where $x = \{x_1, x_2, ...x_n\}$ is a given good with n attributes. The *focus weights* h_s are defined as:

$$h_s = h\left(\max_{x\in C} u_s(x_s) - \min_{x\in C} u_s(x_s)\right),$$

where C is the choice set and $h()$ is the *focusing function,* assumed strictly increasing. In this formalization, an agent overweights the utility generated by the attributes in which her options differ more, where these differences are measured in utility terms.

In a bargaining game in which the players' preferences are distorted by the focusing effect, incomplete agreements may be used by the bargaining parties to eliminate from the bargaining set extreme outcomes with respect to a specific issue, and therefore reduce the salience of that issue. This mechanism is well understood by practitioners, who believe that in negotiations, "the importance of an issue might be lessened by the parties first narrowing the range of possible outcomes on that issue" (Raiffa, 1982, "The art and science of negotiation" p.216).[3] A notable example for this negotiating procedure is the use of so-called "threshold agreements" during the Panama Canal negotiations between the Panamanian government and the US government. The threshold agreements were incomplete agreements guaranteeing to the Panamanian government the achievement of minimum outcomes on three different issues, and were initiated by the American delegation to avoid the break-off of the negotiation.[4]

We show that, in the absence of a "threshold agreement", the negotiating parties may settle only on a subset of the issues on the table, despite the fact that agreeing on all of these issues may be optimal from the ex-ante viewpoint. This outcome is more likely to occur when the issues on the table generate positive but uneven surplus, that is, when there is a major issue that, if resolved, generates a large surplus and a minor issue that, if resolved, generates a positive but smaller surplus. In this case, the minor issue may be left out of the agreement. Instead, by imposing an appropriate

[3]See also Fisher, Ury and Patton (1991) "Getting to yes", p. 172.

[4]The three threshold agreements were on the jurisdiction of the Panamanian canal, the Panamanian participation in its defense, and on the operation of the canal. The American delegations initiated the discussion of three threshold agreements to "avoid the break-off of the negotiations and to demonstrate the good will that would be necessary for later Panamanian concessions." See Raiffa (1982) p.178.

“threshold agreement”, the parties can avoid this outcome and reach an agreement including all issues on the table.

We also consider the possibility of renegotiating and ignoring a previous incomplete agreement, where the latter is defined as waiting one period and then negotiating over the unconstrained bargaining set. The possibility of renegotiating and ignoring constrains the way in which the bargaining parties can manipulate the future bargaining set. Despite this constraint, we show that also in this case incomplete agreements may be used in equilibrium. This result is empirically relevant because incomplete agreement are, for the most part, non binding. When facing a sequence of agreements between two parties, in case a later agreement contradicts an earlier agreement, courts typically enforce the most recent one. When courts are not available (for example, in the case of international negotiations), the same two parties are free to jointly ignore a previous agreement. Nonetheless, negotiations are often structured as a sequence of incomplete agreements, which can be explained within our framework.

We structure the paper in the following way. In the remainder of this section, we discuss the relevant literature. In Section 2, we examine a simple example in which the preferences of one of the two bargaining parties are distorted by the focusing effect. In Section 3, we consider a general bargaining game in which both players have generic context-dependent preferences. In Section 4, we develop an extension to our example, in which incomplete agreements can be renegotiated or ignored. Section 5 concludes. Unless indicated otherwise, proofs are relegated to Appendix A. The online Appendix presents further robustness checks and extensions.

Relevant Literature

The literature on incomplete contracts has argued that behavioral biases and cognitive limitations may explain why agreements are often incomplete and the degree of their incompleteness.[5] However, to the best of our knowledge, all these explanations rely on the resolution of some uncertainty: after signing an incomplete contract, the arrival of new information makes the environment less complex, reduces the number of possible contingencies, and allows the contract to be completed in a final negotiating round. Instead, we consider a deterministic environment. This is justified by the observation that in many negotiations the start of a new bargaining round follows the end of

[5] See, for example, Segal (1999), Battigalli and Maggi (2002), Bolton and Faure-Grimaud (2010), Tirole (2009), Hart and Moore (2008), Herweg, Karle and Müller (2014) and Herweg and Schmidt (2015).

the previous round and is not determined by the arrival of new information. With respect to this literature, our contribution is therefore to provide a foundation for the existence of incomplete agreements in contexts with no uncertainty.[6]

Some authors argue that, if information is perfect but players are prevented from writing a complete contract, then the players may decide to leave some potentially contractible aspects of an agreements unspecified (see Bernheim and Whinston, 1998). Closest to our model, Battaglini and Harstad (2014) study international environmental agreements, and assume that countries cannot perform side payments. They argue that environmental agreements may be left incomplete, as a way of inducing more countries to sign the agreement (see also Harstad, 2007). In our model, instead, every aspect of an agreement is potentially contractible, including side payments.

Some papers demonstrate that if the players' outside options are determined endogenously, agreements may be reached gradually rather than in a single period. Closest to our work, Compte and Jehiel (2003) introduce reference-dependent utility in a game of alternating offers, and show that the history of offers affects the players' preferences and the final outcome of the game. Here we solve each bargaining stage via Nash bargaining, and show that multiple bargaining stages are possible. Hence, our model is a model of sequential agreements, and not of sequential offers.

With this respect, our paper is related to Esteban and Sákovics (2008), who allow the negotiating parties to sign separate agreements on different parts of the surplus. We are also close to the literature on agenda setting in negotiations, which has long argued that when players bargain over multiple issues, the order in which agreements are reached matters for the outcome of the bargaining process.[7] The difference with this literature is that we study negotiations that entail a unique final agreement reached via several incomplete agreements, rather than several agreements that are issue-specific, or only concern fractions of the surplus.

We employ here the model of focusing in economic choice proposed by Kőszegi and Szeidl (2013), in which the decision maker overweights attributes in which her options vary the most. Bushong, Rabin and Schwartzstein (2015) make the opposite assumption: that a decision maker *underweights* attributes in which her options vary

[6] The fact that the information set does not change between bargaining rounds does not imply that there is no uncertainty. However, an environment with uncertainty, perfect contracting, and no change in the information set across bargaining rounds is equivalent to an environment with perfect information. The reason is that the parties can bargain conditionally on the realization of a given state of the world.

[7] See Lang and Rosenthal (2001), Bac and Raff (1996), Inderst (2000), Busch and Horstmann (1999b), Busch and Horstmann (1999a), Flamini (2007).

the most. We develop our argument using Kőszegi and Szeidl (2013) because, as already discussed in the introduction, practitioners believe that reducing the range of possible outcomes on a dimension reduces the importance of this dimension within a negotiation (see Raiffa, 1982, p.216). Bordalo, Gennaioli and Shleifer (2013) also develop a model of salience. They assume that agents overvalue the attributes that differ the most with respect to a reference point. Hence, whereas the framework developed by Kőszegi and Szeidl (2013) can be directly used to describe how preferences change with the bargaining set, performing the same analysis using the framework developed by Bordalo, Gennaioli and Shleifer (2013) would require us to establish how changes in the bargaining set affect the reference point. Despite this modeling choice, we show in Section 3 that the central predictions of our model are robust to using other models of context-dependent preferences including Bushong, Rabin and Schwartzstein (2015) and Bordalo, Gennaioli and Shleifer (2013).

2 Example: a bargaining problem with 2 discrete issues

We start by considering a simple bargaining problem, in which two discrete issues are on the table, together with a continuous one. This problem corresponds, for example, to a supplier and a buyer having to agree on the number of items to be supplied and on a monetary payment. It may also represent two countries engaged in a negotiation in which some dimensions of the problem are discrete (e.g., whether the ownership of the Panama canal is transferred to the Panamanian government, whether the US maintains the right of use of the canal) and one is continuous (e.g., the number of US troops that will be stationed in Panama). For simplicity, we assume that only one player has behavioral preferences, while the other is rational.[8] The goal of this section is to show that the bargaining parties may impose a bound on the set of possible bargaining outcomes before agreeing on a specific bargaining outcome. This type of bounds are similar to the "threshold agreements" discussed in the introduction.

[8]In online Appendix I, we consider the same model described here but assume that both players' are behavioral. We show below that when only one player is behavioral, the parties may reach an agreement on a subset of the issues on the table, despite the fact that agreeing on all of them is the ex-ante efficient outcome (see Proposition 1). In online Appendix I, we show that when both players are behavioral the outcome of the negotiation may be inefficient because too many issues are included in the agreement. Beside this difference, the insights derived with a single behavioral player extend to two behavioral players.

Imposing these bounds may affect the number of issues that are included in the final agreement.

Consider two players a and b ("she" and "he" respectively), bargaining over $i \in \{1, 2\}$ discrete issues and a continuous issue t. Without loss of generality, we interpret t as a transfer, assumed from b to a. If there is an agreement on issue i, player a earns $-c$ and player b earns v_i, with $v_1 > v_2$. If there is no agreement on issue i, the status quo on that issue is maintained, and each player earns zero. We assume that an agreement including both issues is materially efficient, i.e. $v_1, v_2 > c$. Call $q_i \in \{0, v_i\}$ the outcome of the negotiation on issue i, which equals v_i if this issue is included in the final agreement and 0 otherwise. Call $\{q_1, q_2, t\}$ a *bargaining outcome* and $X = \{0, v_i\} \times \{0, v_2\} \times \mathbb{R}_0^+$ the *unconstrained bargaining set*, or the set of all possible bargaining outcomes. The players bargain in two periods. In the first period they negotiate over an incomplete agreement and in the second period they negotiate over the final outcome of the negotiation. There is no time discounting.

Definition 1. An *incomplete agreement* is a set $S \subset X$. A *negotiation structure* $\mathbb{S}$ is the collection of closed and convex S that can be chosen in period 1.

Because there is no time discounting, we interpret the case $S = X$ as a one-step negotiation: no restrictions are imposed on the bargaining set in period 1, and the players bargain over the entire X in period 2. In what follows we always assume that $X \in \mathbb{S}$, so that the players can always choose to bargain in one-period. Note also that the simplest possible type of incomplete agreement (and the one we consider below) is a cap $\hat{t}$ on transfer t. In this case, the set $\mathbb{S}$ contains all the sets that can be written as $S = \{0, v_1\} \times \{0, v_2\} \times [0, \hat{t}]$ for some $\hat{t} \geq 0$.

Assumption 1 (Binding incomplete agreements). *A bargaining outcome* $\{q_1, q_2, t\}$ *is a* feasible solution *to the bargaining problem in period 2 if and only if* $\{q_1, q_2, t\} \in S \cup \{0, 0, 0\}$.

Under this assumption, incomplete agreements are binding: during the last stage of a negotiation, the players can either agree on a bargaining outcome which is element of a previously-agreed S, or they can disagree.[9]

Here we restrict our attention to a specific form of context dependence, namely the focusing effect as modeled by Kőszegi and Szeidl (2013). We assume that both

[9] We relax this assumption in Section 4, where we allow players to renegotiate or ignore any prior incomplete agreement.

players' utilities are additive in the payoff earned on each issue and in the transfer. However, player b is behavioral and the weights he attaches to each issue and to the transfer depend on the set of possible bargaining outcomes. More precisely, player b's utility is:

$$U^b(q_1, q_2, t) = \sum_{i=1}^{2} h(\bar{q}_i - \underline{q}_i)q_i - h(\bar{t} - \underline{t})t,$$

where $h(\bar{q}_i - \underline{q}_i)$ and $h(\bar{t} - \underline{t})$ are the *focus weights*, and $h()$ is a strictly increasing function with $h(0) = 1$. The values of $\bar{q}_i$, $\underline{q}_i$, $\bar{t}$, $\underline{t}$ depend on the set of bargaining outcomes that player b considers possible, which we call the *consideration set*. In particular, $\bar{q}_i$ and $\underline{q}_i$ are the largest and smallest q_i in the consideration set; similarly $\bar{t}$ and $\underline{t}$ are the largest and smallest t in the consideration set. Therefore, the focusing effect causes player b to focus more on, and hence to overweight, the dimension of the bargaining problem with the largest difference in terms of possible bargaining outcomes. Note also that the focus-weighted utility is a *decision utility*, because it describes the decision maker's choice. We will contrast player b's decision utility with his *material utility* corresponding to a rational benchmark in which all the focus weights are equal to one.

Player a is, instead, fully rational. Her utility function is:

$$U^a(q_1, q_2, t) = t - c \cdot \sum_{i=1}^{2} \frac{q_i}{v_i}.$$

Assumption 2 (Consideration set)**.** *A bargaining outcome* $\{q_1, q_2, t\}$ *is in the consideration set if and only if it is feasible and both players satisfy their rationality constraint at* $\{q_1, q_2, t\}$.

In other words, the consideration set coincides with the bargaining set, and is composed of all the feasible bargaining outcomes which are preferred by both players to no agreement at all.[10] Note that finding the consideration set is a fixed point problem, because the focus weights determine player b's preferences, his rationality constraint and the shape of the consideration set. At the same time, the shape of the consideration set determines the focus weights and b's preferences.

Assumption 3. *If at any negotiation stage the parties disagree, the outcome is no agreement.*

[10] Clearly, other definitions of the consideration set are possible. However, we believe that Assumption 2 is the most reasonable way to define the consideration set. See the online Appendix II and III for a discussion.

Assumption 3 implies a form of commitment with respect to the structure of the negotiation. For example, players cannot meet again and negotiate after having failed to agree on an incomplete agreement $S \in \mathbb{S}$. Furthermore, it implies that, at any stage of the negotiation, the lower bounds of the consideration set are always given by the option of not agreeing, so that $\underline{t} = \underline{q}_i = 0$, $\forall i$.[11]

Assumption 4. *Each bargaining round is solved by Nash bargaining.*

Under this assumption, within each bargaining round and for given preferences, irrelevant alternatives do not affect the bargaining outcome and the standard Nash-bargaining axioms apply. Note however that preferences in our model are a function of the entire bargaining set, and therefore the solution to the entire bargaining problem is affected by irrelevant alternatives. Hence, our approach isolates a single channel through which incomplete agreements affect the outcome of the negotiation: the players' preferences.

We solve this example by restricting our attention to a specific type of incomplete agreement: a cap on the transfer t. This restriction allows us to show our main point: when given the option to impose a cap on t, the players may decide to do so.[12] We work backward and solve the model starting from period 2. We first look at the case in which no cap on transfer was imposed in period 1. Then we look at the case in which a cap on transfer was imposed in period 1. Finally, we move to period 1 and consider the choice of whether to impose a cap on transfer, and what cap to impose.

2.1 No cap on transfer in period 2 (one-step negotiation)

If no cap on transfers is imposed in period-1, in period 2 the players bargain over the entire bargaining set. Because there is no time discounting, we interpret this case as a one-step negotiation. By Assumption 2, "no agreement" is always in the consideration set, so that $\underline{q}_i = \underline{t} = 0$ for all i. Hence, a bargaining outcome $\{q_1, q_2, t\}$ is in the consideration set if and only if:

$$t \geq c \cdot \sum_{i=1}^{2} \frac{q_i}{v_i}$$

[11] In online Appendix IV, we discuss what happens if after disagreeing, the players have the option to initiate a new round of negotiations.

[12] We are more general in Section 4, in which we characterize the set of renegotiation-proof incomplete agreements and allow players to choose any agreement in this set.

$$\sum_{i=1}^{2} h(\bar{q}_i) q_i \geq h(\bar{t}) t,$$

To avoid issues of equilibrium multiplicity, we impose the following restriction:

$$v_1 \geq 2c \tag{2.1}$$

This restriction has two implications:

- Agreeing on both issues is in the consideration set. Define $\bar{t}_o$ as the largest transfer achievable in the one-step negotiation when both issues are included:

  $$\bar{t}_o \equiv \bar{t} : h(\bar{t})\bar{t} = \sum_{i=1}^{2} h(v_i) v_i. \tag{2.2}$$

  Under (2.1), we have that $\bar{t}_o \geq 2c$. Agreeing on both issues is a bargaining outcome that satisfies both players' rationality constraints, and therefore $\bar{q}_i = v_i$ for $i \in \{1, 2\}$. If instead (2.2) is violated, then only one (or no) issue can be in the consideration set, and the presence of the second one is irrelevant.

- There can only be two issues in the consideration set. Suppose only one issue is in the consideration set.If this is the case, player b utility is given by

  $$h(v_1)v_1 - h(\bar{t})t,$$

  and $\bar{t}$ solves

  $$h(v_1)v_1 = h(\bar{t})\bar{t},$$

  which implies $\bar{t} = v_1$. However, if $v_1 \geq 2c$, at transfer $t = v_1$, player a is willing to include both issues in the agreement. This is beneficial also for player b. Hence, it is not possible to have only one issue in the consideration set. A similar argument rules out that the largest possible agreement in the consideration set is no agreement.

Given $\bar{t}_o$ as in (2.2), player b's utility is

$$u^b(q_1, q_2, t) = \sum_{i=1}^{k} h(v_i) v_i - h(\bar{t}_o) t,$$

where k is the number of issues included in the agreement. The Nash bargaining problem is

$$\max_{t\leq\bar{t}_o,k\in\{0,1,2\}}\left(\sum_{i=1}^{k}h(v_i)v_i-h(\bar{t}_o)t\right)(t-ck)\,. \tag{2.3}$$

The solution is characterized by a transfer as a function of the number of issues included in the agreement:

$$t(k^\star)=\frac{1}{2}\left(\sum_{i=1}^{k^\star}\frac{h(v_i)}{h(\bar{t}_o)}v_i+ck^\star\right), \tag{2.4}$$

and the number of issues included in the agreement:

$$k^\star=\sharp\{v_i|v_i\cdot h(v_i)-c\cdot h(\bar{t}_o)>0\} \tag{2.5}$$

The key observation is that the number of issues that will be included in the agreement depends on the marginal focus-weighted surplus generated by each issue, that is $v_i\cdot h(v_i)-c\cdot h(\bar{t}_o)$. By (2.2), we know that the average focus-weighted surplus generated by both issues is positive. Despite this, the following proposition shows that the marginal focus-weighted surplus generated by the second issue can be negative, and hence the players may agree on only one issue. In this case, the outcome of the negotiation is materially inefficient.

Proposition 1.

- *If v_1 and v_2 are sufficiently close to each other, then both issues are included in the final agreement, that is $k^\star=2$. The equilibrium transfer is*

  $$\frac{1}{2}\left(\frac{h(v_1)}{h(\bar{t}_o)}v_1+\frac{h(v_2)}{h(\bar{t}_o)}v_2+c\right).$$

  The outcome of the negotiation is materially efficient.

- *If either v_1 is sufficiently large or v_2 is sufficiently close to c (or both), then only the first issue is included in the final agreement, that is $k^\star=1$. The equilibrium transfer is*

  $$\frac{1}{2}\left(\frac{h(v_1)}{h(\bar{t}_o)}v_1+c\right).$$

  The outcome of the negotiation is materially inefficient.

When one of the two issues is more important relative to the other, the minor issue may be left out of the agreement, despite the fact that it would be efficient to include both issues in the agreement. That is because one issue is much more salient than the other. As a consequence, transfers are much more salient than the secondary issue, which will be left out of the agreement.

Note that Kőszegi and Szeidl (2013) argue that, in a decision problem with focusing effect concentrating an option's advantages makes it more preferable, while concentrating its disadvantages makes it less preferable (Proposition 2). This result resonates with the one derived here: making the first issue more valuable relative to the second issue increases the chances that only the first issue is included in the agreement. However, contrary to the decision problem considered in Kőszegi and Szeidl (2013), here the salience of the transfer dimension is endogenous, that is, if either v_1 or v_2 (or both) increase, $\bar{t}_o$ increases as well. This implies that if only v_1 increases the second issue will eventually be considered too costly and won't be included in the final agreement.

2.2 Cap on transfer in period 2

Consider now the case in which a cap $\hat{t}$ was imposed in period 1. In period 2, the maximum possible transfer t is $\min\{\hat{t}, \bar{t}_o\}$, where $\bar{t}_o$ is the largest possible t in case no cap is imposed in period 1, and is defined in (2.2). Therefore, $\hat{t} > \bar{t}_o$ is equivalent to having no cap and solving the one-step negotiation problem in period 2.

The cap $\hat{t}$ determines the number of issues in the players' consideration set $k(\hat{t})$, which is given by:[13]

$$k(\hat{t}) = \begin{cases} 2 & \text{if } 2c \leq \hat{t} \\ 1 & \text{if } c \leq \hat{t} < 2c \\ 0 & \text{otherwise.} \end{cases}$$

Note that the number of issues that are in the consideration set increases with the

[13] A given $\{k, \hat{t}\}$ is in the consideration set if both players satisfy their rationality constraints at $\{k, \hat{t}\}$. However, because of (2.1) and (2.2) we can ignore the rationality constraint of player b. If player b's rationality constraint is satisfied at $\{k = 2, t = \bar{t}_o\}$, it is also satisfied for $k = 2$ and any $t \leq \hat{t}$. Hence, $k = 2$ is in the consideration set if, and only if, there is a transfer that satisfies player a's rationality constraint, that is if, and only if, $\hat{t} \geq 2c$. Similarly, if only one issue is in the consideration set, player b's utility is

$$h(v_1)v_1 - h(\hat{t})t$$

which implies that the player b's rationality constraint is satisfied at transfer $t = \hat{t}$ as long as $\hat{t} \leq v_1$. Again, because $v_1 > 2c$ whether agreeing on one issue and transfer $\hat{t}$ is in the consideration set depends on player a's rationality constraint.

cap. Given this, player b's utility is

$$u^b(q_1, q_2, t) = \sum_{i=1}^{k(\hat{t})} h(v_i)v_i - h(\min\{\hat{t}, \bar{t}_o\})t.$$

Note that, for given $k(\hat{t})$, a cap $\hat{t} < \bar{t}_o$ reduces player b's focus on transfer t. We call this a "peace of mind" effect, because knowing that large transfers are excluded from the set of possible outcomes decreases the salience of this dimension and player b's sensitivity to transfers.

As in the previous case, also here the Nash bargaining solution is characterized by a transfer as a function of the number of issues included in the agreement:

$$t(\hat{t}, k^\star(\hat{t})) = \min\left\{\frac{1}{2}\left(\sum_{i=1}^{k^\star(\hat{t})} \frac{h(v_i)}{h(\min\{\hat{t}, \bar{t}_o\})} v_i + ck^\star(\hat{t})\right), \hat{t}\right\}, \tag{2.6}$$

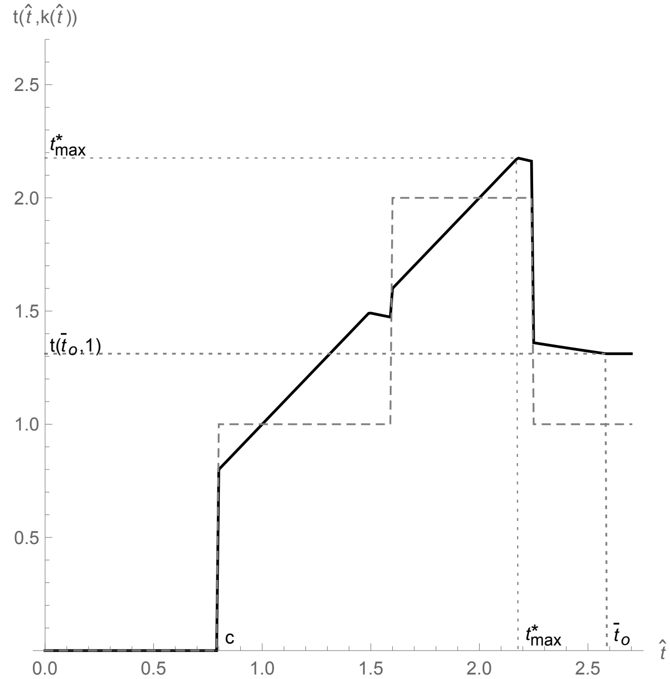
and the number of issues included in the agreement:

$$k^\star(\hat{t}) = \min\{\sharp\{v_i | h(v_i)v_i - h(\min\{\hat{t}, \bar{t}_o\})c \geq 0\}, k(\hat{t})\}. \tag{2.7}$$

Therefore, $\hat{t}$ affects the outcome of the negotiation in three ways. First, the cap could be binding and determine the equilibrium transfer. Second, the number of issues included in the agreement cannot exceed the number of issues in the consideration set $k(\hat{t})$. These two effects cause both the equilibrium transfer and the equilibrium number of issues in the agreement to decrease when $\hat{t}$ is reduced. Third, because of the "peace of mind" effect, the salience of the transfer dimension is reduced when a lower cap is imposed. This last effect may countervail the first two, and cause the equilibrium transfer and the number of issues included in the agreement to increase when $\hat{t}$ is reduced.

Figure 2.1 plots equation 2.6 and 2.7 as a function of $\hat{t}$. Note that for high $\hat{t}$, $t(\hat{t}, k^\star(\hat{t}))$ increases as $\hat{t}$ decreases due to the "peace of mind" effect. For $\hat{t}$ sufficiently low, $\hat{t}$ is binding and therefore $t(\hat{t}, k^\star(\hat{t})) = \hat{t}$. This pattern repeats for every $k(\hat{t})$. Note also that $\hat{t}$ has a non-monotonic effect on $k^\star(\hat{t})$, which is first increasing—when $k(\hat{t})$ is binding—and then decreasing—again because of the "peace of mind" effect.

We call $t^\star_{\max}$ the highest possible transfer achievable in period 2 via a cap on t,



The period-2 transfer $t(\hat{t}, k^{\star}(\hat{t}))$ (solid black line) and the period-2 number of quality levels $k^{\star}(\hat{t})$ (dashed gray line) as a function of the cap on transfer $\hat{t}$ set in period 1. $t(\hat{t}, k^{\star}(\hat{t}))$ reaches its maximum at $t^{\star}_{\max} = t(t^{\star}_{\max}, 2)$. The equilibrium transfer in the one-step negotiation case equals $t(\bar{t}_o, 1)$. Parameter values are $v_1 = 2$, $v_2 = 1$, $c = 0.8$ and $h(x) = x/4 + 1$. $t(\bar{t}_o, 1) = 1.3115$, $t^{\star}_{\max} = 2.1762$, $\bar{t}_o = 2.5826$ and $\bar{k} = 2$.

Figure 2.1: Period–2 transfer as a function of the cap on transfer

implicitly defined as

$$\frac{1}{2}\left(\sum_{i=1}^{k(t^{\star}_{\max})} \frac{h(v_i)}{h(t^{\star}_{\max})} v_i + ck(t^{\star}_{\max})\right) = t^{\star}_{\max}. \tag{2.8}$$

By definition, the transfer $t^{\star}_{\max}$ is achievable by imposing a cap $\hat{t} = t^{\star}_{\max}$. For future reference, note also that $t^{\star}_{\max} < \bar{t}_o$, that is, the largest transfer achievable via a cap on transfer is smaller than the largest possible transfer in the one-step negotiation (defined in 2.2).

The next proposition shows that whether at $\hat{t} = t^{\star}_{\max}$ both issues are included in the agreements depends on c and v_2. On the one hand, c determines the strength of the "peace of mind" effect, that is, how tight can the cap on the transfer be before violating player a's rationality constraint, and therefore how small player b's focus weight on transfer can be. On the other hand, v_2 determines whether a given reduction in player b's focus weight on transfer will lead to the inclusion of the second issue in the agreement.

Lemma 1. • *If* $c \cdot h(2c) \leq v_2 \cdot h(v_2)$*, then* $k^\star(t^\star_{\max}) = 2$*, i.e., when the cap is equal to* $t^\star_{\max}$ *the number of issues included in the agreement is 2.*

- *If instead* $c \cdot h(2c) > v_2 \cdot h(v_2)$*, then* $k^\star(t^\star_{\max}) = 1$*, i.e., when the cap is equal to* $t^\star_{\max}$ *the number of issues included in the agreement is 1.*

We showed in the previous section that in the one-step negotiation the number of issues included in the agreement is 1 whenever v_1 is sufficiently large, which is materially inefficient. The above lemma is relevant because it implies that, if the cost c is sufficiency small so that $c \cdot h(2c) \leq v_2 \cdot h(v_2)$, an appropriate cap on transfers can induce the materially-efficient outcome. On the other hand, if $c \cdot h(2c) > v_2 \cdot h(v_2)$, then the materially efficient outcome cannot be achieved via a cap on transfer. In this case, relative to no cap, imposing a cap $\hat{t} = t^\star_{\max}$ has purely the effect of increasing the transfer from player b to player a .

A useful corollary to the above lemma is the following (the details of which are in the proof of Lemma 1):

Corollary 1. $k^\star(t^\star_{\max}) = \max_{\hat{t}}\{k^\star(\hat{t})\}$*, that is, the number of issues included in the agreement is the largest possible when* $\hat{t} = t^\star_{\max}$*.*

Hence, when $\hat{t} = t^\star_{\max}$ both the transaction and the number of issues included in the agreement are the largest possible.

2.3 Period 1

We now examine which cap on transfer will be implemented in period 1. We consider three different cases. First, as an illustration, we consider a *unilateral pre-bargaining concession*, in which the cap $\hat{t}$ is not the outcome of a bargaining process but is rather chosen by player a before the negotiation starts. Second, we assume that the cap $\hat{t}$ is the outcome of a negotiation in which player b has rational preferences, that is, player b is in a "cold state" in period 1. Finally, we assume that the cap $\hat{t}$ is the outcome of a negotiation in which player b's preferences are distorted by the focusing effect, that is, player b is in a "warm state" in period 1.

Unilateral pre-bargaining concession Suppose that the negotiation happens in one step, but player a can make a pre-negotiation *concession*, i.e., she can credibly announce a $\hat{t}$. This corresponds, for example, to the seller of a car announcing a price, with the understanding that this price can be negotiated downward. From Lemma 1, it follows that player a will announce $\hat{t} = t^{\star}_{\max}$: the cap on transfer leading to the largest possible transfer. Note also that, by definition of $t^{\star}_{\max}$, in equilibrium the final transaction will be equal to player a's announcement.

Whenever $c \cdot h(2c) > v_2 h(v_2)$ the only effect of the announcement is to increase player a's payoff by giving "peace of mind" to player b: there is no effect on the number of issues included in the agreement and on material efficiency. It follows quite immediately that player a maximizes the transfer received by announcing $t^{\star}_{\max}$.

If instead $c \cdot h(2c) \leq v_2 \cdot h(v_2)$ announcing $\hat{t} = t^{\star}_{\max}$ generates higher material surplus than no announcement, because it allows both issues to be included in the agreement. Hence, player a is effectively choosing between agreeing on both issues (for a cost $2c$) and receive the largest possible transfer, and agreeing on only one issue (for a cost c) and receive a lower transfer. Note, however, that the second issue will be included in the agreement if its focus-weighted surplus is positive (see 2.7). Because focus-weighted surplus generated by the second issue is always smaller or equal to its material surplus, player a prefers to include both issues in the final agreement. In this case, the possibility of making a unilateral pre-bargaining concession increases the material surplus achieved in the negotiation.

Bargaining over $\hat{t}$, cold state Consider now the case in which both players bargain over $\hat{t}$ in period 1. Furthermore, assume that in period 1, player b is in a *cold state*, that is, her preferences are rational and her utility function is:

$$u^b(q_1, q_2, t) = \sum_{i=1}^{2} q_i - t$$

However, player b anticipates that in period-2 her preferences will be distorted by the focusing effect.

The change in preferences between period-1 and period-2 could be due to several factors. For example, no matter what is agreed in period-1, in period-2 player b can always trigger the disagreement option. Therefore, when bargaining in period 1, player b may not feel under pressure and behave rationally. However, period 2 is the very last moment in which player b can walk out of the negotiation, which may cause her preferences to be distorted.

Both players correctly anticipate the bargaining outcome in period 2 as a function of the cap on transfer imposed in period 1, which is given by (2.6) and (2.7). The optimal cap on transfer $\hat{t}^*$ maximizes the Nash product:

$$\max_{\hat{t}} \left(\sum_{i=1}^{k(\hat{t})} v_i - t(\hat{t}, k(\hat{t})) \right) \left(t(\hat{t}, k(\hat{t})) - ck(\hat{t}) \right). \tag{2.9}$$

It follows immediately that the players will use the cap to push the outcome of the negotiation closer to the outcome that would be achieved by rational players, that is, the outcome that achieves the highest Nash product with rational preferences. By noting that $\frac{v_1+v_2+2c}{2}$ is the transfer that solves the unconstrained Nash bargaining problem when both issues are included in the agreement, and that $\frac{v_1+c}{2}$ solves the same problem when only the first issue is included in the agreement, we achieve the following result:

Proposition 2. *Assume that player b is in a cold state in period 1.*

- *If $c \cdot h(2c) \leq v_2 \cdot h(v_2)$, in period 1 the players agree to impose a cap on transfers equal to $\min\left\{\frac{v_1+v_2+2c}{2}, t^\star_{\max}\right\}$. In period 2 both issues are included in the agreement and transfers are equal to $\min\left\{\frac{v_1+v_2+2c}{2}, t^\star_{\max}\right\}$.*
- *If $c \cdot h(2c) > v_2 \cdot h(v_2)$, then the players imposes a cap on transfers equal to $\hat{t} = v_1$. In period 2 only one issue is included in the agreement and transfers are equal to $\frac{v_1+c}{2}$.*

Hence, incomplete agreements are used in equilibrium, which is our main result. Incomplete agreements are used to impose on period-2 players the outcome that is optimal from period-1 point of view, under the constraint that this outcome must be achievable via a cap on the transfer. This is done by setting a constraint on the future negotiation. Incomplete agreements are, therefore, akin to commitment devices. The main difference with respect to commitment devices is that an incomplete agreement may be effective in manipulating the outcome of the negotiation also when it is not binding (as in the case $c \cdot h(2c) > v_2 \cdot h(v_2)$ of the proposition). The reason is that incomplete agreement have an effect on the players' preferences.

By manipulating $\hat{t}$, the players manipulate the future preferences of player b and align them with player b's present preferences, i.e., make them closer to rational. This alignment of preferences is beneficial to player a as well, because it leads to higher transfers from player b. Furthermore, if $c \cdot h(2c) \leq v_2 \cdot h(v_2)$, imposing a cap on transfers also increases the number of issues included in the agreement, and with it the material efficiency of the negotiation. Note that this was the rationale for using "threshold agreements" during the Panama Canal negotiations (see the Introduction).

Bargaining over $\hat{t}$, warm state. There are a number of objections that can be made to the assumption that player b is rational in period 1. For example, all negotiations (whether on $\hat{t}$ or on t and k) may be heated, conducted under pressure, or conducted by committee which first have to agree among themselves and then with the other party. In these cases, the players' preferences may be far from rational in period 1.

For this reason, we now assume that player b's preferences are distorted by the focusing effect also in period-1. Following Kőszegi and Szeidl (2013), we assume that player b is fully *consequential* in the sense that—consciously or unconsciously—he reasons backward and evaluates each possible future bargaining set by the bargaining outcome achieved in case this bargaining set is reached. In our example, this implies that player b understands that to every $\hat{t}$ which may be imposed there is a corresponding bargaining outcome given by equation (2.6) and (2.7).

Therefore, player b's period-1 context is given by the set of k and t that are achievable as a function of $\hat{t}$. In particular, by Corollary 1 $\hat{t} = t^{\star}_{\max}$ maximizes both t and k. It follows that player b's period-1 utility is

$$u^b(q_1(\hat{t}), q_2(\hat{t}), t(\hat{t})) = \sum_{i=1}^{k(t^{\star}_{\max})} h(v_i)q_i(\hat{t}) - h(t^{\star}_{\max})t(\hat{t}),$$

The key observation is that because $t^{\star}_{\max} < \bar{t}_o$, when bargaining over $\hat{t}$ player b is less sensitive to t than in the one-step negotiation case. Bargaining over $\hat{t}$ *by itself* eliminates from the set of possible bargaining outcomes extremely high transfers and reduces player b's period-1 sensitivity to the transfer, even before a transfer cap is actually imposed. That is, the possibility of restricting the future bargaining set (which is exogenously given) affects player b's preferences in period 1.

Because the cap matters only to the extent that it delivers a specific final transfer, bargaining in period 1 over $\hat{t}$ is equivalent to bargaining over t under the restriction that t and k can be achieved by setting a specific $\hat{t}$. Therefore, the solution to the period-1 bargaining problem is:

$$\frac{1}{2}\left(\sum_{i=1}^{k(t^\star_{\max})}\frac{h(v_i)}{h(t^\star_{\max})}v_i + c\cdot k(t^\star_{\max})\right).$$

which, by equation (2.8), is equal to $t^\star_{\max}$.

Proposition 3. *Assume that the parties bargain first over the transfer cap and then agree on the transfer. The final transfer $t^\star$ is equal to $t^\star_{\max}$. The number of issues included in the agreement is 2 whenever $c\cdot h(2c)\le v_2\cdot h(v_2)$, and 1 otherwise.*

Proof. The proof follows simply from the fact that when $\hat{t}=t^\star_{\max}$, player-b period-1 preferences are equal to player-b period-2 preferences, period-1 Nash bargaining problem is equal to period-2 Nash bargaining problem, and the solution to the negotiation in period 2 also solves period-1 bargaining problem. □

Also here the players strictly prefer to restrict their future bargaining possibilities using an incomplete agreement. Similar to the intuition discussed in the "cold state" case, the players use incomplete agreements to align preferences between period 1 and period 2. Both in warm and cold state, whenever $c\cdot h(2c)\le v_2\cdot h(v_2)$ in equilibrium the two players will agree on both issues, which is the materially efficient outcome. Otherwise, only one issue will be included in the agreement, which is materially inefficient.

The difference between the cold state and the warm state is in the equilibrium transfer. In the warm state, the equilibrium transfer is $t^\star_{\max}$, which is (weakly) greater than the equilibrium transfer in the cold state. When $c\cdot h(2c)\le v_2\cdot h(v_2)$, this is easily verified by comparing Propositions 2 and 3. When $c\cdot h(2c)>v_2\cdot h(v_2)$, it can be shown that $\frac{v_1+c}{2}<t^\star_{\max}=\frac{(h(v_1)/h(t^\star_{\max}))v_1+c}{2}$, and therefore the transfer in the warm state is strictly greater than the transfer in the cold state.[14] Hence, in equilibrium, the distortion in player b's period-1 preferences causes him to make a larger transfer for the same outcome.

This result is robust to player b failing to correctly anticipate how the future transfer will depend on $\hat{t}$ (i.e., the mapping between t and $\hat{t}$ is different from equation 2.6). This could be because player b is not fully sophisticated with respect to his future focused preferences. As long as player b believes that the largest t that is achievable

[14] This follows from the fact that the period-2 rationality constraint of player b is $v_1 h(v_1) - t\cdot h(t^\star_{\max})>0$, which is satisfied at the equilibrium $t=t^\star_{\max}$ if and only if $t^\star_{\max}<v_1$.

by imposing a cap is below $\bar{t}_o$, when bargaining over $\hat{t}$ he will be less sensitive to the transfer dimension than when negotiating in one period. As a consequence, he is willing to restrict the future bargaining set by imposing a cap. In addition, Proposition 3 also holds whenever player b is fully *naïve* with respect to his focused preferences, and thinks that t does not depend on $\hat{t}$. In this case, player b is indifferent between all possible $\hat{t}$, while player a strictly prefers $t^{\star}_{\max}$.

2.4 Discussion

We showed that when the players negotiate in one step the outcome of the negotiation may be inefficient because it may not include the second issue. This happens whenever the first issue is much more important to player b relative to the other issue. Imposing an incomplete agreement in the form of a cap on transfers then leads to a reduction in player b sensitivity to transfers and affects the outcome of the negotiation in two ways. First, it may increase the material efficiency of the negotiation, by allowing the parties to reach an agreement that includes both issues. Second, it increases the transfers made from player b to player a. This increase is larger if player b is in a warm state than in a cold state in period 1.

The remainder of the paper is dedicated to exploring the robustness of these results. In Section 3, we consider a general bargaining game with generic context-dependent preferences and show that the main insights derived here generalize to that environment. Instead, in Section 4 and in the online Appendix, we show that some of the results derived are specific to the framework considered here. For example, here we assumed that the only type of incomplete agreement the parties can sign is a cap on the transfer, and that this cap is binding. In the Section 4, we assume that players can renegotiate and even ignore a previous incomplete agreement, and allow them to sign any renegotiation-proof incomplete agreement. Furthermore, contrary to the model presented here, in Section 4 the one-step negotiation already achieves the materially efficient outcome. Hence, incomplete agreements are never used if player b is in a cold state in period 1. However, they may be used if player b is in a warm state. The use of incomplete agreements therefore reduces the material efficiency of the negotiation.

3 General Bargaining Problem with Context-Dependent Preferences

Consider the following generalization of the problem analyzed in the previous section. Two players a and b bargain over an outcome $x \in X$. The players can sign an incomplete agreement $S \in \mathbb{S}$ in period 1, and choose an $x \in S$ in period 2. In case of disagreement during either period 1 or period 2, the outcome is the outside option. As before, we assume that $X \in \mathbb{S}$, that is the players can always choose to bargain over the entire bargaining set in period 2.

Rational players When players are rational, for a given incomplete agreement S, in period 2 the bargaining problem is:

$$\max_{x \in S} U^a(x) \cdot U^b(x).$$

Because the players' preferences do not change across periods, in period 1 the bargaining problem is:

$$\max_{S \in \mathbb{S}} \left[\max_{x \in S} U^a(x) \cdot U^b(x) \right],$$

which is maximized for the largest possible S, which is X itself.

Remark 1. *Suppose both players are rational. Suppose, furthermore, that the players have the option to restrict their bargaining set by signing an incomplete agreement. The players can never do better than bargaining in one step.*

Cold state Suppose now that, in period 2, both players have context-dependent preferences of the form $U^a(x, \hat{X})$, $U^b(x, \hat{X})$, where $\hat{X}$ is the players' consideration set defined as:

$$\hat{X} = \left\{ x \in X | \ U^a(x, \hat{X}) \geq 0, \ U^b(x, \hat{X}) \geq 0 \right\}. \tag{3.10}$$

We call the players' outside option $0 \in X$, and normalize its utility to zero, so that $U^a(0, \hat{X}) = U^b(0, \hat{X}) = 0$ for all possible $\hat{X}$. Because we leave the form of context dependence completely unspecified, we cannot address here issues of existence and uniqueness of the consideration set $\hat{X}$. We therefore simply assume that $\hat{X}$ exists and is unique for any bargaining set X. The solution to the bargaining problem over X is:

$$x(X) = \text{argmax}_{x \in X} U^a(x, \hat{X}) U^b(x, \hat{X}),$$

which we also assume to exist and to be unique.

Define set of anticipated bargaining outcomes $\mathbb{C}$ as:

$$\mathbb{C} \equiv \{x(S)|S \in \mathbb{S}\}.$$

Using the fact that, in period 1, preferences are rational, we can write the period-1 bargaining problem as:

$$\max_{x\in\mathbb{C}} U^a(x) \cdot U^b(x),$$

that is, among the outcomes achievable via an incomplete agreement, choose the one that maximizes the Nash product with rational preferences. It follows quite immediately that an incomplete agreement will be imposed if it leads to an outcome that achieves a larger Nash product than the outcome achieved in the one-step negotiation. In this sense, incomplete agreements are used to move the outcome of the negotiation "closer" to the outcome that would be achieved by fully rational players. The next proposition formalizes this intuition.

Proposition 4. *In period 1, the players will agree on the $S \in \mathbb{S}$ that maximizes the Nash product with rational preferences, that is:*

$$S : x(S) = argmax_{x\in\mathbb{C}} U^a(x) \cdot U^b(x)$$

It follows that, if there exist an $\mathbb{S}$ such that $\max_{x\in\mathbb{C}} U^a(x) \cdot U^b(x) > U^a(x(X)) \cdot U^b(x(X))$, the players will strictly prefer to restrict their bargaining set by imposing an incomplete agreement in period 1.

Proof. In the text. □

Warm state When preferences are context dependent on period 1 as well, the set $\mathbb{C}$ determines the period-1 consideration set $\hat{\mathbb{C}}$. It follows that the period-1 bargaining problem has a solution given by:

$$x(\mathbb{C}) = \text{argmax}_{x\in\mathbb{C}} U^a(x, \hat{\mathbb{C}}) U^b(x, \hat{\mathbb{C}}).$$

We want to establish under what conditions the two players choose in period 1 to restrict their future bargaining set, i.e., under what conditions $x(\mathbb{C}) \neq x(X)$.

To start, note that for $x(\mathbb{C}) \neq x(X)$ to hold, it must be the case that $\hat{\mathbb{C}} \neq \hat{X}$, i.e., the period-1 consideration set differs from the unconstrained consideration set.

Note that $\hat{\mathbb{C}} \neq \hat{X}$ holds whenever there are some bargaining outcomes in $\hat{X}$ that cannot be achieved as a solution to the bargaining problem under any $S \in \mathbb{S}$. Second, the fact that $\hat{\mathbb{C}} \neq \hat{X}$ may or may not imply that the players' preferences will be different under the two consideration sets. This will depend on the specific form of context-dependent preferences. Finally, even if preferences are different under the two consideration sets, the players will agree on an incomplete agreement only if there exists an $S \in \mathbb{S}$ delivering an outcome that is jointly preferred to the outcome in case of no incomplete agreement. This will be the case if, for example, $\mathbb{C}$ itself is available in period 1 (i.e., $\mathbb{C} \in \mathbb{S}$). When $\mathbb{C}$ is chosen, period-2 players' context (and preferences) are identical to period-1 players' context (and preferences). Hence, by choosing $\mathbb{C}$ in period 1, the period-2 problem becomes identical to the period-1 problem, and whatever bargaining outcome is chosen in period 2 also solves the period-1 bargaining problem. It follows that, in period 1, if $\mathbb{C}$ is available, it will be chosen over all other available options (including the unconstrained bargaining set X). In such cases, an incomplete agreement will be used in equilibrium to shrink the bargaining set from X to $\mathbb{C}$.

To conclude this section, note that the set of incomplete agreements $\mathbb{S}$ affects the players' preferences in period 1, the choice of incomplete agreements and the final bargaining outcome (cf. also our extension in the next section). Hence, even if for some $\mathbb{S}$ the players will not want to progressively restrict their bargaining set, for some other $\mathbb{S}'$ they may decide to do so. The next proposition formalizes this intuition.

Proposition 5. *Suppose that preferences are context dependent, in the sense that there exist a set $D \subset X$ with $x(X) \in D$ such that $x(X) \neq x(D)$ (that is, when the bargaining set changes from X to D, preferences and bargaining solution change as well). There exist an $\mathbb{S}$ such that the players will strictly prefer to restrict their bargaining set by imposing an incomplete agreement in period 1.*

Proof. Define $\mathbb{S}$ as $\{D, X\}$ plus all singletons in D. In this case $\mathbb{C} = D \subset X$, which implies that $x(X) \neq x(\mathbb{C}) = x(D)$ and the players strictly prefer D to X in period 1, so to make the period-2 problem identical to the period-1 problem. Hence, the players will restrict their bargaining set. □

4 Extension: non-binding incomplete agreements.

As an extension, consider now a variation on the simple bargaining problem discussed in Section 2. There is only a single issue on the table together with a transfer, but the value of an agreement on this issue depends on a continuous, costly action taken by player a. Utilities are:

$$U^a(q,t) = t - \frac{1}{2}q^2, \tag{4.1}$$

$$U^b(q,t) = h(\bar{q} - \underline{q})q - h(\bar{t} - \underline{t})t, \tag{4.2}$$

where $q \geq 0$ is the *quality* of the agreement and $t \geq 0$ is, again, a transfer from player b to player a. Hence, the set of unconstrained bargaining outcomes is $X = \mathbb{R}^2_+$, with $\{0,0\}$ corresponding to no agreement. We maintain Assumptions 2, 3 and 4. Among other things, these assumptions imply that, also here, $\underline{t} = \underline{q} = 0$.

With endogenous quality q the issue of whether—and to what extent—incomplete agreements are binding is particularly important. For example, suppose that a previous incomplete agreement imposes a cap on the transfer t. It is possible that, in period 2, both players are willing to ignore this cap provided that the quality of the agreement is sufficiently high. In other words, it is possible that an incomplete agreement may be renegotiated.

In this section, we assume that previous agreements can be renegotiated or even ignored. Crucially, we assume that ignoring a previous incomplete agreement is possible only by waiting one period (at no cost) and bargaining over the entire bargaining set in a third period. Hence, incomplete agreements are not binding in the sense that they do not affect the set of possible bargaining outcomes achievable within the negotiation. However, incomplete agreements affect the set of possible bargaining outcomes achievable in period 2. The following assumption formalizes this intuition and replaces Assumption 1.

Assumption 5 (Renegotiation). *For given S, a bargaining outcome $x \equiv \{q,t\} \in X$ is feasible if and only if at least one of the following conditions holds:*

- $x \in S \cup \{0,0\}$,
- $U^b(x) \geq U^b(x')$ *and* $U^a(x) \geq U^a(x')$ *for some* x' *element of the Pareto frontier of* S,
- x *is the solution of the bargaining problem over the entire* X.

As earlier, in period 2, players can choose a bargaining solution that is in compliance with the period-1 agreement, or disagree. Novel to Assumption 5, the players can now renegotiate a period-1 agreement by implementing a bargaining outcome that dominates the Pareto frontier of S, or completely ignore the period-1 agreement and go to a fresh round of negotiations. This last possibility amounts to renegotiating the structure of the negotiation, and moving to a one-step negotiation, even if the players are currently engaged in a two-step negotiation. The possibility of ignoring the period-1 agreement implies that, from the viewpoint of period 2, one of the outcomes of the negotiation is the solution to the one-step bargaining problem. In other words, independent of the period-1 incomplete agreement, the players can always choose in period 2 to implement the bargaining outcome x, the solution of the bargaining problem over the entire X.[15]

In the following, we restrict our attention to period-1 agreements that are closed, convex and renegotiation proof: that is, for every $S \in \mathbb{S}$, no element of X that is not in S can Pareto improve upon the Pareto frontier of S. We will characterize the set of renegotiation-proof incomplete agreements, and show that they may involve establishing minimum utility guarantees to the players. In addition, we show that the structure of the negotiation—that is, the specific set of incomplete agreements $\mathbb{S}$ available to the players in period 1—determines the outcome of the negotiation.

4.1 One-step negotiation

In the absence of a previous incomplete agreement, the consideration set is given by the set of $\{q, t\}$ that satisfy the players' rationality constraints. Therefore, the upper bound of the consideration set is given by the specific $\{q, t\}$ which satisfies the two rationality constraints with equality:

$$h(\bar{q})\bar{q} = h(\bar{t})\bar{t} \tag{4.3}$$

$$\frac{\bar{q}^2}{2} = \bar{t}, \tag{4.4}$$

[15] The fact that the third round of negotiations can be triggered only by mutual agreement is not essential to our results but simplifies our derivations because the outside option of the players is always no agreement. In case each player can trigger a third round of negotiations, the outside option when bargaining in period 2 is the outcome of the one-step negotiation. In online Appendix IV, we consider this problem, and show that incomplete agreements may nonetheless be used in equilibrium.

which implies $\bar{q} = \bar{t} = 2$. Given the identical focus weights, here the one-shot bargaining outcome is equivalent to the outcome when player b is rational: $t^\star = \frac{3}{4}$, $q^\star = 1$. This outcome maximizes the material surplus of the negotiation.

It follows immediately that, if player b is in a cold state in period 1, no incomplete agreement will be imposed in period 1 and the surplus-maximizing outcome is achieved by negotiating in one period. For this reason, in the remainder of this section we consider exclusively the "warm state" case.

4.2 The set of renegotiation-proof incomplete agreements

Consider a closed, convex incomplete agreement S, such that all elements of S satisfy the players' rationality constraints.[16] For any such S, there always exist a cap on transfer $\hat{t} \geq 0$, a cap on quality $\hat{q} \geq 0$, and minimum utility levels for both players $\underline{u}^a, \underline{u}^b \geq 0$ such that $\forall \{t, q\} \in S$:

$$h(\bar{q})q - h(\bar{t})t \geq \underline{u}^b \tag{4.5}$$

$$t - \frac{1}{2}q^2 \geq \underline{u}^a \tag{4.6}$$

$$\hat{t} = \max\{t | \{t, q\} \in S \text{ for some } q\} \tag{4.7}$$

$$\hat{q} = \max\{q | \{t, q\} \in S \text{ for some } t\}, \tag{4.8}$$

where (4.5) and (4.6) hold with equality for some $\{t, q\} \in S$.

Remember that a given S can be renegotiated in period 2 if a bargaining outcome on the Pareto frontier of S can be renegotiated, independent of whether or not this outcome is chosen in equilibrium. It follows that an S is renegotiation proof if it includes the elements of the Pareto frontier of X which give at least utility $\underline{u}^a$ to player a and utility $\underline{u}^b$ to player b. When this happens, the Pareto frontier of S coincides with the Pareto frontier of X (excluding the points violating the minimum utility guarantees), and therefore S cannot be renegotiated.

The next proposition shows that this condition is also sufficient to characterize the set of renegotiation-proof S. That is, an S is renegotiation proof if and *only if* it includes the elements of the Pareto frontier of X which give at least utility $\underline{u}^a$ to player a and utility $\underline{u}^b$ to player b. We use the fact that if S will not be renegotiated

[16]Considering only this class of incomplete agreements is without loss of generality, because the elements of S that do not satisfy one of the players' rationality constraints are not part of the period-2 consideration set, and are therefore irrelevant.

in period 2, then the upper bounds of the consideration set are $\bar{q} = \max\{\hat{q}, 1\}$ and $\bar{t} = \max\{\hat{t}, \frac{3}{4}\}$ (remember that the solution to the one-step negotiation can always be reached by waiting one period).

Proposition 6. *The set of $\underline{u}^a$, $\underline{u}^b$, $\hat{t}$, $\hat{q}$ that satisfy the following conditions fully characterizes the set of renegotiation-proof, convex incomplete agreement:*

$$\hat{t} + \frac{\underline{u}^b}{h(\max\{\hat{t}, \frac{3}{4}\})} \geq \left(\frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})}\right)^2 \tag{4.9}$$

$$\hat{q} \geq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})} \tag{4.10}$$

$$\frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})}\hat{q} \geq \hat{t} + \frac{\underline{u}^b}{h(\max\{\hat{t}, \frac{3}{4}\})} \tag{4.11}$$

$$\hat{t} - \frac{1}{2}\hat{q}^2 \geq \underline{u}^a. \tag{4.12}$$

The first two conditions of the proposition state that the caps on transfer and quality $\{\hat{t}, \hat{q}\}$ should be larger than the largest transfer and quality $\{t, q\}$ on the Pareto frontier of X, giving at least utility $\underline{u}^b$ to player b. The last two conditions are the rationality constraints of the players at $\{\hat{t}, \hat{q}\}$.

Next, we derive the solution to the two-step negotiation game under three negotiation structures. First, we assume that, in period 1, the players cannot use minimum utility guarantees, so that the set of possible period-1 agreements is the set of renegotiation-proof agreements with $\underline{u}^a = \underline{u}^b = 0$. Second, we allow the players to also bargain over the minimum utility guaranteed to player b. Third, we allow the players to choose in period 1 any S that is renegotiation-proof and convex. The point that we want to stress here is that the structure of the negotiation is relevant for material efficiency, because the q agreed upon depends on the set of incomplete agreements that can be signed in period 1.

Finally, in order to simplify our derivations, for the remainder of this section we assume that the focusing function has an exponential form: $h(x) = x^\gamma$ for some $\gamma > 0$.

4.3 Solution for different negotiation structures

No minimum utility guarantees We start by assuming that the players cannot use minimum utility guarantees, and therefore bargain in period 1 only over the cap on transfer and quality $\hat{t}, \hat{q}$, under the constraints (4.9) to (4.12) with $\underline{u}^a = \underline{u}^b = 0$. Note that, also here, by setting sufficiently large $\hat{t}, \hat{q}$ the players will bargain over the entire bargaining set in period-2. Hence the choice of whether to impose an incomplete agreement is fully endogenous.

Again, we solve the two-step bargaining problem by backward induction. First, for every focus weight ratio $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$ which satisfies conditions (4.9) to (4.12) with $\underline{u}^a = \underline{u}^b = 0$, we derive the period-2 Nash bargaining solution. It is easy to verify that this solution is

$$q^\star = \frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}, \tag{4.13}$$

$$t^\star = \frac{3}{4}\left(\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}\right)^2. \tag{4.14}$$

Hence, the period-1 focus weights are determined by the highest quality $\bar{q}$ and transfer $\bar{t}$ that satisfy (4.13) and (4.14) for a renegotiation-proof $\{\hat{t}, \hat{q}\}$. Given these focus weights, the period-1 bargaining problem is:

$$\max_{\alpha}\left\{ \left(h(\bar{q})q^\star(\alpha) - h(\bar{t})t^\star(\alpha)\right)\left(t^\star(\alpha) - \frac{1}{2}q^\star(\alpha)^2\right) \text{ s.t. (4.9) to (4.12) and } \underline{u}^a = \underline{u}^b = 0\right\},$$

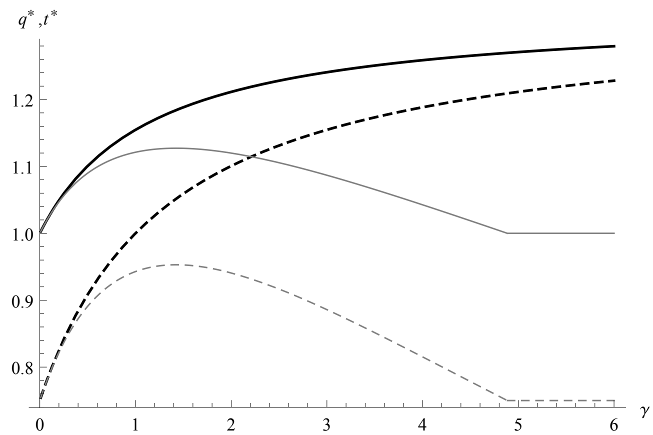
where $\alpha \equiv \frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$ is the ratio of period-2 focus weights, and $q^\star(\alpha) = \alpha$ and $t^\star(\alpha) = 3/4\alpha^2$ are the implied period-2 quality and transfer given by (4.13) and (4.14). Deriving $\bar{q}$ and $\bar{t}$, and solving the above problem we arrive at the following proposition.

Proposition 7. *Suppose that, in period 1, the players can sign only renegotiation-proof agreements with $\underline{u}^a = \underline{u}^b = 0$. The solution to the bargaining problem is:*

$$q^\star = \max\left\{\left(\frac{2^{\frac{3\gamma+4}{2(\gamma+1)}}}{3}\right)^\gamma, 1\right\}$$

$$t^\star = \frac{3}{4}(q^\star)^2.$$

Proposition 7 shows that, for γ sufficiently low, bargaining in two periods will lead to a bargaining outcome that is different from the bargaining outcome reached in the one-shot case. This result is illustrated in Figure 4.1 (see the gray lines). Therefore, for γ low, the structure of the negotiation is relevant for material efficiency, because it affects the quality of the final outcome.



$q^\star(\gamma)$ (solid line), $t^\star(\gamma)$ (dashed line) as a function of the focusing intensity γ. Gray lines represent the equilibrium outcomes under no minimum utility. Black lines represent the equilibrium outcomes with a minimum utility to player b.

Figure 4.1: Period-2 Quality and Transfer

The way the parameter γ affects the outcome of the bargaining is quite complex. It is possible to show that the largest period-2 focus weight ratio $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$ achievable via a period-1 agreement is increasing in γ. In other words, the stronger the focusing effect, the more quality sensitive player b can be made in period 2 via an appropriate period-1 agreement. In addition, when bargaining in period 2, final q and t increase with the focus weight ratio. The key observation is that for low period-2 focus weight ratios the final q will be above the final t, while for large period-2 focus weight ratios the opposite holds.

It follows that, from the viewpoint of period 1, when the focusing effect is very strong the largest transfer achievable by means of a period-1 agreement is above the corresponding largest quality. Hence, in period 1, player b is *more* sensitive to the transfer than in the one-shot case. Because the smallest quality which is achievable via a renegotiation-proof agreement is equal to $q = 1$, the players agree on this quality. When the focusing effect is not strong, the opposite is true and, in period 1, player b is less sensitive to transfers than in the one-shot case. As a consequence, the players agree on $\{q, t\}$ that are larger than the one-step case.

Intuitively, when the focusing effect is strong, player b anticipates that he may accept transfers above quality in the future, and this expectation makes him very transfer sensitive in period 1. Instead when the focusing effect is not very strong, player b knows that he won't accept transfers above quality in period 2, and therefore is less sensitive to transfers in period 1. Interestingly, when the behavioral distortion is sufficiently severe, the solution is identical to the case of a rational agent, and hence the players cannot do better than bargaining in one step.

Minimum utility to player b We now allow players to bargain also over $\underline{u}^b \geq 0$. This problem is similar to the one discussed in the previous case. The only difference is that, for a given period-2 focus weight ratio and period-2 quality (as in 4.13), the period-2 transfer can be below (4.14) by setting an appropriate $\underline{u}^b$.

Proposition 8. *Suppose that, in period 1, the players can sign any renegotiation-proof agreement with* $\underline{u}^b \geq 0$, $\underline{u}^a = 0$. *The solution to the bargaining problem is:*

$$q^* = \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}} \tag{4.15}$$

$$t^* = \left(\frac{4}{3}\right)^{\frac{\gamma-1}{\gamma+1}}. \tag{4.16}$$

By comparing the result of Proposition 7 and 8, we see that the equilibrium transfer and quality are greater when the players also bargain over $\underline{u}^b$ compared to when the players bargain only over $\hat{t}$ and $\hat{q}$. This result is illustrated in Figure 4.1. Compared with Proposition 7, here if a very large utility level is promised to player b in period 1, achieving this utility level in period 2 implies agreeing on a very high quality and a very low transfer. From the viewpoint of period 1, the possibility of such an outcome makes player b particularly focused on the quality dimension rather than on the transfer dimension.

Interestingly, in period 1, the parties agree on $\underline{u}^b = 0$: on the equilibrium path there is no minimum-utility guaranteed to player b. However, because by manipulating $\underline{u}^b$ very high quality and very low transfers can be achieved in period 2, the fact that players could impose a minimum utility guarantee makes player b more quality sensitive, and willing to accept a greater $q^\star$ and $t^\star$. Hence, player a reaches a higher utility level when minimum utility for player b can be imposed (relative to the previous case in which no minimum utility for player b can be imposed).

Minimum utility to both players When the players bargain over minimum utility for both players in period 1, the following bargaining outcome is achieved.

Proposition 9. *Suppose that, in period 1, the players can sign any renegotiation-proof agreement with $\underline{u}^b \geq 0$, $\underline{u}^a \geq 0$. There exist a $\overline{\gamma}$ such that:*

- *for $\gamma > \overline{\gamma}$ the solution to the bargaining problem is*

$$q^* = \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}} \tag{4.17}$$

$$t^* = \left(\frac{4}{3}\right)^{\frac{\gamma-1}{\gamma+1}}. \tag{4.18}$$

  which is the same solution derived in Lemma 8,

- *for $\gamma < \overline{\gamma}$ the solution to the bargaining problem is*

$$q^* < \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}} \tag{4.19}$$

$$t^* = \frac{3}{4}(q^\star)^2. \tag{4.20}$$

Imposing a minimum utility guarantee to player a has two effects on period-2 transfers. For a given quality, when the minimum utility guarantee to player a is binding, the final transfer is increasing with $\underline{u}^a$. At the same time, the possibility of imposing a minimum utility guarantee in period 1 makes the player b more sensitive to transfers, which tends to decrease both final quality and transfer.

The previous proposition shows that for γ large, the second effect dominates: increasing $\underline{u}^a$ above zero lowers the transfer in period 2, so that the highest period-2 transfer and quality are achieved at $\underline{u}^a = 0$. As a consequence, when γ is large, in period 1, player b is as transfer sensitive in case $\underline{u}^a \geq 0$ as in case $\underline{u}^a = 0$. Instead, when γ is small, the first effect dominates: by setting $\underline{u}^a$ positive, it is possible to achieve some large period-2 transfers that are not achievable when $\underline{u}^a = 0$ (period-2 quality is independent of $\underline{u}^a$), which implies that in period 1, player b is particularly sensitive to transfers when $\underline{u}^a > 0$ is possible.

Overall, when the parties bargain over a minimum utility guarantee to player a, player a's utility is weakly lower compared to the case in which only a minimum utility guarantee to player b is discussed. Hence, player a is better off by negotiating over a minimum utility guarantee to the player b, but should avoid negotiating over a minimum utility guarantee to herself. We also show in the proof of the proposition that, also here, the minimum utilities are relevant off-equilibrium but, in equilibrium, the players will set $\underline{u}^b = \underline{u}^a = 0$.

5 Conclusion

We provide a theory of incomplete agreements in the absence of uncertainty. When the players' preferences are context dependent, the presence of previous incomplete agreements which restrict the set of possible bargaining outcomes may affect the bargaining solution. More interestingly, a player who anticipates having context-dependent preferences values the possibility of aligning her present and future preferences. Achieving this alignment may require the players to sign an incomplete agreement restricting the future bargaining possibilities.

We assumed that the players are exogenously given the opportunity to shrink their bargaining set via an incomplete agreement, and showed that the players may want to do it. One open question is where this possibility comes from. That is, one could introduce a period-0 in which players have to adopt a negotiation structure $\mathbb{S}$ out of a set of possible negotiation structures. We speculate that this extension of the game will not significantly change our results. If the players are in a "cold state" in period 0, they will choose the negotiation structure delivering the negotiation outcome that is closer to the negotiation outcome with rational players. If the players are in a "warm state" in period 0, the set of possible $\mathbb{S}$ will determine the period-0 contest, the period-0 preferences, and the negotiation structure adopted by the players. However, the formal analyses of this problem is left for future work.

Finally, incomplete agreements have been shown to be relevant in many contexts. We focus on negotiations because they are often structured as a sequence of incomplete agreements, with no new information expected to arrive between bargaining rounds. Hence, this setting is well suited to illustrate our main result: when preferences are context dependent, incomplete agreements may be used even in the absence of uncertainty. Exploring the implications of this result in other contexts where incomplete agreements are used (such as, for example, the allocation of ownership) is also left for future work.

A Appendix: Mathematical Derivations

Proof of Proposition 1. The first part of the statement follows from the fact that whenever $v_1 = v_2$, condition (2.2) implies that both issues generate positive focus-weighted surplus.

For the second part, note that when v_1 is arbitrarily large, then by (2.2) $\bar{t}_o$ is also arbitrarily large, which implies that the second issue generates negative focus-weighted surplus. Similarly, if v_2 is sufficiently close to c, because by (2.2) $\bar{t}_o > c$ the second issue generates negative focus-weighted surplus. $\square$

Proof of Lemma 1. To prove the first part of the statement, we need to show that whenever $c \cdot h(2c) \leq v_2 \cdot h(v_2)$, then $c \cdot h(t^\star_{\max}) \leq h(v_2) \cdot v_2$ (i.e. the focus-weighted surplus of the second issue is positive) and $2c \leq t^\star_{\max}$ (i.e., player a is willing to agree on both issues if the transfer is $t^\star_{\max}$).

Define $\underline{\hat{t}} = 2c$ and $\bar{\hat{t}}:\ c \cdot h(\bar{\hat{t}}) = v_2 \cdot h(v_2)$. Note that if $c \cdot h(2c) \leq v_2 \cdot h(v_2)$, then $\underline{\hat{t}} \leq \bar{\hat{t}}$. It follows that $\forall \hat{t} \in [\underline{\hat{t}}, \bar{\hat{t}}]$ both players' rationality constraints are satisfied, and $k^\star(\hat{t}) = 2$ $\forall \hat{t} \in [\underline{\hat{t}}, \bar{\hat{t}}]$. Moreover, for $\hat{t} \notin [\underline{\hat{t}}, \bar{\hat{t}}]$ at most one issue is included in the agreement. For $\hat{t} > \bar{\hat{t}}$, the equilibrium transfer is

$$\frac{1}{2}\left(\frac{h(v_1)}{h(\hat{t})} v_1 + c\right)$$

which is maximized for $\hat{t}$ arbitrarily close to $\bar{\hat{t}}$. However, the transfer achieved at $\hat{t} = \bar{\hat{t}}$ is

$$\frac{1}{2}\left(\frac{h(v_1)}{h(\bar{\hat{t}})} v_1 + \frac{h(v_2)}{h(\bar{\hat{t}})} v_2 + 2c\right)$$

and is larger than any transfer achievable at any $\hat{t} > \bar{\hat{t}}$. Similarly, for $\hat{t} < \underline{\hat{t}}$, the largest possible transfer achievable is

$$\hat{t}^{**} : \frac{1}{2}\left(\frac{h(v_1)}{h(\hat{t}^{**})} v_1 + c\right) = \hat{t}^{**} < \underline{\hat{t}}.$$

Because player a's rationality constraint is binding at $\hat{t} = \underline{\hat{t}}$, the equilibrium transfer achieved when the cap is set at $\underline{\hat{t}}$ is again $\underline{\hat{t}}$. It follows that the equilibrium transfer whenever $\hat{t} = \underline{\hat{t}}$ is larger than any equilibrium transfer whenever $\hat{t} < \underline{\hat{t}}$. As a consequence, $t^\star_{\max} \in [\underline{\hat{t}}, \bar{\hat{t}}]$ and $k^\star(t^\star_{\max}) = 2$.

If instead $c \cdot h(2c) > v_2 \cdot h(v_2)$, then there is no $\hat{t}$ for which $c \cdot h(\hat{t}) \leq h(v_2) \cdot v_2$ and $2c \leq \hat{t}$. Hence, it is not possible to include both issues in the agreement. We know from Proposition 1 that for $\hat{t}$ sufficiently large (for example, larger than $\bar{t}_o$), there is a positive transfer at which the number of issues included in the agreement is 1. It follows that at $\hat{t} = t^\star_{\max}$ the number of issues included in the agreement is 1. $\square$

Proof of Proposition 2. Consider the following problem

$$\max_{t,k} \left(\sum_{i=1}^{k} v_i - t \right) (t - ck) \,,$$

that is, the Nash bargaining problem in case in period 1 the players could bargain directly over k and t with rational preferences. The solution to this problem is $k = 2$ and $t = \frac{v_1+v_2+2c}{2}$. If $\frac{v_1+v_2+2c}{2} < t^\star_{\max}$, then by setting $\hat{t} = \frac{v_1+v_2+2c}{2}$ the players are effectively constraining their future selves to set transfers equal to $\frac{v_1+v_2+2c}{2}$.[17] If instead $\frac{v_1+v_2+2c}{2} > t^\star_{\max}$, the players preferred transfer is not achievable. Hence, $t^\star_{\max}$ is constrained optimal.

Finally, note that if $c \cdot h(2c) > v_2 \cdot h(v_2)$, then $k = 2$ is not achievable. The best the players can do is to achieve the transfer t that maximizes

$$\max_t \, (v_1 - t) \, (t - c) \,.$$

By setting $\hat{t} = v_1$, in period 2 player b's focus weight on transfer is $h(v_1)$ which is also his focus weight on v_1. Because of equal focus weights, player b will then behave as rational. Conditional on $k = 1$, the solution achieved in period 2 is optimal from period 1 point of view as well. □

Proof of Proposition 6. Because of conditions (4.5), (4.6), (4.7), and (4.8), an agreement S will not be renegotiated if all $\{t', q'\}$ such that:

$$\frac{h(\max\{\hat{t}, \frac{3}{4}\})}{h(\max\{\hat{q}, 1\})} = \frac{1}{q'} \tag{A.1}$$

$$u^a = t' - \frac{q'^2}{2}, \tag{A.2}$$

for

$$\underline{u}^a \leq u^a \leq \frac{h(\max\{\hat{q}, 1\})q' - \underline{u}^b}{h(\max\{\hat{t}, \frac{3}{4}\})} - \frac{q'^2}{2}, \tag{A.3}$$

are elements of S. In other words, a renegotiation-proof S contains the tangency points between indifference curves of the two players (pinned down by equations (A.1) and (A.2) as a function of u^a), giving utility greater or equal than $\underline{u}^a$ to player a and $\underline{u}^b$ to player b (expressed in equation (A.3) as a range of possible u^a).

Conditions (A.1), (A.2), and (A.3) can be used to show that conditions (4.9), (4.10), (4.11), and (4.12) are necessary and sufficient for S to be renegotiation proof. Conditions (4.9), (4.10) state that $\{\hat{t}, \hat{q}\}$ should be larger than the largest $\{t, q\}$ on the Pareto frontier of

[17] Note that, by (2.1) $\frac{v_1+v_2+2c}{2} > 2c$. Hence transfers $\frac{v_1+v_2+2c}{2}$ satisfy the rationality constraint of player a.

the set S. In particular, condition (4.9) is derived from player b's utility function, and states that $\hat{t}$ should be at least as large as the transfers leaving b on his minimum utility level at the Pareto optimal q, which is given by the focus weight ratio. The last two conditions are the rationality constraints of the players at $\{\hat{t}, \hat{q}\}$.

Lemma 2 (Necessity)**.** *If a closed, convex S is renegotiation proof, then the conditions* (4.9)*,* (4.10)*,* (4.11)*, and* (4.12) *must hold.*

Proof. We next want to show that whenever one of conditions (4.9) - (4.12) is violated, then S is not renegotiation proof. The fact that whenever (4.9) or (4.10) are violated S cannot be renegotiation proof follows immediately from the fact that, for S to be renegotiation proof, the Pareto frontier of $\mathbb{R}^2_+$ giving utility at least $\underline{u}^b$ to player b and $\underline{u}^a$ to player a should be an element of S.

Suppose that $\{\hat{t}, \hat{q}\} \in S$ (i.e., the cap on transfer and quality are achieved at the same bargaining outcome), then again it is easy to see that (4.11) and (4.12) should hold, because otherwise one of the rationality constraints will be violated.

Finally, suppose that $\{\hat{t}, \hat{q}\} \notin S$. Call the two "extreme" bundles $\{t', \hat{q}\} \in S$ and $\{\hat{t}, q'\} \in S$ for $q' < \hat{t}$ and $t' < \hat{t}$. It is easy to see that, whenever (4.11) is violated, it must be the case that player b's rationality constraint is violated at $\{\hat{t}, q'\}$. Similarly, whenever (4.12) is violated, it must be the case that player a's rationality constraint is violated at $\{t', \hat{q}\}$. □

Lemma 3 (Sufficiency)**.** *For every $\underline{u}^a$, $\underline{u}^b$, $\hat{t}$, $\hat{q}$ that satisfy conditions* (4.9)*,* (4.10)*,* (4.11)*, and* (4.12)*, there exist a renegotiation-proof S satisfying conditions* (4.5) *to* (4.8)*.*

Proof. For example, define S as the set of $x \in \mathbb{R}^2$ that satisfy equations (4.5) to (4.8), with $\{t, q\} \leq \{\hat{t}, \hat{q}\}$ for all $\{t, q\} \in S$. □

Finally, note that starting from any closed and convex S satisfying conditions (4.5), (4.6), (4.7) and (4.8) (i.e., a renegotiation-proof S) for the same $\underline{u}^a$, $\underline{u}^b$, $\hat{t}$, $\hat{q}$, the players will reach the same period-2 solution. The reason is that the player b's preferences and the set of possible solutions (the Pareto frontier of the set S) are the same across all renegotiation-proof, closed and convex S having the same $\underline{u}^a$, $\underline{u}^b$, $\hat{t}$, $\hat{q}$. Therefore, in order to characterize the set of possible renegotiation-proof, closed and convex incomplete agreements, we only need to characterize the set of $\underline{u}^a$, $\underline{u}^b$, $\hat{t}$, $\hat{q}$ that satisfy conditions (4.9), (4.10), (4.11), and (4.12). This concludes the proof of Proposition 6. □

Proof of Proposition 7. It is left to derive the highest quality and the highest transfer achievable in period 2 given the set of renegotiation-proof $\{\hat{t}, \hat{q}\}$ in period 1.

$\underline{u}^a = \underline{u}^b = 0$, $\hat{t}, \hat{q}$ are renegotiation proof if

$$\sqrt{\hat{t}} \geq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})}, \tag{A.4}$$

$$\hat{q} \geq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})}, \tag{A.5}$$

$$\frac{\hat{t}}{\hat{q}} \leq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})}, \tag{A.6}$$

$$\frac{\hat{t}}{\hat{q}} \geq \frac{\hat{q}}{2}. \tag{A.7}$$

As mentioned in the main text, for every $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$ which satisfy conditions (A.4), (A.5), (A.6) and (A.7), the outcome from Nash bargaining in period 2 is

$$q^\star = \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})},$$

$$t^\star = \frac{3}{4}\left(\frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})}\right)^2,$$

It is easy to see that the minimum $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$ is reached when both conditions (A.6), (A.7) are binding, so that

$$\min\left\{\frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})}\right\} = 1.$$

Consider a given $\hat{q}' \geq 1$. Condition (A.4) can be written as

$$\left(\max\{\hat{t}, \frac{3}{4}\}\right)^\gamma \sqrt{\hat{t}} \geq (\hat{q}')^\gamma,$$

which implies that $\hat{t} > 1$. Hence, the smallest $\hat{t}$ that satisfies conditions (A.7), (A.5), and (A.4) is

$$\underline{\hat{t}}(\hat{q}') = \max\left\{\frac{(q')^2}{2}, (q')^{\frac{\gamma-1}{\gamma}}, (q')^{\frac{2\gamma}{2\gamma+1}}\right\},$$

so that

$$\max\left\{\frac{h(\hat{q}')}{h(\hat{t})}\right\} = \frac{h(\hat{q}')}{h(\underline{\hat{t}}(\hat{q}'))} = \max\left\{\min\left\{\hat{q}', (\hat{q}')^{\frac{\gamma}{2\gamma+1}}, \left(\frac{2}{\hat{q}'}\right)^\gamma\right\}\right\}.$$

Also, by varying $\hat{q}'$, $\frac{h(\hat{q}')}{h(\underline{\hat{t}}(\hat{q}'))}$ reaches its maximum for

$$\overline{\hat{q}'} : (\hat{q}')^{\frac{\gamma}{2\gamma+1}} = \left(\frac{2}{\hat{q}'}\right)^\gamma,$$

or

$$\overline{\hat{q}'} = 2^{\frac{2\gamma+1}{2\gamma+2}},$$

which implies

$$\max_{q'} \left\{ \frac{h(\hat{q}')}{h(\bar{\hat{t}}(\hat{q}'))} \right\} = \left(\frac{\overline{\hat{q}'}}{2} \right)^{\gamma} = 2^{\frac{\gamma}{2\gamma+2}}.$$

It follows that the highest quality and the highest transfer achievable in period 2 by choosing a renegotiation-proof $\{\hat{t}, \hat{q}\}$ in period 1 are $\bar{q} = 2^{\frac{\gamma}{2\gamma+2}}$ and $\bar{t} = \left(\frac{3}{4}\right) 2^{\frac{\gamma}{\gamma+1}}$ respectively. As a consequence, the period-1 bargaining problem is:

$$\max_{\alpha} \left\{ \left(h\left(2^{\frac{\gamma}{2\gamma+2}}\right) q^{\star}(\alpha) - h\left(\frac{3}{4} \cdot 2^{\frac{\gamma}{\gamma+1}}\right) t^{\star}(\alpha) \right) \left(t^{\star}(\alpha) - \frac{1}{2} q^{\star}(\alpha)^2 t \right) \right.$$
$$\left. \text{s.t. } 1 \leq q^{\star}(\alpha) \leq 2^{\frac{\gamma}{2\gamma+2}} \right\},$$

where α is the ratio of period-2 focus weights $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$. Note that, in period 1, the quality dimension is more salient than the price dimension, i.e.,$\bar{q} > \bar{t}$, for small γ and the opposite is true for large γ. The period-1 solution is:

$$\alpha^{\star} = \max \left\{ h\left(2^{\frac{\gamma}{2\gamma+2}}\right) / h\left(\frac{3}{4} \cdot 2^{\frac{\gamma}{\gamma+1}}\right), 1 \right\} = \max \left\{ \left(\frac{2^{\frac{3\gamma+4}{2(\gamma+1)}}}{3} \right)^{\gamma}, 1 \right\}.$$

Using that $q^{\star}(\alpha) = \alpha$ and $t^{\star}(\alpha) = 3/4\alpha^2$ by (4.13) and (4.14) leads to $q^{\star}$ and $t^{\star}$ in the proposition. This completes the proof. □

Proof of Proposition 8. In this case, a renegotiation-proof agreement has to satisfy

$$\hat{t} + \frac{\underline{u}^b}{h(\max\{\hat{t}, \frac{3}{4}\})} \geq \left(\frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})} \right)^2, \tag{A.8}$$

$$\hat{q} \geq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})}, \tag{A.9}$$

$$\hat{t} + \frac{\underline{u}^b}{h(\max\{\hat{t}, \frac{3}{4}\})} \leq \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})} \hat{q}, \tag{A.10}$$

$$\frac{2}{\hat{q}} \geq \frac{\hat{q}}{\hat{t}}. \tag{A.11}$$

It is easy to see that the smallest possible renegotiation-proof $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$ is determined by condition (A.10) for $\underline{u}^b = 0$ and it is equal to 1. Regarding the largest possible $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$, we again fix $\hat{q}' \geq 1$ and compute the largest $\frac{h(\hat{q}')}{h(\max\{\hat{t},\frac{3}{4}\})}$ satisfying conditions (A.8), (A.9) and (A.11).

The smallest $\hat{t}$ satisfying conditions (A.8), (A.9) and (A.11) is given by:

$$\underline{\hat{t}}(\hat{q}') = \max\left\{t^{\star}_{\max}(\hat{q}',\underline{u}^b), (\hat{q}')^{\frac{\gamma-1}{\gamma}}, \frac{(q')^2}{2}\right\},$$

where $t^{\star}_{\max}(\hat{q}',\underline{u}^b)$ is implicitly defined using condition (A.8) as

$$t^{\star}_{\max}(\hat{q}',\underline{u}^b)^{2\gamma+1} + \underline{u}^b t^{\star}_{\max}(\hat{q}',\underline{u}^b)^{\gamma} = (\hat{q}')^{2\gamma}. \tag{A.12}$$

Note that $t^{\star}_{\max}(\hat{q}',\underline{u}^b)$ is increasing in $\hat{q}$ and decreasing in $\underline{u}$. It follows that

$$\max\left\{\frac{h(\max\{\hat{q}',1\})}{h(\max\{\hat{t},\frac{3}{4}\})}\right\} = \frac{h(\hat{q}')}{h(\underline{\hat{t}}(\hat{q}'))} = \max\left\{\min\left\{\hat{q}', \left(\frac{\hat{q}'}{t(\hat{q}',\underline{u}^b)}\right)^{\gamma}, \left(\frac{2}{\hat{q}'}\right)^{\gamma}\right\}\right\}.$$

Define $\hat{q}(\underline{u}^b)$ as $\hat{q}' : \left(\frac{\hat{q}'}{t(\hat{q}',\underline{u}^b)}\right)^{\gamma} = \left(\frac{2}{\hat{q}'}\right)^{\gamma}$, which by using (A.12), can be expressed as

$$\hat{q}(\underline{u}^b) = \left(\left(1 - \frac{\underline{u}^b}{2^{\gamma}}\right) 2^{2\gamma+1}\right)^{\frac{1}{2\gamma+2}}. \tag{A.13}$$

Note the following:

- $\hat{q}(\underline{u}^b)$ is unique for every $\underline{u}^b$, which implies that $\left(\frac{2}{\hat{q}'}\right)^{\gamma}$ and $\left(\frac{\hat{q}'}{t(\hat{q}',\underline{u}^b)}\right)^{\gamma}$ intercept only once.
- $\left(\frac{\hat{q}'}{t(\hat{q}',\underline{u}^b)}\right)^{\gamma}$ is strictly increasing in $\hat{q}'$ for $\hat{q}' > 1$. By the implicit function theorem:

$$\frac{dt(\hat{q}',\underline{u}^b)}{d\hat{q}'} = \frac{2\gamma\hat{q}'^{2\gamma-1}}{(2\gamma+1)t(\hat{q}',\underline{u}^b)^{2\gamma} + \gamma\underline{u}^b t(\hat{q}',\underline{u}^b)^{\gamma-1}}. \tag{A.14}$$

  Hence,

$$\frac{d\frac{\hat{q}'}{t(\hat{q}',\underline{u}^b)}}{d\hat{q}'} = \frac{t(\hat{q}',\underline{u}^b) - \frac{dt(\hat{q}',\underline{u}^b)}{d\hat{q}'}\hat{q}'}{t(\hat{q}',\underline{u}^b)^2} = \frac{t(\hat{q}',\underline{u}^b) - \frac{2\gamma\hat{q}'^{2\gamma}}{(2\gamma+1)t(\hat{q}',\underline{u}^b)^{2\gamma}+\gamma\underline{u}^b t(\hat{q}',\underline{u}^b)^{\gamma-1}}}{t(\hat{q}',\underline{u}^b)^2}. \tag{A.15}$$

  The above expression is positive at $\underline{u}^b = 0$ (and therefore it is positive for every $\underline{u}^b$) whenever

$$t^{\star}_{\max}(\hat{q}',\underline{u}^b)^{2\gamma+1} > \frac{2\gamma}{2\gamma+1}(\hat{q}')^{2\gamma},$$

  which is true by (A.12).
- $\left(\frac{\hat{q}'}{t(\hat{q}',\underline{u}^b)}\right)^{\gamma} < \hat{q}'$ at $\hat{q}' = \hat{q}(\underline{u}^b)$ for every $\underline{u}^b \leq 2^{\gamma-1}$. But, as we show later, $\underline{u}^b > 2^{\gamma-1}$

implies negative utility for player a, so that we can ignore this case

Taken together, the three facts above imply that the focus weight ratio achieves its maximum at $\hat{q}(\underline{u}^b)$, so that

$$\max_{\hat{q}'}\left\{\frac{h(\hat{q}')}{h(\hat{\underline{t}}(\hat{q}'))}\right\} = \frac{h(\hat{q}(\underline{u}^b))}{h(\hat{t}(\underline{u}^b))} = \left(\frac{2}{\hat{q}(\underline{u}^b)}\right)^{\gamma}, \tag{A.16}$$

where using (A.12) and (A.13) we can define

$$\hat{t}(\underline{u}^b) \equiv \frac{\hat{q}(\underline{u}^b)^2}{2}. \tag{A.17}$$

Finally, note that $\max_{\hat{q}'}\left\{\frac{h(\hat{q}')}{h(\hat{\underline{t}}(\hat{q}'))}\right\}$ is strictly increasing in $\underline{u}^b$.

We now consider the set of possible period-2 outcomes achievable in period 2 by means of a renegotiation proof $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$. In particular, we want to determine the highest transfer and the highest quality achievable in period 2, as these quantities will determine preferences in period 1. Note that, in period 2, the minimum utility $\underline{u}^b$ affects the bargaining solution in two ways. On the one hand, it affects the highest focus weight ratio $\max_{\hat{q}'}\left\{\frac{h(\hat{q}')}{h(\hat{\underline{t}}(\hat{q}'))}\right\}$ achievable. On the other hand, it imposes a constraint on the period-2 bargaining problem.

It is quite straightforward to see that the highest quantity and transfer achievable in period 2 are achieved if the minimum utility constraint is binding. Let's first consider the case in which the constrained solution coincides with the unconstrained solution:

$$q^{\star}(\underline{u}^b) = \frac{h(\max\{\hat{q}(\underline{u}^b),1\})}{h(\max\{\hat{t}(\underline{u}^b),\frac{3}{4}\})} = \left(\frac{2}{\hat{q}(\underline{u}^b)}\right)^{\gamma}, \tag{A.18}$$

$$t^{\star}(\underline{u}^b) = \frac{3}{4}\left(q^{\star}(\underline{u}^b)\right)^2. \tag{A.19}$$

The corresponding minimum-utility constraint equals

$$U^b\left(q^{\star}(\underline{u}^b), t^{\star}(\underline{u}^b), \underline{u}^b\right) = \frac{1}{4}\frac{h(\max\{\hat{q}(\underline{u}^b),1\})^2}{h(\max\{\hat{t}(\underline{u}^b),\frac{3}{4}\})} = \underline{u}^b. \tag{A.20}$$

Note that LHS of condition (A.20) simplifies to

$$\frac{1}{4}\frac{\hat{q}(\underline{u}^b)^{2\gamma}}{\frac{\hat{q}(\underline{u}^b)^{2\gamma}}{2^{\gamma}}} = \frac{2^{\gamma}}{4} = 2^{\gamma-2}.$$

Thus, the minimum-utility constraint is binding for $\underline{u}^b \geq \underline{u}^{b\star} \equiv 2^{\gamma-2}$.

For $\underline{u}^b > \underline{u}^{b\star} \equiv 2^{\gamma-2}$, the final outcome of the constrained Nash bargaining problem in period 2 equals

$$q^{\star\star}(\underline{u}^b) = \left(\frac{h(\max\{\hat{q}(\underline{u}^b), \frac{3}{4}\})}{h(\max\{\hat{t}(\underline{u}^b), 1\})} \right) = \left(\frac{2}{\hat{q}(\underline{u}^b)} \right)^{\gamma}, \tag{A.21}$$

$$t^{\star\star}(\underline{u}^b) = \left(q^{\star\star}(\underline{u}^b) \right)^2 - \left(\frac{\underline{u}^b}{h(\max\{\hat{t}(\underline{u}^b), \frac{3}{4}\})} \right) = \left(q^{\star\star}(\underline{u}^b) \right)^{-2} 2^{\gamma}, \tag{A.22}$$

where the last equality follows from equations (A.13) and (A.17). Therefore $q^{\star\star}(\underline{u}^b)$ is increasing in $\underline{u}^b$ while $t^{\star\star}(\underline{u}^b)$ is decreasing in $\underline{u}^b$. Call

$$u^a(\underline{u}^b) = t^{\star\star}(\underline{u}^b) - \frac{1}{2} q^{\star\star}(\underline{u}^b)^2$$

the utility of player a as a function of the minimum utility for player b for $\underline{u}^b \geq \underline{u}^{b\star}$. The maximum level of $\underline{u}^b$ such that player a will want to trade is $\underline{u}^{b\star\star} \equiv 2^{\gamma-1}$. It follows that highest period-2 quantity and transfer are equal to

$$q^{\star\star}(\underline{u}^{b\star\star}) = \max \left\{ \frac{h(\max\{\hat{q}(\underline{u}^{b\star\star}), 1\})}{h(\max\{\hat{t}(\underline{u}^{b\star\star}), \frac{3}{4}\})} \right\} = \left(\frac{2}{\hat{q}(\underline{u}^{b\star\star})} \right)^{\gamma} = 2^{\frac{\gamma}{\gamma+1}},$$

$$t^{\star}(\underline{u}^{b\star}) = 3/4 \left(\max \left\{ \frac{h(\max\{\hat{q}(\underline{u}^{b\star}), 1\})}{h(\max\{\hat{t}(\underline{u}^{b\star}), \frac{3}{4}\})} \right\} \right)^2 = 3^{\frac{1}{\gamma+1}} 2^{\frac{\gamma-2}{\gamma+1}}.$$

Having established this, the value function of the period-1 bargaining problem is:

$$V(\alpha) \equiv \left\{ \left(h\left(2^{\frac{\gamma}{\gamma+1}}\right) \alpha - h\left(3^{\frac{1}{\gamma+1}} 2^{\frac{\gamma-2}{\gamma+1}}\right) \beta(\alpha) \right) \left(\beta(\alpha) - \frac{1}{2}\alpha^2 \right) \text{s.t. } 1 \leq \alpha \leq 2^{\frac{\gamma}{\gamma+1}} \right\},$$

for

$$\beta(\alpha) = \begin{cases} \frac{3}{4}\alpha^2 & \text{if } 1 \leq \alpha \leq 3^{-\frac{\gamma}{2\gamma+2}} 2^{\frac{3\gamma}{2\gamma+2}} \\ \frac{2^{\gamma}}{\alpha^2} & \text{if } 3^{-\frac{\gamma}{2\gamma+2}} 2^{\frac{3\gamma}{2\gamma+2}} \leq \alpha \leq 2^{\frac{\gamma}{\gamma+1}}. \end{cases}$$

In other words, in period 1 the players agree on a period-2 focus weight ratio α which will determine period-2 transaction transfer and quality. Note that if α is high enough, the period-1 minimum utility will be binding, which will affect the final transfer.

It is easy to see that $V(\alpha)$ is always below

$$\tilde{V}(\alpha, \beta) \equiv \left\{ \left(h\left(2^{\frac{\gamma}{\gamma+1}}\right) \alpha - h\left(3^{\frac{1}{\gamma+1}} 2^{\frac{\gamma-2}{\gamma+1}}\right) \beta \right) \left(\beta - \frac{1}{2}\alpha^2 \right) \text{s.t. } 1 \leq \alpha \leq 2^{\frac{\gamma}{\gamma+1}} \right\},$$

where β can be any positive number. In practice, $\tilde{V}(\alpha, \beta)$ is equivalent to choosing a quality

α and a transfer β, without worrying about whether this transfer and quality will be implementable in period 2 via a renegotiation-proof period-1 agreement. Maximizing $\tilde{V}(\alpha, \beta)$ with respect to α and β, the solution is

$$\alpha^\star = \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}},$$

$$\beta^\star = \frac{3}{4}(\alpha^\star)^2,$$

which is always no lower than 1 and smaller than $3^{-\frac{\gamma}{2\gamma+2}} 2^{\frac{3\gamma}{2\gamma+2}}$, and therefore also maximizes $V(\alpha)$. Given this, the final bargaining outcome is

$$q^* = \left(\frac{4}{3}\right)^{\frac{\gamma}{\gamma+1}}, \tag{A.23}$$

$$t^* = \left(\frac{4}{3}\right)^{\frac{\gamma-1}{\gamma+1}}. \tag{A.24}$$

Interestingly, this result can be achieved by setting $\underline{u}^b = 0$ in period 1. To see this, note that highest focus weight ratio that can be achieved in period 2 when $\underline{u}^b = 0$ is $2^{\frac{\gamma}{2\gamma+2}}$, which also corresponds to the highest possible quality achievable in period 1 by setting $\underline{u}^b = 0$. This quality is below the equilibrium quality derived in A.23. Therefore, when bargaining in period 1, player a and player b will agree on $\underline{u}^b = 0$. □

Proof of Proposition 9. Quality in period 2 only depends on $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$, and is given by

$$q^\star = \frac{h(\max\{\hat{q}, 1\})}{h(\max\{\hat{t}, \frac{3}{4}\})} = \left(\frac{\max\{\hat{q}, 1\}}{\max\{\hat{t}, \frac{3}{4}\}}\right)^\gamma.$$

We show that the set of $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$ that is achievable whenever $\underline{u}^a \geq 0$ is the same set of $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$ that is achievable whenever $\underline{u}^a = 0$. Note that $\underline{u}^a$ matters only for constraint (4.12), which can be rewritten as:

$$\frac{1}{2}\left(\frac{\hat{q}}{\hat{t}}\right)^2 \leq \frac{1}{\hat{t}}\left(1 - \frac{\underline{u}^a}{\hat{t}}\right).$$

Suppose that, for some $\underline{u}^a > 0$, at $\max\left\{\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}\right\}$ the above constraint is binding. By lowering $\underline{u}^a$ and increasing $\hat{q}$, it is always possible to achieve an even higher $\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}$. Hence, whatever $\max\left\{\frac{h(\max\{\hat{q},1\})}{h(\max\{\hat{t},\frac{3}{4}\})}\right\}$ can be achieved by allowing $\underline{u}^a$ to be greater or equal to zero can also be achieved by fixing $\underline{u}^a = 0$. Compared with the case analyzed in the previous proposition, allowing for $\underline{u}^a \geq 0$ does not change the set of possible period-2 quality levels.

Nonetheless, a minimum utility guaranteed to player a may affect the final transfer if it is binding in period 2. For given preferences, whenever the minimum utility guaranteed to player a is binding, the period-2 transfer equals

$$t^\star = \underline{u}^a + \frac{1}{2}\left(\frac{\max\{\hat{q},1\}}{\max\{\hat{t},\frac{3}{4}\}}\right)^{2\gamma},$$

where $\underline{u}^a$ binding implies (using the structure in (A.18) and (A.19))

$$\underline{u}^a \geq \frac{1}{4}\left(\frac{\max\{\hat{q},1\}}{\max\{\hat{t},\frac{3}{4}\}}\right)^{2\gamma}.$$

It follows that there is a potential trade-off. A higher minimum utility guarantee to player a increases the final transfer because it shifts bargaining power to player a. At the same time, a minimum utility guarantee to player a affects player b's preferences and may make player b more transfer sensitive.

Following steps that are similar to the ones used in the previous proposition, we can derive $\left(\frac{\hat{q}(\underline{u}^b,\underline{u}^a)}{\hat{t}(\underline{u}^b,\underline{u}^a)}\right)^\gamma$, i.e., the highest renegotiation-proof focus weight ratio for given $\underline{u}^b, \underline{u}^a$. Consider a given $\hat{q}' \geq 1$. The smallest renegotiation-proof $\hat{t}$ is given by:

$$\underline{\hat{t}}(\hat{q}') = \max\left\{t^\star_{\max}(\hat{q}',\underline{u}^b), (\hat{q}')^{\frac{\gamma-1}{\gamma}}, \frac{(q')^2}{2} + \underline{u}^a\right\},$$

where $t^\star_{\max}(\hat{q}',\underline{u}^b)$ is, again, implicitly defined by (A.12). It follows that, for given $\hat{q}'$

$$\max\left\{\frac{h(\hat{q}')}{h(\hat{t})}\right\} = \frac{h(\hat{q}')}{h(\underline{\hat{t}}(\hat{q}'))} = \max\left\{\min\left\{\hat{q}', \left(\frac{\hat{q}'}{t(\hat{q}',\underline{u}^b)}\right)^\gamma, \left(\frac{\hat{q}'}{2} + \frac{\underline{u}^a}{\hat{q}'}\right)^{-\gamma}\right\}\right\}.$$

The same argument discussed in the proof of the previous proposition (see the three bullet points) guarantees that the above expression reaches its maximum at

$$\hat{q}(\underline{u}^b,\underline{u}^a): \left(\frac{\hat{q}'}{t(\hat{q}',\underline{u}^b)}\right)^\gamma = \left(\frac{\hat{q}'}{2} + \frac{\underline{u}^a}{\hat{q}'}\right)^{-\gamma},$$

so that

$$\max_{\hat{q}'}\left\{\frac{h(\hat{q}')}{h(\hat{t}(\hat{q}'))}\right\} = \frac{\hat{q}(\underline{u}^b,\underline{u}^a)^\gamma}{\hat{t}(\underline{u}^b,\underline{u}^a)^\gamma} = \left(\frac{\hat{q}(\underline{u}^b,\underline{u}^a)}{2} + \frac{\underline{u}^a}{\hat{q}(\underline{u}^b,\underline{u}^a)}\right)^{-\gamma}.$$

Note that, using (A.12), $\hat{q}(\underline{u}^b, \underline{u}^a)$ can be implicitly defined as

$$\left(\frac{\hat{q}(\underline{u}^b, \underline{u}^a)^2}{2} + \underline{u}^a\right)^{2\gamma+1} + \underline{u}^b \left(\frac{\hat{q}(\underline{u}^b, \underline{u}^a)^2}{2} + \underline{u}^a\right)^{\gamma} = \hat{q}(\underline{u}^b, \underline{u}^a)^{2\gamma}. \tag{A.25}$$

By implicit differentiation, we can show that:

$$\frac{\partial \left[\left(\frac{\hat{q}(\underline{u}^b, \underline{u}^a)}{2} + \frac{\underline{u}^a}{\hat{q}(\underline{u}^b, \underline{u}^a)}\right)^{-\gamma}\right]}{\partial \underline{u}^b} = -\gamma \left(\frac{\hat{q}}{2} + \frac{\underline{u}^a}{\hat{q}}\right)^{-\gamma-1} \left(\left[\frac{1}{2} - \frac{\underline{u}^a}{\hat{q}(\underline{u}^b, \underline{u}^a)^2}\right] \frac{d\hat{q}(\underline{u}^b, \underline{u}^a)}{d\underline{u}^b}\right)$$

which is positive if $\underline{u}^a < \frac{\hat{q}(\underline{u}^b, \underline{u}^a)^2}{2}$, and negative otherwise.

Given this, there are two cases

1. if $\underline{u}^a > \frac{\hat{q}(\underline{u}^b=0, \underline{u}^a)^2}{2} \equiv \frac{\hat{q}(\underline{u}^a)^2}{2}$, then the highest renegotiation-proof focus weight ratio for given $\underline{u}^a$ binding is reached at $\underline{u}^b = 0$, and is equal to

$$\left(\frac{\hat{q}(\underline{u}^a)}{\hat{t}(\underline{u}^a)}\right)^{\gamma} = \hat{q}(\underline{u}^a)^{\frac{\gamma}{2\gamma+1}},$$

where, using equation (A.25), $\hat{q}(\underline{u}^a)$ is implicitly defined as

$$\left(\frac{\hat{q}(\underline{u}^a)}{2} + \frac{\underline{u}^a}{\hat{q}(\underline{u}^a)}\right)^{2\gamma+1} \cdot \hat{q}(\underline{u}^a) = 1.$$

Using the fact that $\underline{u}^a > \frac{\hat{q}(\underline{u}^a)^2}{2}$, the above expression implies

$$\hat{q}(\underline{u}^a) < 1,$$

which is not possible, as $\hat{q}(\underline{u}^a)$ is that $\hat{q}' \geq 1$ at which the focus weight ratio reaches its maximum as a function of $\underline{u}^a$.

2. if $\underline{u}^a < \frac{\hat{q}(\underline{u}^b, \underline{u}^a)^2}{2}$, then the highest renegotiation-proof focus weight for given $\underline{u}^a$ binding is reached at $\underline{u}^b$ that is also binding:

$$\underline{u}^b = \frac{\hat{q}(\underline{u}^a)^{2\gamma}}{\hat{t}(\underline{u}^a)^{\gamma}} - \hat{t}(\underline{u}^a)^{\gamma} \left(\underline{u}^a + \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^a)}{\hat{t}(\underline{u}^a)}\right)^{2\gamma}\right). \tag{A.26}$$

Condition (A.26), together with the fact that $\underline{u}^a$ and $\underline{u}^b$ are binding (i.e., conditions (4.11) and (4.12) are binding), implies that conditions (4.9) and (4.10) are also binding: that is, the highest possible quality is the actual quality exchanged, and the highest possible transfer is the actual transfer exchanged so that:[18]

$$\hat{q}(\underline{u}^a) = \left(\frac{\hat{q}(\underline{u}^a)}{\hat{t}(\underline{u}^a)}\right)^{\gamma}, \tag{A.29}$$

$$\hat{t}(\underline{u}^a) = \underline{u}^a + \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^a)}{\hat{t}(\underline{u}^a)}\right)^{2\gamma}. \tag{A.30}$$

Dividing the LHS of equation (A.29) with the LHS of equation (A.30), and the RHS of equation (A.29) with the RHS of equation (A.30) leads to:

$$\left(\frac{\hat{q}(\underline{u}^a)}{\hat{t}(\underline{u}^a)}\right)^{\gamma-1} = \underline{u}^a + \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^a)}{\hat{t}(\underline{u}^a)}\right)^{2\gamma}.$$

Finally, we derive the cap on transfer by solving for

$$\max_{\underline{u}^a} \left\{ \underline{u}^a + \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^a)}{\hat{t}(\underline{u}^a)}\right)^{2\gamma} \right\} \tag{A.31}$$

$$\text{s.t. } \frac{1}{4}\left(\frac{\hat{q}(\underline{u}^a)}{\hat{t}(\underline{u}^a)}\right)^{2\gamma} \leq \underline{u}^a \leq \frac{1}{2}\left(\frac{\hat{q}(\underline{u}^a)}{\hat{t}(\underline{u}^a)}\right)^{2\gamma}. \tag{A.32}$$

Where the constraint requires that $\underline{u}^a$ is binding, and that $\underline{u}^a$ is such that player b enjoys non-negative utility. We preform this last step numerically, and we compare the solution to the cap on transfer achievable when $\underline{u}^a = 0$ (see the previous proposition). The results of the computation are reported in Figure A.1. Note that, for low γ the cap on transfer is achieved for $\underline{u}^a > 0$, while for high γ the cap on transfer is achieved for $\underline{u}^a = 0$.

When γ is large, the cap on transfer that can be achieved in period 2 by setting $\underline{u}^a > 0$ is below the cap on transfer that can be achieved in period 2 by setting $\underline{u}^a = 0$ (which is the maximum period-2 transfer derived in the Proposition 8). We argued earlier that the maximum quality achievable in period 2 is achieved at $\underline{u}^a = 0$. Hence, for γ large the period-1 focus weights on transfer and quality are equal to the period-1 focus weight on transfer and quality derived in the previous proposition, and hence the final negotiation outcome is the same as in Proposition 8.

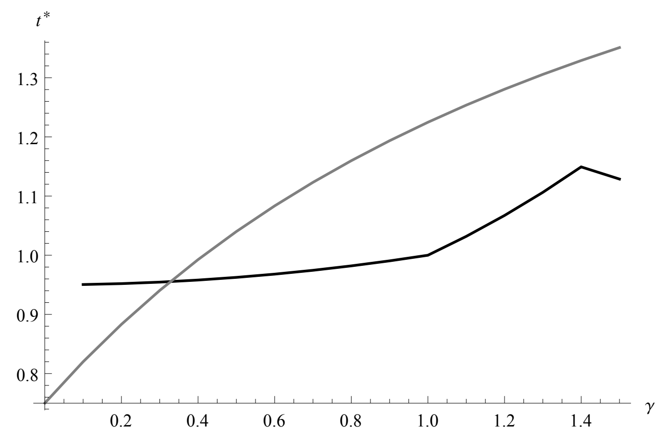
[18]Consider a $\hat{q}' \geq (\hat{q}'/\hat{t}')^{\gamma}$. Conditions (A.26) and (4.12) imply

$$\underline{u}^b = \frac{\hat{q}'^{2\gamma}}{\hat{t}'^{\gamma}} - \hat{t}'^{\gamma+1}, \tag{A.27}$$

while condition (4.11) implies

$$\underline{u}^b = \hat{q}'^{\gamma+1} - \hat{t}'^{\gamma+1}. \tag{A.28}$$

Taking the difference between these two equations leads to $\hat{q}'^{2\gamma}/\hat{t}'^{\gamma} = \hat{q}'^{\gamma+1}$, which is equivalent to condition (A.29). Condition (A.30) follows from (A.29) and the fact that $\underline{u}^a$ is binding.



$t^*(\gamma)$: maximum transaction transfer for $\underline{u}^a > 0$ (black line) and for $\underline{u}^a = 0$ (gray line) as a function of the focusing intensity γ.

Figure A.1: Cap on transfer

If instead γ is low, the cap on transfer that can be achieved in period 2 by setting $\underline{u}^a > 0$ is *above* the cap on transfer that can be achieved in period 2 by setting $\underline{u}^a = 0$. Compared to the solution in Proposition 8, in period 1 the focus weight on quality is unchanged, but the focus weight on transfer is higher. Hence, in period 1 the agent is relative more transfer sensitive here compared to the case considered in Proposition 8, leading to lower transaction transfer and quality.

Finally, we showed in Proposition 8 that, in equilibrium, $\underline{u}^b = 0$: the possibility of setting a minimum utility to player b affects player b's focus weights but is not used in equilibrium. The reason is that all period-2 $\frac{h(\hat{q})}{h(\hat{t})}$ lower than some threshold can be achieved with $\underline{u}^b = 0$, while higher $\frac{h(\hat{q})}{h(\hat{t})}$ require to set $\underline{u}^b > 0$. The equilibrium transfer and quality derived in Proposition 8 can be implemented by choosing in period 1 a period-2 $\frac{h(\hat{q})}{h(\hat{t})}$ achievable with $\underline{u}^b = 0$. The same thing happens here: in equilibrium $\underline{u}^b = \underline{u}^a = 0$. This result is immediate when γ is large, because the solution to this case is the same as in Proposition 8. When γ is low, however, the final transfer and quality here are below the final transfer and quality derived in Proposition 8. Hence, the period-2 $\frac{h(\hat{q})}{h(\hat{t})}$ that implements the equilibrium transfer and quality here is below the period-2 $\frac{h(\hat{q})}{h(\hat{t})}$ that implements the equilibrium transfer and quality derived in Proposition 8. Hence, also the period-2 $\frac{h(\hat{q})}{h(\hat{t})}$ that implements the equilibrium transfer and quality here can be achieved by setting $\underline{u}^b = \underline{u}^a = 0$ in period 1. □

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Appendix: further extensions (not for publication)

I Both players are behavioral

We modify the model presented in Section 2 by assuming that also player a is behavioral. Call $h_a()$ player a focus function and $h_b()$ player b focus function, both assumed strictly positive and strictly increasing. The utility functions are

$$U^a(q_1, q_2, t) = h_a(\bar{t})t - \sum_{i=1}^{2} h_a\left(\frac{\bar{q}_i c}{v_i}\right)\frac{q_i c}{v_i},$$

$$U^b(q_1, q_2, t) = \sum_{i=1}^{2} h_b(\bar{q}_i)q_i - h_b(\bar{t})t,$$

where, again, $\bar{q}_i = v_i$ if agreeing on issue i is in the consideration set, and zero otherwise. We define the *focus wedge* as

$$\Delta(x) \equiv \frac{h_a(x)}{h_b(x)},$$

which measures the relative salience of an outcome.

We maintain the assumption $v_1 > c$, but we consider both the case $v_2 > c$ and the case $v_2 < c$. Hence, it is now possible that the materially efficient outcome is to agree only on one issue. We also impose the following restriction:

$$\lim_{x \to 0} \Delta(x) = 1,$$

which, with a slight abuse of language, can be interpreted as saying that the two players are approximately equally focused over arbitrarily small stakes. Finally, in what follows, we will confine our attention to two cases:

- $\Delta(x)$ increasing, which implies that player a is always more focused than player b, the more so the larger x.
- $\Delta(x)$ decreasing, which implies that player b is always more focused than player a, the more so the larger x.

One step negotiation Also here we assume that

$$\bar{t}_o \equiv \bar{t} : h_b(\bar{t})\bar{t} = \sum_{i=1}^{2} h_b(v_i)v_i \text{ with } h_a(\bar{t}_o)\bar{t}_o \geq 2h_a(c)c, \tag{A.33}$$

that is, the largest transaction player b is willing to make for agreeing on both issues satisfies player a's rationality constraint for both issues. Hence, agreeing on both issues is in the consideration set.[19]

The Nash bargaining solution of the problem solves:

$$\max_{t \leq \bar{t}, k \in \{0,1,2\}} \left(\sum_{i=1}^{k} h_b(v_i)v_i - h_b(\bar{t}_o)t \right) (h_a(\bar{t}_o) \cdot t - h_a(c) \cdot ck), \tag{A.34}$$

and is characterized by two conditions:

$$t(k^\star) = \frac{1}{2} \left(\sum_{i=1}^{k^\star} \frac{h_b(v_i)}{h_b(\bar{t}_o)} v_i + \frac{h_a(c)}{h_a(\bar{t}_o)} ck^\star \right), \tag{A.35}$$

$$k^\star = \sharp \left\{ v_i | \frac{h_b(v_i)}{h_b(\bar{t}_o)} v_i - \frac{h_a(c)}{h_a(\bar{t}_o)} c > 0 \right\} \tag{A.36}$$

The introduction of a focused player a has two effects. First, for given $k^\star$, the equilibrium transfer is now lower than when player a is rational. That is because $h_a(c) < h_a(\bar{t}_o)$ player a focuses more on the transfer t than on the disutility c. Hence, relative to the rational case, a smaller transfer is required to compensate player a for the disutility generated by the agreement.

The second effect is a change in the set of issues that will be included in the agreement. By (A.33), issue 1 is always included in the agreement. However, by (A.36), the second issue will be included in the agreement if and only if

$$\frac{\frac{h_a(\bar{t})}{h_b(\bar{t})}}{\frac{h_a(c)}{h_b(c)}} \equiv \frac{\Delta(\bar{t}_0)}{\Delta(c)} \geq \frac{h_a(c) \cdot c}{h_b(v_2) \cdot v_2}.$$

Because $\bar{t}_0 > c$, simple inspection leads to the following results.

Proposition 10. • *Whenever $\Delta(x)$ is increasing (player a more focused than*

[19] We ignore the possibility that other consideration sets may exist.

player b), then the second issue may be included in the agreement also when it is not materially efficient to do so. This will happen whenever $v_2 < c$ but v_2 arbitrarily close to c.

- *Whenever $\Delta(x)$ is decreasing (player b more focused than player a), then the second issue may be left out of the agreement when it would be materially efficient to include it. This will happen whenever $v_2 > c$ but v_2 arbitrarily close to c.*

Proof. In the text. □

Hence, introducing a focused player a creates the possibility that an issue that should not be included in the agreement is instead included. This is due to the fact that player a now overvalues the transfer dimension, and may agree to include the second issue even when it is not efficient to do so.

Two-step negotiation. Suppose a cap $\hat{t}$ was imposed in period 1. Again, $\hat{t} > \bar{t}_0$ is equivalent to no cap being imposed. Furthermore, the consideration set as a function of the cap is given by[20]

$$k(\hat{t}) = \begin{cases} 2 & \text{if } 2h_a(c)c \leq h_a(\hat{t})\hat{t} \\ 1 & \text{if } h_a(c)c \leq h_a(\hat{t})\hat{t} \leq 2h_a(c)c \\ 0 & \text{otherwise.} \end{cases}$$

Hence, the player's utilities are

$$u^a(q_1, q_2, t) = h_a(\min\{\hat{t}, \bar{t}_o\})t - \sum_{i=1}^{k(\hat{t})} h_a\left(\frac{\bar{q}_i c}{v_i}\right)\frac{q_i c}{v_i},$$

$$u^b(q_1, q_2, t) = \sum_{i=1}^{k(\hat{t})} h_b(v_i)v_i - h_b(\min\{\hat{t}, \bar{t}_o\})t.$$

A cap $\hat{t} < \bar{t}_o$ reduces both player's focus on transfer t relative to no cap. Furthermore, under our assumptions,

$$\frac{h_a(\min\{\hat{t}, \bar{t}_o\})}{h_b(\min\{\hat{t}, \bar{t}_o\})} = \Delta(\min\{\hat{t}, \bar{t}_o\})$$

[20] For the same argument made in footnote 13, also here the consideration set is determined by the rationality constraint of player a.

converges to 1 as $\hat{t}$ decreases. That is, the player's sensitivities to the transfer dimension become similar as $\hat{t}$ decreases.

The Nash bargaining solution is

$$t(\hat{t}, k^\star(\hat{t})) = \min\left\{\frac{1}{2}\left(\sum_{i=1}^{k^\star(\hat{t})}\frac{h_b(v_i)}{h_b(\min\{\hat{t},\bar{t}_o\})}v_i + \frac{h_a(c)}{h_a(\min\{\hat{t},\bar{t}_o\})}ck^\star(\hat{t})\right), \hat{t}\right\}, \quad \text{(A.37)}$$

$$k^\star(\hat{t}) = \min\left\{\sharp\left\{v_i | \frac{\Delta(\min\{\hat{t},\bar{t}_o\})}{\Delta(c)} \geq \frac{h(c)\cdot c}{h(v_i)v_i}\right\}, k(\hat{t})\right\}. \quad \text{(A.38)}$$

The key observation is that, as $\hat{t}$ decreases, $\frac{\Delta(\min\{\hat{t},\bar{t}_o\})}{\Delta(c)}$ tends to one, which implies that, as long as $k(\hat{t})$ is not binding, the number of issues included in the agreement will eventually correspond to the efficient one. We can again define the largest transfer achievable via a cap $\hat{t}$ as

$$t^\star_{\max} : \frac{1}{2}\left(\sum_{i=1}^{k^\star(t^\star_{\max})}\frac{h_b(v_i)}{h_b(\min\{t^\star_{\max},\bar{t}_o\})}v_i + \frac{h_a(c)}{h_a(\min\{t^\star_{\max},\bar{t}_o\})}ck^\star(t^\star_{\max})\right) = t^\star_{\max}$$

Note that, analogously to what discussed in the main text, $\max_{\hat{t}} k^\star(\hat{t}) = k^\star(t^\star_{\max})$. That is $\hat{t} = t^\star_{\max}$ achieves both the largest transfer t and the largest number of issues included in the agreement.

Period-1, cold state. To analyze the cold state case, we simply rely on the results derived in Section 3: the players will use incomplete agreement to achieve the outcome that is closer to the outcome that would be achieved by rational players. Improving the efficiency of the agreement may require including both issues in the agreement, which could be achieved by imposing an appropriate cap $\hat{t}$.[21] Specific to this section, here players may want to limit the number of issues included in the agreement to 1, which also can be achieved via an appropriate cap. For example, whenever

$$\frac{v_1 + c}{2} \leq 2h_a(c)c$$

then at $\hat{t} = \frac{v_1+c}{2}$ only one issue is in the consideration set. At the same time $\frac{v_1+c}{2}$—the transfer that solves the bargaining problem with rational preferences—is achievable in period 2. $\hat{t} = \frac{v_1+c}{2}$ is therefore preferred to no cap whenever the players are trying

[21] This follows simply by continuity with the case discussed in the body of the paper.

to exclude the second issue from the agreement.

Period-1, warm state. In period 1 the two players utilities are

$$u^a(q_1, q_2, t) = h_a(t^\star_{\max})t - \sum_{i=1}^{k(t^\star_{\max})} h_a\left(\frac{\bar{q}_i c}{v_i}\right)\frac{q_i c}{v_i},$$

$$u^b(q_1, q_2, t) = \sum_{i=1}^{k(t^\star_{\max})} h_b(v_i)v_i - h_b(t^\star_{\max})t.$$

Again, when $\hat{t} = t^\star_{\max}$, period-1 and period-2 preferences are identical for both players. Therefore in equilibrium the players will impose $\hat{t} = t^\star_{\max}$, whether this cap maximizes material efficiency or not. This is the same result derived in the body of the text, and has the same exact intuition.

II Non-cooperative bargaining game

In the body of the paper, we solve each bargaining round using the Nash bargaining solution. This modeling choice allows us to focus on the sequence of agreements rather than on how each agreement is reached. In this section we argue that our results continue to hold when the bargaining game is modeled explicitly.

Assume that the bargaining parties play Rubinstein's game of alternating offers during each stage of the negotiation. Clearly, for a given consideration set (and given preferences) the Rubinstein (1982) result applies: if the players are arbitrarily patient the only subgame-perfect equilibrium of the game is the Nash bargaining solution. Hence, the main question of interest is how to define the players' consideration set in the game of alternating offers.[22]

The consideration is defined as the set of outcomes which are possible but will not arise in equilibrium. For this reason, we assume that in the game of alternating offers the consideration set of a player is given by:[23]

[22] More precisely, the relevant question is how to define the consideration set when making the first offer. The reason is that, in equilibrium, the first offer is accepted and the game ens immediately. Note also that, if the game does not end immediately (which never happens in equilibrium), the game repeats itself identically every other period. The consideration set of each player therefore does not change over time.

[23] To the best of our knowledge, we are the first to address the issue of how to define the consideration set in strategic situations.

1. The set of Nash equilibria of the game, which in the Rubinstein (1982) game of alternating offers coincides with the Pareto frontier of the bargaining set.
2. All allocations that are achievable from a Nash equilibrium via a unilateral deviation of this player.

Similar to a decision problem, whenever an outcome is achievable via the unilateral decision of a player, this outcome should be in his/her consideration set. This implies that, for example, the disagreement outcome is always in the consideration set of both players. Similarly, each player can always propose a bargaining solution to her opponent that is (weakly) better for the opponent compared to the equilibrium offer, and hence any such deviation should be in the consideration set. More interestingly, here the set of possible equilibria in the consideration set is the set of Nash equilibria of the game. Intuitively, among the set of possible equilibria, only one is subgame perfect and will be played. However, the players know that they could coordinate on another equilibrium, and this awareness affects the players' preferences.

Finally, note that as long as the bargaining set is closed and convex, the set of bargaining outcomes that can be achieved starting from the Pareto frontier and making a unilateral concession to the other player coincides with the entire bargaining set. Hence, for both players the consideration set of the game of alternating offers coincides with the bargaining set, which is what we assumed in the body of this paper (Assumption 2).[24] As a consequence, all of our results are robust to a non-cooperative implementation of the bargaining game.

III The consideration set

So far, we have assumed that the consideration set is equivalent to the entire bargaining set. Other alternative assumptions are possible. For example, the consideration set could be equal to the Pareto frontier of the bargaining set. In this case, the lower bounds of the consideration set are the minimum possible transfer and the minimum quality. It follows that period-1 agreements imposing a floor on transfer and quality may be used in order to manipulate the final bargaining outcome. In other words, under some alternative assumptions on the shape of the consideration set, the basic logic we derive goes through but the space of possible period-1 agreements changes.

However, we think that the consideration set should be equal to the bargaining set. Consider a generic bargaining problem with context-dependent preferences (such

[24]See also Appendix III for further discussions on how to define the consideration set.

as the one discussed in Section 3), solved using a Generalized Nash Bargaining (GNB) solution:

$$\max_{x \in C} \left\{ U^a(x,C)^{\nu_a} U^b(x,C)^{\nu_b} \right\},$$

subject to the players' rationality constraint, where the weights ν_a and ν_b represent the player's ability to influence the bargaining process. The GNB solution is equivalent to the Nash bargaining solution whenever $\nu_a = \nu_b$. The GNB solution is obtained starting from the same axioms underlying the Nash bargaining solution, and weakening the symmetry axiom (see Roth, 1979).

At the limit case $\nu_a = 0$, the bargaining problem is equivalent to a choice problem: choose the x preferred by player b subject to both players' rationality constraints. Hence, the players' rationality constraints define the choice set of the problem. As discussed in the Introduction, in a choice problem the consideration set is equivalent to the choice set. Therefore, to be consistent with the choice problem, when $\nu_a = 0$ the consideration set should be equal to the bargaining set.

Here we assume that the consideration set is independent of ν_a and ν_b, so that the players' *preferences* over each bargaining outcome do not depend on ν_a and ν_b (on the other hand, the solution to the bargaining problem will clearly depend on ν_a and ν_b). If the consideration set is independent of ν_a and ν_b, by consistency with the limit case $\nu_a = 0$ the consideration set should be equivalent to the entire bargaining set for all ν_a and ν_b, including for $\nu_a = \nu_b$.

Finally, note that a similar argument holds if we modify the bargaining problems analyzed in Sections 2 and 4 by adding a second player a, and assuming that player b signs an agreement with only one of the two players a. Due to Bertrand competition, the resulting bargaining problem is equivalent to a choice problem for player b. Again, if we assume that preferences over bargaining outcomes do not change with the number of players, then we should also assume that for any number of players the consideration set is equivalent to the bargaining set.

IV One-step negotiation as an outside option

Up to this point, we have assumed that if player a and b disagree in period 1, the negotiation ends and each player receives his/her outside option. That is, after disagreeing the players cannot go back to the negotiating table. Here we consider the opposite case: we assume that the disagreement outcome in period 1 is the one-step negotiation. In other words, if the players disagree over what incomplete agreement to sign, they can wait one period (at no cost) and negotiate over the entire bargaining set. Note that, in the model considered in the body of the paper, the players always have the option to jointly decide to bargain over the entire bargaining set in the following period. The difference here is that each player can trigger this outcome unilaterally.

For simplicity, we address this problem in the context of our extension of the simple bargaining problem. In that extension, when negotiating in one step player b is equally focused on transfer and quality and behaves as rational. However, in period 1 player b may be relatively more quality sensitive or relatively more transfer sensitive. In these cases, player b may prefer to reach a solution that is different from that of the one-step negotiation.

More formally, suppose that player a and b bargain in two steps over q and t. Assume that the negotiation has already reached its final stage, and that the boundaries of the consideration set are $\bar{t}$, $\underline{t}$, $\bar{q}$, $\underline{q}$. Because the solution to the one-step negotiation is $q = 1$ and $t = \frac{3}{4}$, a given bargaining outcome is in the consideration set if:

$$h(\bar{q} - \underline{q})(q - 1) - h(\bar{t} - \underline{t}) \left(t - \frac{3}{4} \right) \geq 0,$$

$$t - \frac{1}{2}q^2 \geq \frac{1}{4}.$$

In this bargaining round, surplus is positive if there exists a q such that when $t = \frac{1}{4} + \frac{1}{2}q^2$ (so that player a is indifferent), player b strictly prefers to reach an agreement, i.e.,

$$h(\bar{q} - \underline{q})(q - 1) - h(\bar{t} - \underline{t}) \frac{(q^2 - 1)}{2} > 0,$$

or

$$\frac{h(\bar{q} - \underline{q})}{h(\bar{t} - \underline{t})} \begin{cases} > \frac{q+1}{2} & \text{if } q > 1 \\ < \frac{q+1}{2} & \text{if } q < 1. \end{cases} \tag{A.39}$$

Therefore, as long as $\frac{h(\bar{q}-\underline{q})}{h(\bar{t}-\underline{t})} \neq 1$, it is always possible to find a $q \neq 1$ such that the two players strictly prefer to agree on q rather than disagreeing. If $\frac{h(\bar{q}-\underline{q})}{h(\bar{t}-\underline{t})} > 1$, player b would prefer to achieve a higher quality and transfer compared to the outside option. If instead $\frac{h(\bar{q}-\underline{q})}{h(\bar{t}-\underline{t})} < 1$, then player b prefers to achieve a lower quality and transfer compared to the outside option. Hence, as long as the focus weights are not equal, the players strictly prefer to reach an agreement within the current negotiation round rather than going to a fresh round of negotiations. If instead $\frac{h(\bar{q}-\underline{q})}{h(\bar{t}-\underline{t})} = 1$, player b's preferences in the last period of the two-step negotiation are identical to her preferences in the case of a one-period negotiation. Hence, the solution to the two-step negotiation is the same as that achieved in a fresh round of negotiations. Therefore, assuming an appropriate tie-breaking argument, the players will not disagree in period 1.