Benevolent mediation in the shadow of conflict*

Andrea Canidio[†] *and Joan Esteban*[‡]

Abstract

Before the start of a negotiation, the negotiating parties may try to affect the disagreement outcome by making socially-wasteful investments, such as purchasing weapons or asking for legal opinions. The incentives to make such investments depend on how the negotiation is conducted. We study the problem of a benevolent mediator who wishes to minimize pre-negotiation wasteful investments. Or main result is that the mediator should favor the strongest player, who has the lowest incentive to make wasteful investments. This result is robust to different specifications of the information available to the mediator. We therefore highlight a conflict between fairness and efficiency arising in negotiations.

JEL classification: D74; F51; J51.

Keywords: Bargaining; negotiation; mediation; conflict.

*We are grateful to Attila Ambrus, Mohamed Belhaj, Yann Bramoullé, Georgy Egorov, Thomas Gall, Sidartha Gordon, Patrick Legros, Dilip Mookherjee, Massimo Morelli, Andy Newman and Debraj Ray, seminar and conference participants at AMSE, Paris Dauphine, IMT Lucca, ThReD 2017, VIII IBEO workshop for their comments and suggestions. Joan Esteban gratefully acknowledges financial support from the AXA Research Fund, the Generalitat de Catalunya, and the Ministry of Economy and Competitiveness grant ECO2015-66883-P, and the National Science Foundation grant SES-1629370.

[†]Corresponding Author. IMT School for Advanced Studies, Piazza S. Francesco, Lucca, Italy; INSEAD, Boulevard de Constance, Fontainebleau, France; andrea.canidio@imt.it.

[‡]Institut d'Anàlisi Econòmica and Barcelona GSE, Campus UAB, 08193 Bellaterra, Barcelona, Spain; joan.esteban@iae.csic.es.

1 Introduction

Negotiations are often conducted under the shadow of conflict: in case an agreement is not reached, the negotiating parties will fight in a non-cooperative game. For example, two firms may negotiate a settlement under the shadow of a lawsuit; two countries may negotiate a treaty under the shadow of war; a government and a rebel group may negotiate a peace agreement under the shadow of a civil conflict; a wage negotiation may be conducted under the shadow of an industrial action.

Because the equilibrium payoffs of the potential conflict define the bargaining power of the negotiating parties, these parties may try to manipulate them. For example, prior to sitting at the bargaining table, a trade union may create a fund to support striking workers. Consequently, all the parties now anticipate a longer period of strikes and industrial actions in case the negotiation breaks down, thus shifting the disagreement point of the negotiation. Similarly, before the start of a negotiation over a territorial dispute, a country may invest in military equipment, or spend resources to conquer part of the disputed territory.[1] Also, before negotiating a legal settlement, a firm may ask for an additional legal opinion. These investments are a form of rent seeking, because they do not increase the total payoff to be shared during the negotiation, but only how this surplus is split. They are, therefore, socially wasteful.[2]

In this paper we consider the problem of a benevolent mediator who wishes to maximize social welfare, that is, to efficiently share the "peace dividend" (i.e, the aggregate benefit of finding an agreement rather than triggering a conflict) in a way that minimizes wasteful pre-negotiation investments. This benevolent mediator could be a person, an institution, or a country called in to mediate, for example, a civil conflict. In our model, the mediator immediately announces how he/she will conduct the negotiation.[3] After that, the negotiating parties make their investment,

[1] There is ample evidence that conflicts are reactivated prior to the beginning of peace negotiations. For example, the mass killing of civilians (thus permanently weakening the opponents) is significantly more probable during the process of democratization of a country. See Esteban, Morelli, and Rohner (2015).

[2] For a review of the literature arguing that the bargaining parties may spend resources before the start of the negotiation see Jackson and Morelli (2011) and the literature review in Meirowitz, Morelli, Ramsay, and Squintani (2015).

[3] Instead of an explicit announcement, we can equivalently think of established norms or protocols followed by a mediator in case of intervention. In this case, we want to know which norms or

and then the negotiation starts. The players' incentives to invest depend on the mediator's announcement, which will be chosen so to minimize pre-negotiation wasteful investments.

During the negotiation, the role of the mediator is to regulate the flow of offers between players. We show that, by deciding which players makes an offer in each period, the mediator can establish the share of peace dividend accruing to each player. We then study the extent to which the mediator can affect wasteful, pre-negotiation investments by setting the share of peace dividend allocated to each player, possibly conditional on the players' actions. We characterize the waste minimizing sharing rule. When the mediator cannot observe the investments made by the bargaining parties, the waste minimizing sharing rule is asymmetric, giving a larger share to the strongest player and inducing the weakest player not to invest at all, where "weak" and "strong" are defined by the outcome of the potential conflict in absence of investments. The intuition is that the weakest player has the strongest incentive to invest. Hence, the mediator reduces the marginal benefit of investing for this player by allocating a larger share of surplus to the strongest player. Furthermore, the more uneven is the distribution of initial strength, the lower the level of pre-negotiation, wasteful investment the mediator can achieve.

Therefore, the model highlights a trade off between equity and efficiency: to minimize wasteful, pre-negotiation investment the mediator should be biased toward the strongest player. This trade off is affected by the information structure. We show that the less the mediator can observe, the more the mediator should be biased in favor of the strongest player. This also implies that, when it comes to the choice of the mediator, the player who is expected to be the strongest will prefer the least informed mediator possible, while the weakest player will have the opposite interest. However, the mediator's actions may have unintended consequences when the player's power is endogenous. If the players expect the mediator to favor the strongest player, they may have an incentive to spend resources to increase their initial power. Hence, rather than decreasing wasteful investment, the actions of the mediator may simply shift wasteful investment to an earlier stage of the game.

We show that the mediator may prevent this from happening by organizing a pre-negotiation contest over the sharing rule. At the start of the negotiation, the mediator can require each player to make *concessions* to the other player, that is,

---

protocols minimize wasteful pre-negotiation investments.

take visible costly actions that benefit the other player. The level of concessions are then used to determine the sharing rule. If appropriately built, this contest can reduce the level of wasteful investment made by the player. Furthermore, under some conditions, the rules of the contest can be made independent from the player's initial power and therefore avoid the shifting of the investment to an earlier period.

Our main contribution is to embed the mediation in a game that includes not only the negotiation but also pre-negotiation actions by the contenders. This is in sharp contrast with the existing studies of mediation in economics, in which the mediator's sole role is to maximize surplus *within the negotiation*.[4] Introducing pre-negotiation actions changes the role of the mediator, because maximizing surplus requires considering also how the mediator can affect pre-negotiation wasteful investments. In order to study this problem in the most direct way, our modeling choices will imply that the mediator can always easily eliminate all inefficiencies arising at the negotiation stage. The only relevant problem will therefore be how to eliminate inefficiencies arising before the negotiation starts.[5]

Due to a lag between the moment the negotiation is announced and the moment the negotiation starts, in our model the parties negotiate only after having performed their investments. We are therefore related to the literature studying hold-up problems. In that literature, because two parties cannot write a complete contract some aspects of their relationship are negotiated ex-post—that is, after some productive investments are made. Crucially, the outside options of the ex-post negotiation can be decided contractually ex-ante in order to induce the efficient level of investments. For example, in the seminal work of Grossman and Hart (1986) the allocation of ownership determines the players' outside options in the ex-post negotiation. Closer to our paper, in Aghion, Dewatripont, and Rey (1994) the players can induce the

[4] See the review by Jackson and Morelli (2011) on the different reasons why bargaining failures and an inefficient war may occur. Recent papers that explore the role of third party intervention in reducing the probability of an inefficient breakdown of the negotiations are Goltsman, Hörner, Pavlov, and Squintani (2009), Hörner, Morelli, and Squintani (2015), Balzer and Schneider (2015). Recent papers that explore the role of third party intervention in reducing the time required to reach an agreement are Fanning (2016) and Basak (2017), who model the negotiation as a war of attrition.

[5] Of course, an interesting extension would be a model in which there is a meaningful interaction between inefficiencies at the negotiating stage and those arising before the negotiation, and where the mediator may need trade off one type of inefficiency for the other. See the conclusion for a discussion.

efficient level of investment by agreeing ex-ante on the outside option and the surplus share accruing to each player in the ex-post negotiation. Similarly to Aghion et al. (1994), also in our model, the surplus share accruing to each player in the ex-post negotiation is chosen so to maximize welfare. However, differently from Aghion et al. (1994), in our model the outside options are fully determined by the players' investment and therefore cannot be chosen ex ante. That is, in our framework if the investments cannot be contracted upon ex-ante, neither can the outside options of the players in the ex-post negotiation.

A number of authors already noted that the way the negotiation is conducted can affect pre-negotiation actions by the players. Both Esteban, Morelli, and Rohner (2015) and Garfinkel, McBride, and Skaperdas (2012) show that the surplus share accruing to each player can have an effect on decisions made prior to the beginning of the negotiation. In Esteban et al. (2015) the surplus share obtained by each party in a negotiation may affect the intensity of the pre-negotiation conflict. They show that an equal surplus-split rule may be welfare decreasing relative to an asymmetric surplus-split rule. Here, we build on this observation and derive the welfare-maximizing surplus split. Garfinkel et al. (2012) notice that by investing in arms players influence the probability of winning in a conflict—and hence the disagreement point—and the share of surplus in the case of a peaceful agreement. Their main result is that when fighting is not sufficiently destructive, arming will be unavoidable within the class of distribution rules they consider. Also related is the model in Anbarci, Skaperdas, and Syropoulos (2002), where each party starts by making wasteful investments in armaments. The paper compares the waste produced by three cooperative bargaining solutions: equal sacrifice, equal benefit, and Kalai-Smorodinski. The main result is that if players are symmetric equal sacrifice is the solution generating the lowest waste.

Closer to our paper, Meirowitz, Morelli, Ramsay, and Squintani (2015) consider the role of the mediator in reducing pre-negotiation wasteful investments. They compare mediated and unmediated negotiation and argue that mediated negotiation generates lower pre-negotiation wasteful investment in arms. In their framework, the mediator is only concerned with maximizing the probability that a settlement is reached. To achieve this objective, he will regulate the flow of information among parties, therefore affecting the precision of each player's belief relative to the other player's strength and the incentive to affect this strength via pre-bargaining investments. Hence, our model differs from Meirowitz et al. (2015) in two fundamental issues. First, in our model the goal of the mediator is to achieve an efficient outcome, which includes reducing wasteful pre-negotiation investments. Second, Meirowitz et al. (2015) assume that the bargaining players are ex-ante identical, while our paper is mostly concerned with how ex-ante differences affect the optimal negotiation procedure.

Finally, our paper contributes to an important debate in political science and international relations regarding the merits of biased mediation (see Svensson, 2014 for a review). This literature argues that a mediator who is biased may be more likely to achieve an agreement, where "bias" typically refers to a distortion in the mediator's preferences. For example, in Kydd (2003) a bargaining party is more likely to believe the information transmitted by a mediator if the mediator is biased in favor of this party. In our paper, the mediator is, in principle, unbiased because his goal is to maximize welfare. However, he may choose to be *strategically* biased in order to decrease total waste. Hence, here bias is an equilibrium outcome.

The remainder of the paper is organized as follows. The next section describes the model. The following three sections analyze the mediator's problem under different assumptions regarding what the mediator can observe: in Section 3 the mediator has full information; in Section 4 the mediator does not observe the wasteful investments made by players prior to the start of the negotiation; in Section 5 the mediator observes neither players' investments nor their initial power levels. Section 6 discusses what happens when the players' power levels are endogenous. Section 7 allows the mediator to organize a pre-negotiation contest over the sharing rule. The last section concludes. Unless otherwise noted, all proofs are in Appendix.

2 The model

A total payoff S needs to be shared between two players, 1 and 2. In the first stage of the game, the players make offensive and defensive investments. Then, in the second stage of the game, the players negotiate over the total payoff S with the help of a mediator. In case the negotiation breaks down, a conflict between the two players will break out. The players' payoffs in case of conflicts are determined by the initial offensive and defensive investments.

Period 1

In the first period of the game, both players can make an offensive investment o_i and a defensive investment d_i. An offensive investment by player i decreases the payoff of player $-i$ in case of a conflict. A defensive investment by player i increases the payoff of this same player in case of a conflict. As examples of offensive investment, a player may purchase ballistic missiles or collect evidence against the opponent to be used in a court case. As examples of defensive investments, a player may purchase antimissile system and bunkers, or move assets to jurisdictions where they are harder to seize in case the outcome of a lawsuit is negative. In the case of industrial conflict, the use of a resistance fund in a labor strike reduces the harm of conflict to workers and hence should be considered "defensive" accordingly with our conceptualization. In the same context, a firm may invest resources to make the relocation of the factory a credible threat, which should be considered "offensive." We assume that these two types of investments are always socially wasteful, in the sense that they have no effect on the total payoff to be allocated S.

Note that many investments are simultaneously offensive and defensive, in the sense that they simultaneously increase a player's utility and decrease the opponents utility in case of conflict (for example, hiring a very expensive but competent lawyer or purchasing tanks). Some other investments decrease both players utilities in case of conflicts, and are therefore mutually destructive (for example, nuclear weapons). Whether an investment is offensive, defensive (or both) depends also on the details of the conflict game. For example, in a tournament, each player's investment is simultaneously offensive and defensive, because when a player increases his/her probability of winning the opponent's probability of winning automatically decreases. In a war of attrition, some investments may increase the length of a conflict and therefore be mutually destructive. In general, for a given model of conflict, any type of investment can be expressed as a combination of offensive and defensive investments based on its effect on the players' payoffs. For this reason, as a first approximation here we only consider purely offensive or purely defensive investments.

Note also that these investments can be interpreted as pre-bargaining costly actions aimed at appropriating part of the surplus. For example, before the start of a negotiation over a territorial dispute, a country may invade part of the disputed territory. This costly action is formally identical to a defensive investment because it increases the payoff of the invading party in case of breakdown of the negotiation over the allocation of the remaining territory. Similarly, an offensive investment is a costly action that transforms a territory formerly in the hands of a given country into a contested area to be allocated via a negotiation.[6]

Formally, call ϕ_i the ex-ante "power" of player i—the payoff achieved by this player in the conflict game in case no investment is made. The ex-ante "power" of a player may depend on natural elements (e.g. the presence of mountains may make one country harder to attack) or by the merit of the legal dispute. It may also depend on prior investments in offensive/defensive technology (see Section 6 for more on this interpretation). Let us denote by $\underline{u}_i$ the payoff of player i in the conflict game, taking into account her ex-ante power ϕ_i, own defensive investment d_i and the opponent's offensive investment o_{-i}, that is, $\underline{u}_i = u(\phi_i, d_i, o_{-i})$. We specify this payoff function to

$$\underline{u}_i = \phi_i e^{-o_j}(2 - e^{-d_i}),\ i, j = 1, 2.$$

The key feature of this expression is weak separability: the marginal rate of substitution between d_i and ϕ_i is independent of o_{-i} and the one between o_{-i} and ϕ_i is independent of d_i. This implies that the rate at which player i can compensate for being weak ex-ante (i.e., having low ϕ_i) by making an investment in defense is independent on o_{-i}. Similarly, the rate at which player i can compensate for player $-i$ being strong ex-ante (i.e., having high ϕ_{-i}) by making an investment in offense is independent on d_{-i}. This key feature is justified by the fact that ϕ_i can be the result of investments in offensive and defensive technology made in prior periods, the marginal effect of which should not depend on the current level of investments.[7] Clearly, weak separability is satisfied if and only if $u_i = \psi[f(\phi_i)g(d_i)h(o_{-i})]$. Our assumed payoff function is a member of this class, that we use for the sake of convenience.

Finally, we impose the restriction $S \geq 2(\phi_1 + \phi_2)$, so that sharing S in a negotiation dominates the conflict payoff *for every possible level of defensive and offensive*

[6] We thank Attila Ambrus for suggesting this interpretation.

[7] One can alternatively assume that the marginal rate of substitution between d_i and ϕ_i is a function of o_{-i}. But then o_{-i} should affect the relative benefit not only of current defensive investments, but also of past defensive investments. It follows that ϕ_i should be a function of the investments o_{-i} and d_i, a complication that we wish to avoid. Note also that our specification abstracts away issues related to risk, that are not central to our analysis.

investment, even when offensive investments are both zero and defensive investments are at their maximum. The marginal cost of investing in defensive and offensive technology are c_d and c_o, assumed constant.

Period 2

In period 2, with the help of a mediator the players negotiate over how to split the ex-post surplus (also called "peace dividend")

$$S - \underline{u}_1 - \underline{u}_2.$$

In line with the existing literature, the mediator's role is to avoid inefficient negotiating outcomes. However, contrary to most of the existing literature, here we assume that there is full information among players. Inefficient bargaining outcomes are due to the lack of a proper bargaining protocol, which the mediator has the power to impose. Inefficient negotiating outcomes are thus easily eliminated by the mediator, so that the only remaining inefficiencies are those emerging before the negotiation starts (which are the subject of this study).

Negotiating without the mediator. A number of authors showed that bargaining games may have inefficient outcomes also under complete information among the players.[8] The simplest possible model of this kind is the finitely-repeated version of the Nash demand game (see Nash, 1953). Absent the mediator, in each period of the negotiation, both players make offers. After observing both offers, either player can trigger a conflict. If no conflict is triggered and the offers are not feasible (i.e. the sum of the utilities exceeds S), the players start a new period. If instead the offers are feasible, the negotiation ends with an agreement. If after n periods there is no agreement, the conflict is triggered. Players' discount factors are $\beta_i \in (0, 1)$, so that any payoff achieved by player i in t periods from the start of the negotiation is discounted today by the factor β_i^t. The discount factors β_i, the outside options $\underline{u}_i$ and S are common knowledge among the players.

[8] For example, Sákovics (1993) and Perry and Reny (1993) consider continuous-time bargaining problems in which the players can time the offers strategically, which therefore could be simultaneous or alternated. Avery and Zemsky (1994) allow the players to destroy surplus between periods. Ponsatí and Sákovics (1998) allow the players to quit the negotiating table anytime. Kambe (1999), assumes that players need to agree in each period on who will make the offer. All these papers show that inefficient subgame perfect Nash equilibria (SPNEs) exist.

This game has multiple subgame perfect Nash equilibria (SPNEs), including very inefficient ones (see Binmore (1987, Section 4), who argues that this game has an "embarrassment of equilibria"). To see this, note that there exists a SPNE in which, in every period and independently on prior history, player 1 demands a share $\gamma \in [0, 1]$ of the ex-post surplus and player 2 demands a share a share $1 - \gamma$ of the ex-post surplus (this is actually the set of Nash equilibria of the static Nash demand game). It follows that there is a SPNE in which an agreement is reached immediately and player 1 earns his outside option, and another SPNE in which an agreement is reached immediately and player 2 earns his outside option. These SPNEs can be used to build inefficient SPNEs in which a given surplus split is achieved with delay. In these SPNEs, each player demands the entire ex-post surplus until a given period, and then agree on some sharing rule. In case player i deviates from this strategy profile, the players will switch to playing the SPNE leaving player i to his outside option. Any such strategy profile is an equilibrium provided that each player prefers to follow it rather than earning his outside option immediately.[9]

Negotiating with the help of a mediator. The role of the mediator is therefore to eliminate simultaneous offers so that the unique SPNE of the bargaining game is efficient. To this effect, the mediator imposes the following structure to the negotiation: in every period one player makes an offer and the other player can accept, reject and move to the next period, or reject and trigger a conflict. Hence, the mediator here is a third party—a large country, an arbitrator with some legal power—who can coerce players into respecting a specific negotiating protocol.[10] Using the language of the negotiation literature, we say that the mediator uses his power to transform the negotiation from *no-caucus* negotiation—in which all parties meet simultaneously, discuss freely and can make offers at any time—to impose *caucus mediation*—a mediation in which the mediator meets separately with the

[9] Chatterjee and Samuelson (1990) consider an infinitely repeated version of the model considered here (but with outside options normalized to zero) and shows a stronger result: that the set of *perfect* equilibrium outcomes consists of every possible individually rational outcome.

[10] The negotiations between the Colombian government and the FARC provide a good example. They have taken place in Cuba because the assumed leverage of this government on the FARC. In Germany, for many years the negotiations between unions and employers have been chaired by the government because its leverage on the parties.

parties involved and relays offers and information between them.[11]

Crucially, by choosing the probability that each player makes an offer, the mediator can determine the surplus share accruing to each player in the unique SPNE of the negotiation game. Suppose that the mediator announces that one of the two players will propose with probability one in each period. If the last stage of the negotiation is reached, the permanent proposer will earn the entire surplus. In the second-to-last period, therefore, because of discounting, the responder prefers to allocate all surplus to the proposer rather than to go to the following period, and hence if this stage is reached the proposer, again, earns the entire surplus. By backward induction, in the unique SPNE, an agreement is reached immediately and the proposer earns the entire surplus.

By exploiting the players' risk neutrality, the mediator can achieve any surplus split by randomizing which player becomes the permanent proposer. That is, if the mediator announces that player 1 will be the permanent proposer with probability $\gamma \in [0,1]$, the mediator is effectively allocating a share γ of the ex-post surplus to player 1 (see Figure 1).[12] Hence, given the mediator's choice, the players payoffs are

$$U_1 = \underline{u}_1 + \gamma \left(S - \underline{u}_1 - \underline{u}_2\right), \qquad U_2 = \underline{u}_2 + (1-\gamma)\left(S - \underline{u}_1 - \underline{u}_2\right).$$

Notice that by choosing the sharing rule the mediator can determine the payoff achieved by each player only if he observes $\underline{u}_1$ and $\underline{u}_2$. When this is the case, the mediator's role is similar to that of an arbitrator. For most of the paper we will instead assume that the outside options $\underline{u}_i$ and S are not known by the mediator, who is therefore an outsider knowing less about the dispute than the players. In this case, the players' payoffs will depend both on the sharing rule announced by the

[11] Hoffman (2011) discusses the choice between caucus and non-caucus mediation in various environments, from commercial and family mediation, to international relations. In the context of international negotiations, caucus mediation is called *shuttle diplomacy.*

[12] If instead the players are risk-averse, the mediator may maximize welfare by announcing a fixed probability of proposing in each period, or by transforming the game into one of alternating offers. Doing so, however, creates the incentive for the players to perform another type of wasteful, pre-bargaining investment: one that increase their discount factors so to be more patient in the negotiation. A famous historical example of this type of investment is the 2-year lease for a house paid by the Vietnamese delegation at the onset of the Paris negotiations to end the Vietnam war (see Raiffa, 1982, page 16). Studying how the mediator should solve the tradeoff between reducing the investments in becoming more patient and reducing players' exposure to risk is left for future work.

mediator and on the players' actions. Using the taxonomy of methods of mediation in Fisher (2012), we therefore model *power mediation*, that is, a mediation in which the mediator has some power over the outcome of the mediation but cannot fully determine it (as opposed to an arbitrator).[13]

Finally, note that choosing the sharing rule is equivalent to choosing the weight attached to a player's utility in a Generalized Nash Bargaining (GNB) problem. Hence, equivalently, we can think of the mediator as choosing a specific bargaining solution among all those that satisfy Pareto optimality, independence of irrelevant alternatives, invariance to rescaling of utility (which are the axioms that characterize the GNB solution). Interestingly Luce and Raiffa (1957) interpreted the axioms of a bargaining solution as normative properties of a negotiation/arbitration scheme (see Luce and Raiffa, 1957, pp. 122–123). According to this interpretation, starting from the axioms of the Generalized Nash Bargaining solution we introduce an additional normative statement about how the mediator should behave (i.e., minimize pre-negotiation waste).

Information structure

We assume that the players are able to observe everything: the ex-ante power ϕ_1 and ϕ_2, the total payoff to be shared S, the level of investment in offensive and defensive technology set by each player. We therefore abstract away from the usual role of the mediator as a filter of the information flow between the two players, and only focus on its role in determining the negotiation structure. We will analyze the mediator's problem under different assumptions regarding what the mediator observes. First, we will consider the case of full information. Then we will assume that only ϕ_1 and ϕ_2 are observable. Finally, we will assume that the mediator does not observe anything, but has beliefs over ϕ_1 and ϕ_2.

[13] Because a given division of the peace dividend is achieved by regulating the communication between players, we establish an interesting connection between what Fisher (2012) calls *conciliation*, that is, the role of the mediator in establishing a communication link between the players, and power mediation. See also a similar taxonomy by Bercovitch (1997), who distinguishes between a mediator's communication strategies (i.e., transmitting messages) and manipulative strategies (i.e., influencing the outcome of the negotiation).

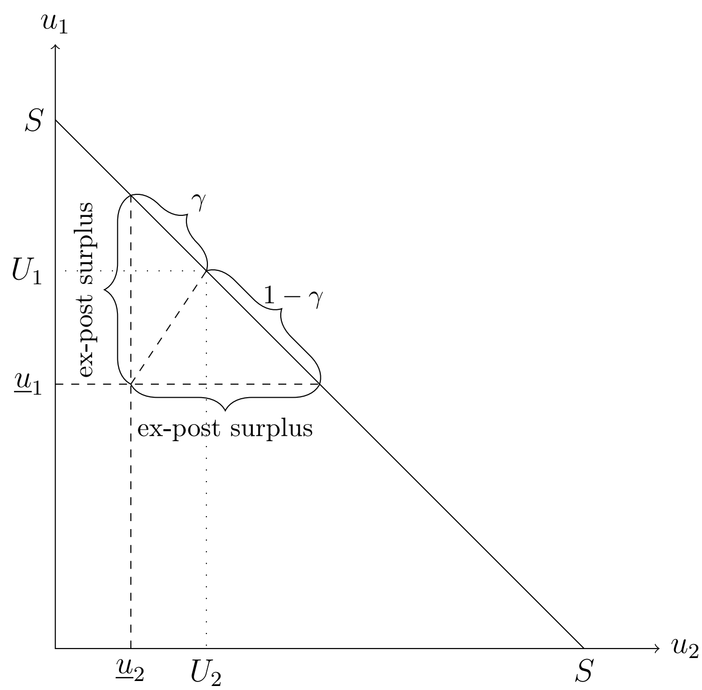

Fig. 1: Solution to the negotiation for given γ

The mediator's objective

We assume that the mediator is benevolent: his objective is to maximize welfare. The mediator cannot commit to destroying welfare ex-post (that is, by implementing an inefficient negotiation protocol). Hence, the mediator's objective is to minimize socially-wasteful expenditure in offensive and defensive technology, $c_o(o_1 + o_2) + c_d(d_1 + d_2)$, and his only instrument is γ. Depending on what the mediator can observe, the ex-post surplus split can be contingent on the initial power ϕ_1 and ϕ_2, and/or on the investment made by each player. Figure 2 summarizes the timing of the game.

3 Full information

Without loss of generality, assume that $\phi_1 \geq \phi_2$. When the mediator observes everything, he can announce that he will implement a sharing rule γ in case of zero investment, and simultaneously announce that he will allocate zero ex-post surplus to the player making positive investments in either defensive or offensive technology.

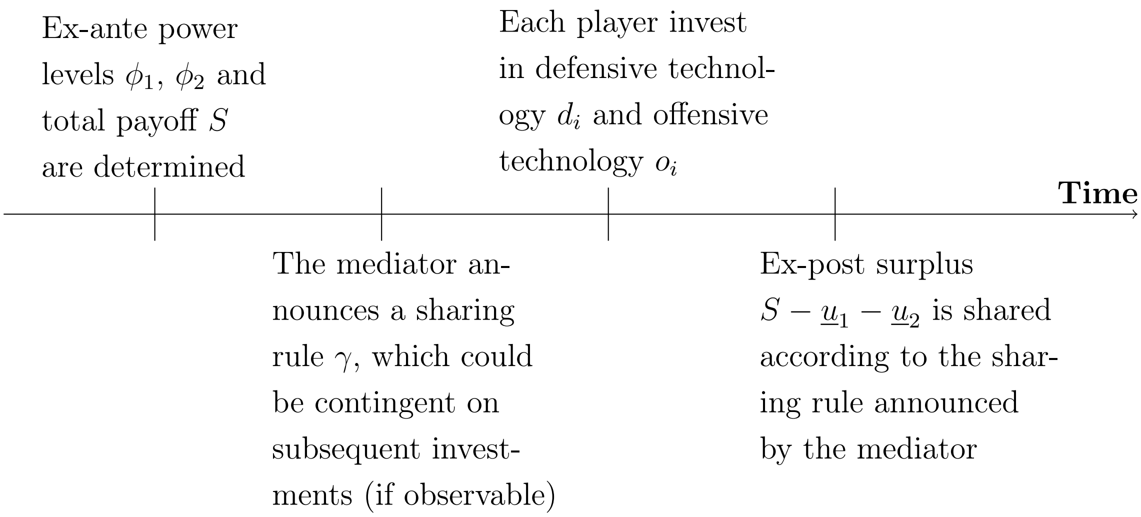

Fig. 2: Timeline

Given this announcement, there is an equilibrium in which the ex-post surplus share is γ and there is no investment whenever no level of defensive investment delivers higher payoff to either player, that is whenever:

$$\begin{aligned}\phi_1 + \gamma(S - \phi_1 - \phi_2) &\geq \max_{d_1} \left\{(2 - e^{-d_1})\phi_1 - c_d d_1\right\} \\ &= 2\phi_1 - \min\left\{\phi_1, c_d\left(1 + \log\left(\frac{\phi_1}{c_d}\right)\right)\right\}\end{aligned} \tag{1}$$

$$\begin{aligned}\phi_2 + (1-\gamma)(S - \phi_1 - \phi_2) &\geq 2\max_{d_2} \left\{(2 - e^{-d_2})\phi_2 - c_d d_2\right\} \\ &= 2\phi_2 - \min\left\{\phi_2, c_d\left(1 + \log\left(\frac{\phi_2}{c_d}\right)\right)\right\}.\end{aligned} \tag{2}$$

The mediator may also announce that in case *both* players make a positive investment, player 1 will receive $\phi_1(1 - e_1^d)$, that is, player 1's outside option had player 2 not invested. The rest of the surplus will go to player 2. Given this announcement, if (1) holds, player 1 will never want to invest, and hence in equilibrium we never have both players investing.

It follows that when the above two inequalities hold at a specific γ there exists an announcement for which no investment is the unique equilibrium. It is also easy to see that, when either (1) or (2) are violated at a specific γ, then there is no announcement for which no investment is an equilibrium, the reason being that allocating zero ex-post surplus to a player in case of positive investment is the harshest punishment that the mediator can impose. Simple algebra show that, under the assumption $S > 2(\phi_1 + \phi_2)$, there always exists a γ that satisfies both conditions and therefore generates zero investment.[14]

Proposition 1. *In equilibrium, the mediator can implement any sharing rule:*

$$\gamma \in \left[\frac{\phi_1 - \min\left\{\phi_1, c_d\left(1+\log\left(\frac{\phi_1}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2}, 1 - \frac{\phi_2 - \min\left\{\phi_2, c_d\left(1+\log\left(\frac{\phi_2}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2}\right] \neq \emptyset$$

and generate zero investment. No other sharing rule can be implemented and generate zero investment.

Proof. In the text. □

We can now address the question of whether the mediator can eliminate all investment *and* be fair at the same time, that is whether

$$\frac{1}{2} \in \left[\frac{\phi_1 - \min\left\{\phi_1, c_d\left(1+\log\left(\frac{\phi_1}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2}, 1 - \frac{\phi_2 - \min\left\{\phi_2, c_d\left(1+\log\left(\frac{\phi_2}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2}\right]$$

Simple algebra shows that

$$\frac{1}{2} < 1 - \frac{\phi_2 - \min\left\{\phi_2, c_d\left(1+\log\left(\frac{\phi_2}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2},$$

which implies that, there is a *tradeoff between efficiency and fairness* if and only if

$$\frac{\phi_1 - \min\left\{\phi_1, c_d\left(1+\log\left(\frac{\phi_1}{c_d}\right)\right)\right\}}{S-\phi_1-\phi_2} > \frac{1}{2}$$

or

$$3\phi_1 + \phi_2 > S + 2\min\left\{\phi_1, c_d\left(1+\log\left(\frac{\phi_1}{c_d}\right)\right)\right\}.$$

Hence, the tradeoff between efficiency and fairness is more likely to emerge whenever the cost c_d is small, S is small, or ϕ_2 is large. The reason is that a small S or a large ϕ_2 reduce the ex-post surplus to be shared in the negotiation and the benefit

[14] This is easy to check by considering the largest possible defensive investment $d_1 = d_2 = 2$ (or alternatively the smallest possible cost of investing $c_d = 0$).

for player 1 of accepting $\gamma = \frac{1}{2}$ rather than deviating. This tradeoff is also more likely to emerge whenever ϕ_1 is large. Again, as ϕ_1 increases the ex-post surplus decreases. Furthermore, ϕ_1 large also increases the benefit for player 2 to deviate and make a large defensive investment. When this tradeoff emerges, to eliminate the incentives to invest the mediator needs to be biased in favor of player 1, who is the strongest player and therefore has the strongest incentive to invest.

4 Unobservable investments

If instead the mediator does not observe the investment made by the players, he can only announce an unconditional surplus split γ, determined after the ex-ante power levels ϕ_1 and ϕ_2 are realized.[15]

Player 1's problem and Player 2's problem are, respectively:

$$\max_{o_1,d_1} \left\{ \phi_1 e^{-o_2}(2-e^{-d_1}) + \gamma \Big[S - \phi_1 e^{-o_2}(2-e^{-d_1}) - \phi_2 e^{-o_1}(2-e^{-d_2}) \Big] \right\} - c_o o_1 - c_d d_1.$$

$$\max_{o_2,d_2} \left\{ \phi_2 e^{-o_1}(2-e^{-d_2}) + (1-\gamma) \Big[S - \phi_1 e^{-o_2}(2-e^{-d_1}) - \phi_2 e^{-o_1}(2-e^{-d_2}) \Big] \right\} - c_o o_2 - c_d d_2.$$

Figures 3 and 4 illustrate the relevant tradeoffs in the choice of investment mix. Figure 3 compares the effect of a given offensive investment by player 1 on his final payoff, for different values of γ. Figure 4 does the same for a given defensive investment by player 1. They show that if γ is large—i.e., the sharing rule is extremely biased in favor of player 1—player 1's investment in offensive weapons produces a much higher benefit than defensive weapons. We have the opposite effect if γ is low. Intuitively, as the share of ex-post surplus received increases, a player's payoff depends more and more on the opponent's outside option rather than on his own outside option. In the limit case in which all ex-post surplus is allocated to player i, the final payoff for both players only depends on player $-i$'s outside

[15] Because the players observe each other's investment, the mediator may try to elicit this information from them. Note, however, that in case the players' reports on the investment levels do not match (so that at least one of the two players is lying), the mediator is unable to punish both players at the same time. The reason is that the mediator cannot comity to destroying surplus ex-post. Hence, his only instrument is γ, and therefore if one player is punished by receiving a low surplus share, the other player must be rewarded. It follows that there is no equilibrium in which the players report truthfully. We do, however, show in Section 7 that the mediator may benefit from organizing a contest for γ in which the sharing rule depends on some costly actions taken by the players.

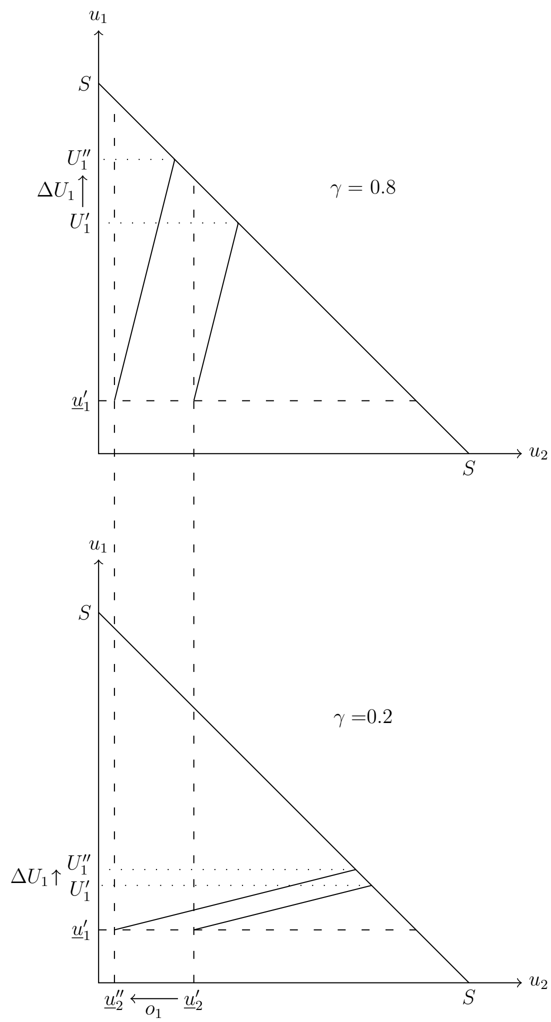

Fig. 3: Benefit of player 1's offensive investment for different values of γ.

option. As a consequence, the incentive to degrade the opponent and make an offensive investment increases with the share of ex-post surplus received. Similarly, as the share of ex-post surplus received *decreases*, a player's payoff depends more and more on his own outside option rather than on his opponent's. It follows that, as the share of ex-post surplus received decreases, the incentive to make a defensive investment increases.

The first-order conditions of the problem yield:

$$o_1 = \log\left(\frac{\gamma\phi_2(2-e^{-d_2})}{c_o}\right) \text{ if } \frac{\gamma\phi_2(2-e^{-d_2})}{c_o} \geq 1 \text{ else } o_1 = 0, \tag{3}$$

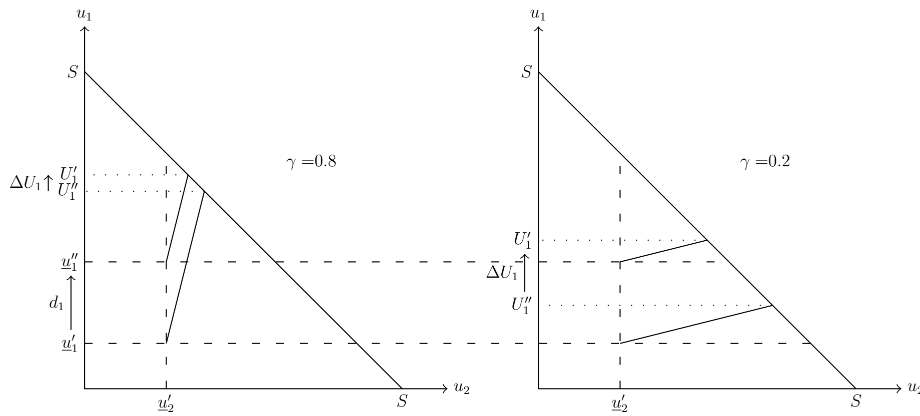

Fig. 4: Benefit of player 1's defensive investment for different values of γ.

$$d_1 = \log\left(\frac{(1-\gamma)\phi_1 e^{-o_2}}{c_d}\right) \text{ if } \frac{(1-\gamma)\phi_1 e^{-o_2}}{c_d} \geq 1 \text{ else } d_1 = 0, \tag{4}$$

$$o_2 = \log\left(\frac{(1-\gamma)\phi_1(2-e^{-d_1})}{c_o}\right) \text{ if } \frac{(1-\gamma)\phi_1(2-e^{-d_1})}{c_o} \geq 1 \text{ else } o_2 = 0, \tag{5}$$

$$d_2 = \log\left(\frac{\gamma\phi_2 e^{-o_1}}{c_d}\right) \text{ if } \frac{\gamma\phi_2 e^{-o_1}}{c_d} \geq 1 \text{ else } d_2 = 0. \tag{6}$$

Note that taking (3) and (6) together we have that o_1 and d_2 are best response of each other and similarly if we take (4) and (5) together we define the reciprocal response of d_1 and o_2.

By (3) and (5), the incentive to invest in offensive technology increases in the ex-post surplus share received, in the opponent's ex-ante power, and in the opponent's defensive investment. As discussed earlier, as the share of ex-post surplus received increases, a player's payoff depends more and more on the opponent's outside option, increasing the incentive to make offensive investment. Conditions (3) and (5) show that this effect is stronger the higher the opponent's outside option—which is determined by the combination of his ex-ante power and his defensive investment. That is, an increase in the share received by player i has a smaller impact on this player's utility when player $-i$ is weak than when player $-i$ is strong.

Similarly, by 4 and 6, the incentive to invest in defensive technology decreases with the share of ex-post surplus received, increases with own ex-ante power and decreases with the opponent's offensive investment. Again, as the surplus share received decreases, a player's payoff depends more and more on his own outside option, increasing the level of defensive investment. This effect is stronger the stronger is the player—where his strength is determined by his ex-ante power and the opponent's investment in offensive technology.

Putting the best responses together, we can characterize the Nash equilibrium of the game:

Lemma 1. *The Nash equilibrium of the game is as follows:*

- *If $c_o \leq c_d$,*

$$o_1 = \max\left\{\log\left(\frac{\gamma\phi_2}{c_o}\right), 0\right\}, o_2 = \max\left\{\log\left(\frac{(1-\gamma)\phi_1}{c_o}\right), 0\right\}$$

  and $d_1 = d_2 = 0$.

- *If $c_o > c_d$,*
  - *for (o_1, d_2)*
    - *for $\gamma \geq \frac{c_o+c_d}{2\phi_2}$ we have $o_1 = \log\left(\frac{2\gamma\phi_2}{c_o+c_d}\right)$ and $d_2 = \log\left(\frac{c_o+c_d}{2c_d}\right)$;*
    - *for $\frac{c_d}{\phi_2} \leq \gamma \leq \frac{c_o+c_d}{2\phi_2}$ we have $o_1 = 0$ and $d_2 = \log\left(\frac{\gamma\phi_2}{c_d}\right)$.*
    - *for $\gamma < \frac{c_d}{\phi_2}$, we have $o_1 = 0$ and $d_2 = 0$.*
  - *for (d_1, o_2)*
    - *for $1-\gamma \geq \frac{c_o+c_d}{2\phi_1}$ we have $o_2 = \log\left(\frac{2(1-\gamma)\phi_1}{c_o+c_d}\right)$ and $d_1 = \log\left(\frac{c_o+c_d}{2c_d}\right)$;*
    - *for $\frac{c_d}{\phi_1} \leq 1-\gamma \leq \frac{c_o+c_d}{2\phi_1}$ we have $o_2 = 0$ and $d_1 = \log\left(\frac{(1-\gamma)\phi_1}{c_d}\right)$.*
    - *for $1-\gamma < \frac{c_d}{\phi_1}$, we have $o_2 = 0$ and $d_1 = 0$.*

The Nash equilibrium depends on the cost of offensive and defensive investments. When $c_d \geq c_o$, in equilibrium there never is any investment in defensive technology. Remember that defensive investment is decreasing in offensive investment. When the cost of offensive investment is low relative to the cost of defensive investment, each player will make a large investment in offensive technology and, in equilibrium, drive the incentive to invest in defensive technology of the other player to zero.

If instead $c_d < c_o$, for extreme sharing rules, (i.e. $\gamma \geq \frac{c_o+c_d}{2\phi_2}$ or $\gamma \leq 1-\frac{c_o+c_d}{2\phi_1}$), one player invests only in offensive technology while the other invests only in defensive technology. Instead, for intermediate sharing rules, players only make defensive investments and no offensive investments.[16] Intuitively, because of the cost advantage, players are more likely to make a defensive investment, the more so the larger the share of ex-post surplus going to the other player. However, the offensive investment made by player i increases with the defensive investment made by player $-i$, which implies that for extreme sharing rules one player makes a defensive investment while the other player makes an offensive investment.

The Nash equilibrium also depends on γ. Suppose the mediator allocates most of the ex-post surplus to player 2 by setting γ close to zero. Depending on c_o and c_d, Player 1 may make positive defensive investment but no offensive investment. Player 2 will make a positive offensive investment and no defensive investment. As γ increases, the players substitute one type of investment with the other. For γ close to one, the situation is reversed, with Player 1 making only an offensive investment, and Player 2 possibly only a defensive investment. The key observation is that the choice of γ determines whether the fight will be over Player 1's outside option (with Player 2 attacking it and Player 1 defending it) or over Player 2's outside option (with Player 1 attacking it and Player 2 defending it). The following proposition characterizes the solution to the mediator's problem.

Proposition 2. *Whenever*

$$\min\{c_o, c_d\} \geq \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}, \tag{7}$$

then any

$$\gamma^\star \in \left[1 - \frac{\min\{c_o, c_d\}}{\phi_1}, \frac{\min\{c_o, c_d\}}{\phi_2}\right]$$

drives wasteful investment to zero. Whenever (7) *is violated, the mediator minimizes waste by setting*

$$\gamma^\star = 1 - \frac{\min\{c_o, c_d\}}{\phi_1}.$$

At the waste-minimizing γ player 2's offensive investment and player 1's defensive investments are zero. If $c_o \leq c_d$ player 1's offensive investment is positive but

[16] The reader may wonder why a player may make a defensive investment when the other player is not making any offensive investment. To understand this, remember that players may have made offensive or defensive investments before the game start. These investments are embedded into the initial power levels ϕ_1 and ϕ_2. Hence, the result here is that, for intermediate sharing rule players make additional defensive investment but no additional offensive investment.

player 2's defensive investment is zero. If $c_d < c_o$, player 2's defensive investment is positive.

Condition (7) implies that, for given $\phi_1 + \phi_2$, the distribution of initial power is sufficiently uneven. In this case, the mediator can completely eliminate the players' incentives to invest. Instead, whenever the distribution of initial power levels is sufficiently equal so that (7) is violated, the mediator is unable to eliminate wasteful investment. As we discussed earlier, the choice of the sharing rule determines whether the fight is over player 1 or player 2 outside option. The proposition shows that total waste is minimized when the mediator sets $\gamma^\star = 1 - \frac{\min\{c_o, c_d\}}{\phi_1}$, which is the sharing rule that eliminates the fight over player 1's outside option. This sharing rule may generate a fight over player 2's outside option. However, because of the difference in initial power levels, player 1's incentive to perform offensive investment is lower than player 2, and the opposite holds for the incentives to perform defensive investments. Hence, total waste is minimized when the fight is over player 2's outside option, with player 1 attacking and player 2 defending, rather than player 1's.

If the distribution of power is uneven so that 7 holds, $\gamma^\star = \frac{1}{2}$ eliminates all investments if and only if $\phi_1 \leq 2\min\{c_o, c_d\}$. If instead (7) is violated simple algebra shows that $\gamma^\star$ is greater than $1/2$. Hence, there is no trade off between fairness and efficiency if and only if $\phi_1 \leq 2\min\{c_o, c_d\}$. In this case, both players' initial power levels are low (remember that $\phi_2 \leq \phi_1$), and the incentives to invest of both players are also low. If instead $\phi_1 > 2\min\{c_o, c_d\}$ fairness and efficiency are mutually exclusive, and the mediator minimizes waste by favoring the strongest player.[17]

5 Unobservable ex-ante power levels

Assume now that the mediator does not observe neither the investment levels, nor the ex-ante power levels, nor S. For simplicity, let us only consider the case $c_o \leq c_d$.

[17] Contrary to the case of observable investments (Section 3), here S and ϕ_2 play no role in determining the emergence of the tradeoff between fairness and efficiency. The reason is that, here, the sharing rule does not change with the investments made by the players. Hence, the benefit of shifting a player's outside option is independent on the other player's outside option or the total payoff S. When the sharing rule reacts to the investment as in Section 3, instead, the size of the ex-post surplus is a relevant element in determining the existence of the tradeoff.

For given γ, the optimal investments by each player are the same as derived in Section 4. It follows that, for a given belief over the distribution of ex-ante power levels, the mediator solves:[18]

$$\min_{\gamma}\left\{\Pr\left(\phi_2>\frac{c_o}{\gamma}\right)E\left[\log\left(\frac{\gamma\phi_2}{c_o}\right)\Big|\phi_2>\frac{c_o}{\gamma}\right]+\Pr\left(\phi_1>\frac{c_o}{1-\gamma}\right)E\left[\log\left(\frac{(1-\gamma)\phi_1}{c_o}\right)\Big|\phi_1>\frac{c_o}{1-\gamma}\right]\right\}.$$

That is, the mediator minimizes each player's probability of investing times the level of investment in case a player invests. The mediator's objective function can be written explicitly whenever ϕ_1 and ϕ_2 are drawn from two Pareto distributions.

Lemma 2. *Assume that ϕ_1 and ϕ_2 are drawn from two Pareto distributions with parameters $\kappa_1>0$ and $\kappa_2>0$, and minimum values $\underline{\phi}_1>0$ and $\underline{\phi}_2>0$ respectively. Then, the mediator minimizes*

$$\begin{cases}\left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\kappa_2}+\left(\log\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)+\frac{1}{\kappa_1}\right) & \text{if } \gamma\le\min\{1-\frac{c_o}{\underline{\phi}_1},\frac{c_o}{\underline{\phi}_2}\}\\ \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\kappa_2}+\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{\kappa_1} & \text{if } 1-\frac{c_o}{\underline{\phi}_1}\le\gamma\le\frac{c_o}{\underline{\phi}_2}\\ \left(\log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right)+\frac{1}{\kappa_2}\right)+\left(\log\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)+\frac{1}{\kappa_1}\right) & \text{if } \frac{c_o}{\underline{\phi}_2}\le\gamma\le1-\frac{c_o}{\underline{\phi}_1}\\ \left(\log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right)+\frac{1}{\kappa_2}\right)+\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{\kappa_1} & \text{if } \max\{1-\frac{c_o}{\underline{\phi}_1},\frac{c_o}{\underline{\phi}_2}\}<\gamma\end{cases} \quad (8)$$

Without loss of generality, we assume that $\underline{\phi}_1>\underline{\phi}_2$. As the parameters κ_1 and κ_2 increase, the masses of the two distributions become more and more concentrated near their minimum value. It follows that for κ_1 and κ_2 arbitrarily large, the mediator's problem converges to the one studied in the previous section. On the other hand, as the parameters κ_1 and κ_2 decrease, the tails of the two Pareto distributions become thicker, with higher ϕ_i becoming more likely and therefore increasing the expected investment levels by the two players. In particular, when $\kappa_i<2$ the tails of the distribution are so thick that $Var[\phi_i]$ is not well defined; when $\kappa_i<1$ the tails of the distribution are even thicker and also $E[\phi_i]$ is not well defined. As $\kappa_1\to0$, the mediator's belief becomes an improper prior.

We interpret κ_i as a measure of how informed the mediator is about player i. If the mediator is well informed, then κ_i is large, the tail of the Pareto distribution is thin and the probability that player i turns out to be extremely powerful is low. On the other hand, the mediator could be completely uninformed: the only thing

[18] S is also unobserved by the mediator. However, the way in which it is determined is not relevant as long as for every realization of ϕ_1 and ϕ_2 the realization of S is such that $2(\phi_1+\phi_2)\le S$, so that also here finding an agreement is preferred to war for every level of investment.

he may know is that player i's power is above a certain threshold. In this case κ_i is small, the tail of the Pareto distribution is thick and there is a non-negligible probability that player i turns out to be extremely powerful.

The following proposition characterizes the solution to the mediator's problem.

Proposition 3. *The waste-minimizing sharing rule is weakly increasing in $\underline{\phi}_1$, weakly decreasing in $\underline{\phi}_2$, weakly increasing in κ_2, weakly decreasing in κ_1.*[19] *Furthermore:*

- *for $\kappa_1,\ \kappa_2 \to \infty$, the players power levels are $\underline{\phi}_1$ and $\underline{\phi_2}$ with almost certainty, and the waste minimizing sharing rule converges to the one derived in Proposition 2.*
- *for $\kappa_1,\ \kappa_2 \leq 1$ the waste minimizing sharing rule is*

$$\gamma^\star = \begin{cases} 1 & \text{if } \frac{1}{\kappa_2} - \frac{1}{\kappa_1} < \log\left(\frac{\underline{\phi}_1}{\underline{\phi}_2}\right) \\ 0 & \text{otherwise,} \end{cases} \tag{9}$$

- *for $\underline{\phi}_1,\ \underline{\phi}_2 \to \infty$ the waste minimizing sharing rule converges to (9),*
- *for $\underline{\phi}_1,\ \underline{\phi}_2 \leq c_o$ (so that for every γ there is a positive probability that neither player invests), the waste minimizing sharing rule is*

$$\gamma^\star : \left(\frac{\underline{\phi}_2}{c_o}\right)^{\kappa_2} \gamma^{\star\kappa_2 - 1} = \left(\frac{\underline{\phi}_1}{c_o}\right)^{\kappa_1} (1-\gamma^\star)^{\kappa_1 - 1}$$

Also here, keeping κ_1 and κ_2 constant, as the expected strength of player i relative to player $-i$ increases the mediator will increase the share of ex-post surplus received by player i. Furthermore, keeping $\underline{\phi}_1$ and $\underline{\phi}_2$ constant, the surplus share received by player i decreases with κ_i and increases with κ_{-i}. That is, each player prefers when the mediator has a precise belief about his opponent's power level, but an uninformative belief about his own power level. This is again due to the fact that for given $\underline{\phi}_1$ and $\underline{\phi}_2$, the expected strength of player i relative to player $-i$ decreases with k_i and increases with k_{-i}.

Finally, as $\underline{\phi}_1$, $\underline{\phi}_2$ increase or κ_1, κ_2 decrease, the two players become more likely to invest. Hence, for $\underline{\phi}_1$, $\underline{\phi}_2$ sufficiently high or κ_1, κ_2 sufficiently low, the

[19] If the solution to the mediator's problem is not unique, then both the smallest and the largest waste-minimizing γ are weakly increasing in $\underline{\phi}_1$ and κ_2, and weakly decreasing in $\underline{\phi}_2$ and κ_1.

waste minimizing sharing rule becomes extreme, allocating all ex-post surplus to one of the two players. In less extreme cases, the waste-minimizing sharing rule is intermediate, because each player has a low probability of investing.

A related question is whether the players would prefer to have a more knowledgeable mediator, who has more precise information about both players' power levels. For example, the mediator may be given the ability to gather intelligence and inspect both players, leading to an increase in both κ_1 and κ_2.[20] To explore this possibility, let us assume $\kappa_1 = \kappa_2 \equiv \kappa$, meaning that the mediator's prior beliefs over ϕ_1 and ϕ_2 are equally precise.

By the previous Proposition, if $\underline{\phi}_1 \leq c_o$ and $\underline{\phi}_2 \leq c_o$, the solution to the mediator's problem is simply

$$\gamma^\star = \begin{cases} \left[\left(\frac{\underline{\phi}_2}{\underline{\phi}_1} \right)^{\frac{\kappa}{\kappa-1}} + 1 \right]^{-1} & \text{if } \kappa > 1 \\ 1 & \text{otherwise.} \end{cases}$$

Again, the player expected to be stronger receives a larger share of ex-post surplus. Note also that, as the mediator belief becomes more imprecise (lower κ), player 1 receives a larger share of ex-post surplus. Whenever $\kappa \leq 1$, the objective function is strictly concave and the mediator's problem has a corner solution $\gamma = 1$.

Hence, the player who is expected to be stronger prefers a less informative belief (in the sense of lower κ and thicker tails), while the opposite is true for the player who is expected to be weaker. This implies that, for example, the player expected to be weaker would want the mediator to have the ability to gather information and inspect both players, so to have a more precise belief about their power levels. The player expected to be stronger instead would oppose this.

6 Endogenous “originary" power levels.

The fact that in order to reduce wasteful investment the mediator should favor the strongest player opens a possible issue. If the initial power levels are endogenous, then each player has the incentive to become the strongest player in order to obtain a more favorable share of the ex-post surplus, potentially leading to very high level of wasteful investment.

[20] See, for example, the inspections of IRAN's nuclear sites prior to the 2015 framework agreement.

Consider, for example, the model discussed in Section 4 in which the initial power levels are observable by players and mediator. Assume now that the investments are done in two steps: the players can invest before the mediator announces the sharing rule as well as after the announcement. The initial investments are observable by the mediator. By Proposition 2, the mediator sets $\gamma^\star = 1 - \frac{\min\{c_d, c_o\}}{\max\{\phi_1, \phi_2\}}$, where we allow for either player 1 or 2 to be the strongest depending on their initial investment.

Hence, if the players have the opportunity to make an investment before the mediator announces the sharing rule, the player who is the weakest ex-ante will benefit from making an offensive investment. This way, in the moment the mediator announces the sharing rule, the other player will be weaker than at the start of the game and therefore will receive a lower share of ex-post surplus. Possibly, the weakest player may become the strongest player and receive the majority of the ex-post surplus. The other player will anticipate this and may invest as well. The expectation of the mediator's intervention leads to wasteful investments before the intervention of the mediator.

The same logic applies also when the mediator does not observe the initial power levels. To show this, we add a stage to the model presented in the previous section: before the power levels are realized, each player can spend resources to affect their minimum power levels. The timeline is now as follows:

1. each player makes his initial offensive and defensive investments. Call these investments $\hat{o}_i$ and $\hat{d}_i$ respectively, and assume that they are observable by the mediator. The minimum power levels are determined by

$$\underline{\phi}_i = \hat{\phi}_i e^{-\hat{o}_{-i}}(2 - e^{-\hat{d}_i})$$

where $\hat{\phi}$ is each player's "original ex-ante minimum power."

2. the mediator announces the sharing rule,
3. the power levels ϕ_1, ϕ_2 are drawn from two Pareto distributions with minimum values $\underline{\phi}_1$, $\underline{\phi}_2$ and parameters κ_1, κ_2. The power levels are not observed by the mediator,[21]

[21] For simplicity, we focus on the case in which players do not know their power levels when making their initial, observable investment. Otherwise the game becomes a signaling game in which the level of investment may reveal something about each player's power level. The fact that in signaling games players may perform socially wasteful investments is well understood, and not a point we wish to reiterate here.

4. the parties make additional investments d_i and o_i, also not observable by the mediator,
5. the negotiation starts.

By Proposition 3 whenever $\kappa_1 = \kappa_2 \equiv \kappa > 1$, investing in $\hat{d}_i$ and $\hat{o}_i$ always results in an increase in the share of ex-post surplus received. As a consequence, if the costs $\hat{c}_o$ and $\hat{c}_d$ are not too large, the mediator's intervention may induce the players to invest in $\hat{d}_i$ and $\hat{o}_i$. As κ decreases the sharing rule implemented by the mediator becomes more and more sensitive to the players' relative power, and therefore the incentive to invest in $\hat{d}_i$ and $\hat{o}_i$ increases. In the limit case $\kappa \leq 1$ the mediator allocates the entire ex-post surplus to the player who is expected to be stronger. It follows that in the equilibrium of the ex-ante investment game each player invests with positive probability for any value of $(\hat{c}_o, \hat{c}_d)$. The reason is that, if a player expects the other player not to invest, this player has the incentive to invest a tiny amount and capture the entire ex-post surplus.

To conclude, notice that the results derived in this section hold also when the mediator can announce the sharing rule at the beginning of the game (i.e., before $\hat{d}_i$ and $\hat{o}_i$ are set) but cannot commit to it. That is because the mediator will always revise the sharing rule after observing $\hat{d}_i$ and $\hat{o}_i$, leading to the same conclusions we have obtained above. This implies the following remark.

Remark 1. *A benevolent mediator who lacks the power to commit to a sharing rule may cause a higher level of social waste than a mediator who simply implements an exogenously given sharing rule.*

Note that, by definition, a mediator who can commit to a sharing rule at the beginning of the game ought to be able to achieve a (weakly) lower level of wasteful investment than a mediator who lacks commitment. Hence, the remark holds whenever the exogenously given sharing rule is the one that would be chosen by a benevolent mediator with the power to commit.

7 Contest for γ: pre-negotiation concessions

We saw that when the mediator has full information he can always eliminate all waste, but this outcome may not be achievable if he does not observe the players' investments. In this section we explore whether the mediator can compensate for this lack of information by conducting a contest for γ, that is, by announcing that the sharing rule will depend on visible, costly actions taken by the players. We allow these costly actions to benefit the other player, and therefore interpret them as *concessions*.

Before the negotiation begins the mediator asks each player to make concessions to the other player. Call b_1 the level of concessions made by player 1 and b_2 the level of concessions made by player 2. If player i makes concessions b_i, player i bears a cost equal to b_i while player $-i$ enjoys a benefit equal to $\alpha \cdot b_i \geq 0$, where $\alpha \in [0, 1]$.[22] Note that whenever $\alpha < 1$, making a concession generates a welfare loss. The concessions are used by the mediator to set the sharing rule γ, which is now

$$\gamma = f(b_1, b_2)$$

with f continuous and differentiable in both arguments, increasing and concave in b_1, decreasing and convex in b_2. We assume that the function $f(b_1, b_2)$ and α are announced by the mediator at the beginning of the game. That is, the mediator can require the players to make specific concessions (corresponding to a given α; given costs/benefits to the player making/receiving the concession) and then use the level of concessions to determine γ in a way that is fully anticipated by the players at the beginning of the game.

We first present our argument under the assumption that the initial power levels ϕ_1 and ϕ_2, and S are observed by the mediator and can be used to design the function $f(b_1, b_2)$. We later argue that if the mediator does not observe the initial power levels, the contest for γ can be constructed in such a way to induce the players to truthfully reveal ϕ_1, ϕ_2. and S.

Only offensive investments

In the choice of concession levels, player 1 solves

$$\max_{b_1} \left\{ f(b_1, b_2)(S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1}) - b_1 + \alpha b_2 \right\},$$

[22] Our results can be easily extended to more general expressions for the cost and benefit of concessions. However, for ease of notation, here we assume simple linear functions. Also, in our specification, the cost and benefits of concessions are independent from the underlying conflict. In a more general environment, one could allow the cost and benefit of a concession to depend on the players' initial power levels and on their investments

with FOC:

$$\frac{\partial f(b_1,b_2)}{\partial b_1}(S-\phi_1 e^{-o_2}-\phi_2 e^{-o_1})=1. \tag{10}$$

Similarly, player 2 solves:

$$\max_{b_2}\left\{(1-f(b_1,b_2))(S-\phi_1 e^{-o_2}-\phi_2 e^{-o_1})-b_2+\alpha b_1\right\},$$

with FOC

$$-\frac{\partial f(b_1,b_2)}{\partial b_2}(S-\phi_1 e^{-o_2}-\phi_2 e^{-o_1})=1. \tag{11}$$

Assuming that both players maximization problems have an internal solution, conditions (10) and (11) define the equilibrium level of concessions $b_1(o_1,o_2)$ and $b_2(o_1,o_2)$.[23] Note that both $b_1(o_1,o_2)$ and $b_2(o_1,o_2)$ must be increasing in the offensive investments o_1 and o_2. The reason is that the higher the offensive investments, the larger the ex-post surplus to be shared in the negotiation, the benefit of obtaining a more favorable surplus split, and therefore the intensity of the competition over γ.

Given this, when deciding on the level of offensive investment, player 1 solves

$$\max_{o_1}\left\{\phi_1 e^{-o_2}+f(b_1(o_1,o_2),b_2(o_1,o_2))(S-\phi_1 e^{-o_2}-\phi_2 e^{-o_1})\right.$$
$$\left.-c_o o_1-b_1(o_1,o_2)+\alpha b_2(o_1,o_2)\right\},$$

with FOC[24]

$$\frac{\partial f(.,.)}{\partial b_2}\frac{\partial b_2(.,.)}{\partial o_1}(S-\phi_1 e^{-o_2}-\phi_2 e^{-o_1})+\gamma\phi_2 e^{-o_1}+\alpha\frac{\partial b_2(.,.)}{\partial o_1}=c_o.$$

Using equation 11, the above FOC becomes:

$$\gamma\phi_2 e^{-o_1}-\frac{\partial b_2(.,.)}{\partial o_1}(1-\alpha)=c_o.$$

Therefore, player 1 anticipates that by investing in o_1, he will increase the concessions made by player 2 during the contest. This has two effects. First, it directly increases player 1 utility. Second, it increases the share of ex-post surplus accruing to player 2, therefore hurting player 1. If $\alpha<1$ the negative effect dominates, and player 1 decreases his investment in offensive technology to reduce the intensity of

[23] Remember that $f(b_1,b_2)$ is chosen by the mediator. Hence, making sure that the equilibrium exists, is unique, and each player's problem is interior will be part of the mediator's problem.

[24] By the envelope theorem, we can ignore the effect of o_1 on b_1.

the contest over γ. If instead $\alpha = 1$ the two effects cancel out and the contest for γ has no impact on o_1.

Similarly, player 2's FOC is

$$(1-\gamma)\phi_1 e^{-o_2} - \frac{\partial b_1(.,.)}{\partial o_2}(1-\alpha) = c_o.$$

Also in this case, if $\alpha < 1$ the introduction of the contest reduces the players incentive to invest in offensive technology, while if $\alpha = 1$ the introduction of the contest has no impact on player 2 investment.

The key observation is that the shape of the function $f(b_1, b_2)$ matters in two ways. The first derivatives of $f(b_1, b_2)$ determine the players' concessions and, whenever $\alpha < 1$, the welfare loss generated by the contest for γ. The second derivatives of $f(b_1, b_2)$ determine $\frac{\partial b_1(.,.)}{\partial o_2}$ and $\frac{\partial b_2(.,.)}{\partial o_1}$, that, in turn, determine how each player's concessions react to the other player's investment and the incentive to make offensive investments. The next proposition shows that these two channels can be controlled separately by the mediator in order to achieve zero waste in equilibrium.

Proposition 4. *For any $\alpha < 1$, the mediator can achieve full efficiency and implement any sharing rule $\gamma \in [0, 1]$.*

The mediator can achieve zero concessions in equilibrium by setting

$$\frac{\partial f(0,0)}{\partial b_1} = -\frac{\partial f(0,0)}{\partial b_2} = \frac{1}{S - \underline{u}_1 - \underline{u}_2}$$

At the same time the mediator sets both $\frac{\partial^2 f(0,0)}{\partial b_1^2}$ and $\frac{\partial^2 f(0,0)}{\partial b_2^2}$ low, so that concessions are very sensitive to the players' offensive investment. That is, both players expect that if they set positive offensive investment, they will generate a large concession from the opponent. If $\alpha < 1$ this expectation draws offensive investment to zero. This mechanisms works for any γ, including $\gamma = \frac{1}{2}$. It follows that, here, there is no conflict between efficiency and fairness.

Few points are worth noting. First, despite the fact that there is no waste in equilibrium, the contest is effective only if $\alpha < 1$. That is, concessions need to be an inefficient way to transfer surplus among players. They cannot be monetary transfers, but should rather be "in kind" transfers. The proposition shows that the mediator can achieve full efficiency as long as he can require the players to make concessions that are wasteful.

Second, although there is no welfare loss in equilibrium, the contest should generate inefficiencies off equilibrium (i.e., for positive offensive investment). Interestingly, here the mediator can easily commit to destroying welfare off equilibrium. The reason is that the mediator does not observe the player's offensive investment. Hence, following a positive offensive investment, the mediator has no incentive to modify the function $f(b_1, b_2)$ so to avoid costly concessions.

Third, whereas in the absence of the contest the mediator favors the strongest player potentially leading to a wasteful race to become the strongest player (see Section 6), here any sharing rule can be implemented and achieve zero waste. Hence, the mediator can credibly announce that, even if the players spend resources to modify their initial power level, the sharing rule will not change. This, therefore, eliminates any incentive to become the strongest player. Not only, but similarly to what has been discussed in Section 3, the mediator can credibly announce that the sharing rule will penalize the players in case they spend resources to modify their initial power levels. Hence, the mediator can achieve zero waste also when the initial power level is endogenous.

Finally, the fact that the mediator can observe S, ϕ_1 and ϕ_2 is here without loss of generality, because the mediator can elicit them from the two players. The mediator can announce that $\frac{\partial f(.,.)}{\partial b_1} = -\frac{\partial f(.,.)}{\partial b_2}$ (so that the player's equilibrium concessions levels are always identical) and $f(b_1, b_2)|_{b_1=b_2} = \gamma$ (so that the sharing rule implemented is constant). It follows that the players cannot manipulate the allocation of the ex-post surplus by misreporting S, ϕ_1 or ϕ_2. The only effect of misreporting is to, potentially, cause positive concessions and positive offensive investment in equilibrium. However, it is easy to see that no player can benefit from inducing positive offensive investments and positive concession. Suppose a player expects his report to have an effect on the function $f(.,.)$. Because player i's concessions and investment are optimal given $f(.,.)$, by an envelope argument manipulating $f(.,.)$ affects player i's utility only because it may induce player $-i$ to change his behavior. It is however evident that player i cannot do better than reporting truthfully and inducing player $-i$ to set both concessions and investments to zero.

Offensive and defensive investment.

In this case player 1 solves

$$\max_{b_1} \left\{ f(b_1, b_2)(S - \phi_1 e^{-o_2}(2 - e^{-d_1}) - \phi_2 e^{-o_1}(2 - e^{-d_2})) - b_1 + \alpha b_2 \right\}$$

with FOC

$$\frac{\partial f(b_1,b_2)}{\partial b_1}(S-\phi_1 e^{-o_2}(2-e^{-d_1})-\phi_2 e^{-o_1}(2-e^{-d_2}))=1. \tag{12}$$

Similarly, the FOC corresponding to b_2 os:

$$-\frac{\partial f(b_1,b_2)}{\partial b_2}(S-\phi_1 e^{-o_2}(2-e^{-d_1})-\phi_2 e^{-o_1}(2-e^{-d_2}))=1. \tag{13}$$

Again, if both players maximization problems have an internal solution, conditions (12) and (13) define the equilibrium level of concessions $b_1(o_1,o_2,d_1,d_2)$ and $b_2(o_1,o_2,d_1,d_2)$. Note that both $b_1(o_1,o_2,d_1,d_2)$ and $b_2(o_1,o_2,d_1,d_2)$ are increasing in the offensive investments o_1 and o_2, but are decreasing in the defensive investments d_1 and d_2. The intuition is the same discussed earlier. The incentive to make concessions increases with the size of the ex-post surplus to be shared in the negotiation. The ex-post surplus increases with the players' offensive investment but decreases with the players' defensive investment.

Hence, when deciding on the investments levels, player 1 solves

$$\max_{o_1,d_1}\left\{\left\{\phi_1 e^{-o_2}(2-e^{-d_1})+f(b_1(o_1,o_2),b_2(o_1,o_2))(S-\phi_1 e^{-o_2}(2-e^{-d_1})-\phi_2 e^{-o_1}(2-e^{-d_2})\right\}\right.$$
$$\left.-c_o o_1-c_d d_1-b_1(o_1,o_2)+\alpha b_2(o_1,o_2)\right\},$$

with FOC for o_1:

$$\frac{\partial f(.,.)}{\partial b_2}\frac{\partial b_2(.,.)}{\partial o_1}(S-\phi_1 e^{-o_2}(2-e^{-d_1})-\phi_2 e^{-o_1}(2-e^{-d_2}))+\gamma\phi_2 e^{-o_1}(2-e^{-d_2})+\alpha_2\frac{\partial b_2(.,.)}{\partial o_1}=c_o$$

which, again, using 13 becomes:

$$\gamma\phi_2 e^{-o_1}(2-e^{-d_2})-\frac{\partial b_2(.,.)}{\partial o_1}(1-\alpha)=c_o.$$

The FOC for d_1 is:

$$\frac{\partial f(.,.)}{\partial b_2}\frac{\partial b_2(.,.)}{\partial d_1}(S-\phi_1 e^{-o_2}(2-e^{-d_1})-\phi_2 e^{-o_1}(2-e^{-d_2}))+(1+\gamma)\phi_1 e^{-o_2}e^{-d_1}+\alpha\frac{\partial b_2(.,.)}{\partial d_1}=c_d$$

$$(1-\gamma)\phi_1 e^{-o_2}e^{-d_1}-\frac{\partial b_2(.,.)}{\partial d_1}(1-\alpha)=c_d$$

Similarly for player 2, the FOCs for o_2 is:

$$(1-\gamma)\phi_1 e^{-o_2}(2-e^{-d_1})-\frac{\partial b_1(.,.)}{\partial o_2}(1-\alpha)=c_o,$$

and for d_2 is:

$$\gamma\phi_2 e^{-o_1} e^{-d_2} - \frac{\partial b_1(.,.)}{\partial d_2}(1-\alpha) = c_d.$$

Also here, when $\alpha < 1$ the contest over γ decreases the benefit of making an offensive investment. However, the contest over γ simultaneously increases the benefit of making a defensive investment. The intuition is the reverse of what discussed in the previous section. A defensive investment decreases the ex-post surplus to be shared in the contest and therefore the incentive of both players to perform monetary payments. Hence, by making a defensive investment, player i can decrease b_{-i} and obtain a higher surplus share during the negotiation. If instead $\alpha_i = 1$, the contest for γ has no impact on the investment made by player 1. The next proposition shows that, because of these tradeoffs, contrary to the previous case here the mediator may not be able to eliminate all waste.

Proposition 5. *The mediator is able achieve full efficiency and implement the surplus split $\gamma \in [0,1]$ if and only if there exists $A_1 \geq 0$ and $A_2 \geq 0$ such that*

$$\begin{aligned} 1 - \min\left\{\frac{c_d}{\phi_1} - (1-\alpha)A_1, \frac{c_o}{\phi_1} + (1-\alpha)A_2\right\} \leq \\ \gamma \leq \min\left\{\frac{c_o}{\phi_2} + (1-\alpha)A_1, \frac{c_d}{\phi_2} - (1-\alpha)A_2\right\} \end{aligned} \tag{14}$$

We showed earlier that without the contest for γ there exists a sharing rule that eliminates all waste if and only if $\frac{\phi_1\phi_2}{\phi_1+\phi_2} \leq \min\{c_o, c_d\}$ (see Proposition 2). Unsurprisingly therefore, under this same condition there exist γ, A_1, A_2 for which (14) holds, so that the mediator can eliminate all waste via a contest for γ in all situations in which he can eliminate all waste also without the contest for γ. More interestingly, as long as as concessions are socially wasteful (that is, $\alpha < 1$) the possibility of running the contest for γ increases the mediator's ability to eliminate waste.[25] However, contrary to the case of only offensive investment, here there are parameter values for which the mediator may not be able to achieve efficiency.[26]

Whenever there is a range of ϕ_1 and ϕ_2 for which the mediator can credibly maintain the same sharing rule and achieve zero waste, the mediator may be able to eliminate the incentives to manipulate the initial power levels. If the players

[25] For example, if $\frac{\phi_1\phi_2}{\phi_1+\phi_2} \leq \frac{c_o+c_d}{2}$ there are A_1, A_2 and γ that satisfy (14).

[26] For example, if $\phi_1 = \phi_2$ and $c_o = c_d$ sufficiently small, then (14) is violated for every $\gamma \in [0,1]$ and it is not possible to achieve efficiency.

manipulate their initial power levels and remain within this range, the surplus share allocated to each player will not change, and hence there is no "race" to become the most powerful player. Not only, but also here the mediator may credibly announce that the sharing rule will penalize a player if he spends resources to modify his initial power levels, eliminating the incentives to spend resources in this type of manipulation. However, here the ability of the mediator to achieve this outcome depends on the parameters of the model. It is easy to see that when c_o and c_d are large, the range of γ that can be implemented is large, and the mediator is better able to eliminate the players incentives to manipulate their initial power levels. The opposite is true if c_o and c_d are small.

Finally, when S, ϕ_1 and ϕ_2 are not observed by the mediator, the possibility of achieving zero waste depends on the parameters. If (14) holds at the same γ for every possible S, ϕ_1 and ϕ_2, then the logic discussed for the case of only offensive investment continues to hold. The mediator can ask the players to report S, ϕ_1 and ϕ_2, which are then used to determine the shape of $f(.,.)$ so to generate zero waste. Because the sharing rule can be made independent from the reports, the players have no incentive to misreport. If instead there is no γ that satisfies (14) for all possible values of S, ϕ_1 and ϕ_2, then this argument will fail, because the mediator cannot commit to maintain the same sharing rule for all possible reports, making truthful reporting impossible.

8 Conclusions

We analyze the problem of a benevolent mediator who can set the sharing rule of the mediation so to minimize total pre-negotiation waste. The main result is that the mediator should penalize the weakest player, who is the one with the strongest incentive to undertake wasteful investments. This results remains true under different assumptions on what the mediator can observe, and highlights a conflict between fairness and efficiency. However, the fact that the mediator will favor the strongest player, by itself may provide incentives for wasteful investment prior to the intervention of the mediator. We discuss how the mediator may avoid this problem by organizing a contest for the sharing rule. Relative to the existing literature on mediation in economics, our paper argues that the mediator's actions have effects on the players' behavior not only within the negotiation but also prior to it, and by doing so highlights a conflict between efficiency and fairness arising in negotiations. Relative to the existing literature on mediation in political science, our paper shows that the mediator can be biased not because of his preferences, but strategically to minimize social waste.

In order to focus on the inefficiencies arising before the negotiation stage, we have assumed away inefficiencies arising during the negotiation. However, several authors have drawn a connection between pre-bargaining wasteful investments and inefficiencies arising within the negotiation.[27] For example, an arms build up prior to the negotiation may increase the chance that an agreement is found and therefore increase the efficiency of the negotiation, either because it makes war more costly or because it reduces the asymmetry of information between players. On the other hand, a military mobilization may decrease the probability of reaching an agreement and the efficiency of the negotiation because it generates a *hands-tying effect*: a decrease in the cost of starting a war that operates as a public commitment device. Introducing a benevolent mediator in a model in which inefficiencies arising in the pre-negotiation stage affect the inefficiencies arising during the negotiation is left for future work.

Our analysis suggests several additional lines for future research. For example, our framework can be used to explore the choice between mediated and unmediated negotiation. Despite the fact that the mediator will favor the strongest player, the weakest player may nevertheless prefer a mediated negotiation over an unmediated one, because of the reduction in wasteful investment. Also, we showed that the precision of the mediator's information affects the sharing rule implemented. Our results suggest that the weakest player benefits from a more informed mediator while the opposite is true for the strongest player, but the full analyses of the strategic choice of transparency remains to be completed. Finally, the ability of the mediator to commit may be key to the reduction of wasteful investment. Analyzing different ways in which the mediator can acquire this commitment remains an open problem.

[27] See, for example, Powell (1993), Kydd (2000), Slantchev (2005), Meirowitz and Sartori (2008), Jackson and Morelli (2009).

Appendix: mathematical derivations

Proof of Proposition 2. For $c_d \geq c_o$, we argued in the text that there is no defensive investments. Also, $o_1^\star$ is zero if $\gamma \leq \frac{c_o}{\phi_2}$, and $o_2^\star$ is zero if $\gamma \geq 1 - \frac{c_o}{\phi_1}$. Hence, wasteful investment can be completely eliminated with any $\gamma \in [1 - \frac{c_o}{\phi_1}, \frac{c_o}{\phi_2}]$ whenever

$$c_o \geq \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}. \tag{15}$$

Suppose now that

$$c_o < \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}.$$

For $\gamma < \frac{c_o}{\phi_2}$ we have that $o_1 = 0$ and o_2 is strictly decreasing in γ. For $\gamma > 1 - \frac{c_o}{\phi_1}$ we have that $o_2 = 0$ and o_1 is strictly increasing in γ. Therefore, it has to be that the waste minimizing $\gamma \in [\frac{c_o}{\phi_2}, 1 - \frac{c_o}{\phi_1}]$.

For this range of values, the mediator solves:

$$\min_{\gamma \in [\frac{c_o}{\phi_2}, 1 - \frac{c_o}{\phi_1}]} \left\{ \log\left(\frac{\gamma \phi_2}{c_o}\right) + \log\left(\frac{(1-\gamma)\phi_1}{c_o}\right) \right\} =$$

$$\min_{\gamma \in [\frac{c_o}{\phi_2}, 1 - \frac{c_o}{\phi_1}]} \left\{ \log\left(\gamma(1-\gamma)\right) + \log\left(\frac{\phi_1 \phi_2}{c_o^2}\right) \right\}.$$

Hence, the mediator minimizes $\gamma(1-\gamma)$ over the relevant interval. It can be verified that when $\phi_1 \geq \phi_2$—as we assume throughout—and $c_o < \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}$ this minimum is always reached at $\gamma^\star = 1 - \frac{c_o}{\phi_1}$.

For $c_d < c_o$, consider the total expenditure fighting over player 2's outside option, with player 1 attacking and player 2 defending:

$$c_o \cdot o_1 + c_d \cdot d_2 = \begin{cases} 0 & \text{if } \gamma \leq \frac{c_d}{\phi_2} \\ c_d \left(\log(\gamma) + \log\left(\frac{\phi_2}{c_d}\right)\right) & \text{if } \frac{c_d}{\phi_2} \leq \gamma \leq \frac{c_o + c_d}{2\phi_2} \\ c_d \log\left(\frac{c_o + c_d}{2c_d}\right) + c_o\left(\log(\gamma) + \log\left(\frac{2\phi_2}{c_o + c_d}\right)\right) & \text{otherwise,} \end{cases}$$

Similarly, consider the total expenditure fighting over player 1's outside option:

$$c_o \cdot o_2 + c_d \cdot d_1 = \begin{cases} 0 & \text{if } 1 - \gamma \leq \frac{c_d}{\phi_1} \\ c_d \left(\log(1-\gamma) + \log\left(\frac{\phi_1}{c_d}\right)\right) & \text{if } \frac{c_d}{\phi_1} \leq 1 - \gamma \leq \frac{c_o + c_d}{2\phi_1} \\ c_d \log\left(\frac{c_o + c_d}{2c_d}\right) + c_o\left(\log(1-\gamma) + \log\left(\frac{2\phi_1}{c_o + c_d}\right)\right) & \text{otherwise,} \end{cases}$$

It is easy to verify that whenever $c_d \geq \frac{\phi_1\phi_2}{\phi_1+\phi_2}$, then any $\gamma \in [1-\frac{c_d}{\phi_1}, \frac{c_d}{\phi_2}]$ achieves zero waste. If instead $c_d < \frac{\phi_1\phi_2}{\phi_1+\phi_2}$, then $\frac{c_d}{\phi_2} < 1-\frac{c_d}{\phi_1}$ and we have

$$c_o \cdot (o_1+o_2) + c_d \cdot (d_1+d_2) = \begin{cases} \text{strictly decreasing} & \text{if } \gamma \leq \frac{c_d}{\phi_2} \\ \text{strictly concave} & \text{if } \frac{c_d}{\phi_2} \leq \gamma \leq 1-\frac{c_d}{\phi_1} \\ \text{strictly increasing} & \text{otherwise.} \end{cases} \tag{16}$$

Hence, total expenditure $c_o \cdot (o_1+o_2)+c_d \cdot (d_1+d_2)$ is minimized either at $\gamma = \frac{c_d}{\phi_2}$, where the expenditures fighting over 2's outside options is zero, or at $\gamma = 1-\frac{c_d}{\phi_1}$ where the expenditures fighting over 1's outside options is zero. At these two values total expenditures are

$$[c_o\cdot(o_1+o_2)+c_d\cdot(d_1+d_2)]|_{\gamma=\frac{c_d}{\phi_2}} = c_d \log\left(\min\left\{\frac{c_o+c_d}{2c_d}, \phi_1\left(\frac{1}{c_d}-\frac{1}{\phi_2}\right)\right\}\right)+c_o\log\left(\max\left\{0, \phi_1\left(\frac{1}{c_d}-\frac{1}{\phi_2}\right)\frac{2c_d}{c_o+c_d}\right\}\right)$$

$$[c_o\cdot(o_1+o_2)+c_d\cdot(d_1+d_2)]|_{\gamma=1-\frac{c_d}{\phi_1}} = c_d \log\left(\min\left\{\frac{c_o+c_d}{2c_d}, \phi_2\left(\frac{1}{c_d}-\frac{1}{\phi_1}\right)\right\}\right)+c_o\log\left(\max\left\{0, \phi_2\left(\frac{1}{c_d}-\frac{1}{\phi_1}\right)\frac{2c_d}{c_o+c_d}\right\}\right)$$

Because $c_d < \frac{\phi_1\phi_2}{\phi_1+\phi_2}$, then $\phi_2\left(\frac{1}{c_d}-\frac{1}{\phi_1}\right) < \phi_1\left(\frac{1}{c_d}-\frac{1}{\phi_2}\right)$ and total waste is minimized whenever $\gamma = 1-\frac{c_d}{\phi_1}$.

□

Proof of Lemma 2. To start, note that if ϕ_i is Pareto-distributed with minimum x and parameter κ, then $\log\left(\frac{\phi_i}{x}\right)$ is exponentially distributed with parameter κ. To see this, consider

$$\Pr\left\{\log\left(\frac{\phi_i}{x}\right) \leq y\right\} = \Pr\left\{\phi_i \leq e^y x\right\}$$

Because ϕ_i is distributed according to a Pareto distribution, the above expression becomes

$$1-\left(\frac{x}{xe^y}\right)^\kappa = 1-e^{-y\kappa}$$

which is the CDF of an exponential distribution with parameter κ.

Knowing this, we can compute

$$E\left[\log\left(\frac{\gamma\phi_2}{c_o}\right)\Big|\phi_2 > \frac{c_o}{\gamma}\right] = \begin{cases} \frac{1}{\kappa_2} & \text{if } \underline{\phi_2} \leq \frac{c_o}{\gamma} \\ E\left[\log\left(\frac{\gamma\phi_2}{c_o}\right)\right] = \log\left(\frac{\underline{\phi_2}\gamma}{c_o}\right) + E\left[\frac{\phi_2}{\underline{\phi_2}}\right] = \log\left(\frac{\underline{\phi_2}\gamma}{c_o}\right)+\frac{1}{\kappa_2} & \text{otherwise} \end{cases}$$

and similarly for $E\left[\log\left(\frac{(1-\gamma)\phi_1}{c_o}\right)\middle|\phi_1 > \frac{c_o}{(1-\gamma)}\right]$. Finally, using the definition of Pareto distribution we compute

$$\Pr\left(\phi_2 > \frac{c_o}{\gamma}\right) = \begin{cases} \left(\frac{\underline{\phi}_1\gamma}{c_o}\right)^{\kappa_2} & \text{if } \underline{\phi}_2 \leq \frac{c_o}{\gamma} \\ 1 & \text{otherwise} \end{cases}$$

and similarly for $\Pr\left(\phi_1 > \frac{c_o}{1-\gamma}\right)$. □

Proof of Proposition 3. The mediator minimizes

$$\begin{cases} A(\gamma) \equiv \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\kappa_2} + \left(\log\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right) + \frac{1}{\kappa_1}\right) & \text{if } \gamma \leq \min\{1-\frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} \\ B(\gamma) \equiv \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\kappa_2} + \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{\kappa_1} & \text{if } 1-\frac{c_o}{\underline{\phi}_1} \leq \gamma \leq \frac{c_o}{\underline{\phi}_2} \\ C(\gamma) \equiv \left(\log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right) + \frac{1}{\kappa_2}\right) + \left(\log\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right) + \frac{1}{\kappa_1}\right) & \text{if } \frac{c_o}{\underline{\phi}_2} \leq \gamma \leq 1-\frac{c_o}{\underline{\phi}_1} \\ D(\gamma) \equiv \left(\log\left(\frac{\underline{\phi}_2\gamma}{c_o}\right) + \frac{1}{\kappa_2}\right) + \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{\kappa_1} & \text{if } \max\{1-\frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} < \gamma \end{cases}$$

Whenever $\kappa_1, \kappa_2 \to \infty$, the uncertainty about the players power level disappears. The solution to the mediator's problem is the one derived in Section 4.

Taking the derivative of the mediator's objective function with respect to γ we get:

$$\begin{cases} A'(\gamma) \equiv \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\gamma} - \frac{1}{1-\gamma} & \text{if } \gamma \leq \min\{1-\frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} \\ B'(\gamma) \equiv \left(\frac{\underline{\phi}_2\gamma}{c_o}\right)^{\kappa_2}\frac{1}{\gamma} - \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{1-\gamma} & \text{if } 1-\frac{c_o}{\underline{\phi}_1} \leq \gamma \leq \frac{c_o}{\underline{\phi}_2} \\ C'(\gamma) \equiv \frac{1}{\gamma} - \frac{1}{1-\gamma} & \text{if } \frac{c_o}{\underline{\phi}_2} \leq \gamma \leq 1-\frac{c_o}{\underline{\phi}_1} \\ D'(\gamma) \equiv \frac{1}{\gamma} - \left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right)^{\kappa_1}\frac{1}{1-\gamma} & \text{if } \max\{1-\frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2}\} < \gamma. \end{cases} \tag{17}$$

which is continuous in γ. We solve the mediator's problem by considering few separate cases:

- $\kappa_1, \kappa_2 \leq 1$. In this case $A(\gamma)$, $B(\gamma)$, $C(\gamma)$ and $D(\gamma)$ are all concave. By continuity of 17, the solution can only be at the extremes, and hence the waste-minimizing sharing rule is

$$\gamma^* = \begin{cases} 1 & \text{if } \frac{1}{\kappa_2} - \frac{1}{\kappa_1} < \log\left(\frac{\underline{\phi}_1}{\underline{\phi}_2}\right) \\ 0 & \text{otherwise} \end{cases}$$

- $\kappa_1, \kappa_2 > 1$. In this case $A'(0) < 0$ and $D'(1) > 0$ and therefore the solution is never an extreme value. If, furthermore $c_o > \underline{\phi}_1 > \underline{\phi}_2$, then the mediator problem is to minimize $B(\gamma)$, which is convex. Hence the solution to the mediator's problem is

$$\gamma^* : B'(\gamma^*) = 0$$

If instead $\underline{\phi}_1, \underline{\phi}_2 \to \infty$, the mediator's objective function converges to $C(\gamma)$, which is concave. By continuity, the solution to the mediator's problem converges to

$$\gamma^* = \begin{cases} 1 & \text{if } \frac{1}{\kappa_2} - \frac{1}{\kappa_1} < \log\left(\frac{\underline{\phi}_1}{\underline{\phi}_2}\right) \\ 0 & \text{otherwise} \end{cases}$$

which is the γ minimizing $C(\gamma)$.

To characterize the solution to the mediator's problem in all other cases, we take the derivative of 17 with respect to $\underline{\phi}_1$, $\underline{\phi}_2$, κ_1, κ_2 and then invoke Topkis's theorem.

The derivative of 17 with respect to $\underline{\phi}_1$ is:

$$\begin{cases} 0 & \text{if } \gamma < 1 - \frac{c_o}{\underline{\phi_1}} \\ -\kappa_1 \left(\underline{\phi}_1(1-\gamma)\right)^{\kappa_1 - 1} \left(\frac{1}{c_o}\right)^{\kappa_1} & \text{otherwise} \end{cases}$$

which is weakly negative. The derivative of 17 with respect to $\underline{\phi}_2$ is:

$$\begin{cases} \kappa_2 \left(\underline{\phi}_2 \gamma\right)^{\kappa_2 - 1} \left(\frac{1}{c_o}\right)^{\kappa_2} & \text{if } \gamma < \frac{c_o}{\underline{\phi_2}} \\ 0 & \text{otherwise} \end{cases}$$

which is weakly positive. The derivative of 17 with respect to κ_2 is

$$\begin{cases} \left(\frac{\underline{\phi}_2}{c_o}\right)^{\kappa_2} \gamma^{\kappa_2 - 1} \log\left(\frac{\underline{\phi}_2 \gamma}{c_o}\right) & \text{if } \gamma \leq \frac{c_o}{\underline{\phi}_2} \\ 0 & \text{otherwise.} \end{cases}$$

which is weakly negative. The derivative of 17 with respect to κ_1 is

$$\begin{cases} -\left(\frac{\underline{\phi}_1}{c_o}\right)^{\kappa_1} (1-\gamma)^{\kappa_1 - 1} \log\left(\frac{\underline{\phi}_1(1-\gamma)}{c_o}\right) & \text{if } 1 - \frac{c_o}{\underline{\phi}_1} \leq \gamma \\ 0 & \text{otherwise.} \end{cases}$$

which is weakly positive. By Topkis's theorem, therefore, the waste-minimizing sharing rule is weakly increasing in $\underline{\phi}_1$, κ_2; weakly decreasing in $\underline{\phi}_2$, κ_1.

□

Proof of Proposition 4. If the mediator announces a $f(b_1, b_2)$ such that

$$\frac{\partial f(0,0)}{\partial b_1} = -\frac{\partial f(0,0)}{\partial b_2} = \frac{1}{S - \phi_1 - \phi_2}$$

and offensive investments are zero, then both equilibrium concessions are zero.

Furthermore, by the implicit function theorem

$$b_1'(o_2)\frac{\partial^2 f(b_1, b_2)}{\partial b_1^2} = \frac{-\phi_1 e^{-o_2}}{(S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1})}$$

$$-b_2'(o_1)\frac{\partial^2 f(b_1, b_2)}{\partial b_2^2} = \frac{-\phi_2 e^{-o_1}}{(S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1})}$$

which, evaluated at zero offensive investment and zero concessions become

$$b_1'(o_2) = A_1\phi_1$$

$$b_2'(o_1) = A_2\phi_2$$

where

$$A_1 \equiv -\frac{1}{\frac{\partial^2 f(0,0)}{\partial b_1^2}(S - \phi_1 - \phi_2)} > 0$$

$$A_2 \equiv \frac{1}{\frac{\partial^2 f(0,0)}{\partial b_2^2}(S - \phi_1 - \phi_2)} > 0.$$

Note that the mediator can set A_1 and A_2 to any strictly positive value by manipulating $\frac{\partial^2 f(0,0)}{\partial b_1^2}$ and $\frac{\partial^2 f(0,0)}{\partial b_2^2}$

It follows that offensive investment by player 2 is zero whenever

$$(1 - \gamma)\phi_1 - A_1\phi_1(1 - \alpha) \leq c_o$$

which is always satisfied for any $\gamma = f(0,0)$ by setting A_1 sufficiently large, that is, if $-\frac{\partial^2 f(0,0)}{\partial b_1^2}$ is sufficiently small (remember that $\frac{\partial^2 f(0,0)}{\partial b_1^2} < 0$). Similar steps show that player 1 offensive investment is also zero if

$$\gamma\phi_2 - A_2\phi_2(1 - \alpha) \leq c_o$$

which is also satisfied if A_2 is sufficiently large, that is, if $\frac{\partial^2 f(0,0)}{\partial b_2^2}$ is sufficiently small. $\square$

Proof of Proposition 5. Following the same steps as in Proposition 4, we can show that, also here, if offensive investment is zero concessions are zero whenever

$$\frac{\partial f(0,0)}{\partial b_1} = -\frac{\partial f(0,0)}{\partial b_2} = \frac{1}{S - \phi_1 - \phi_2}$$

Furthermore, by the implicit function theorem we have that, when all investments are zero and concessions are zero:

$$\frac{\partial b_1(.,.,.,.)}{\partial o_2} = \phi_1 A_1$$
$$\frac{\partial b_1(.,.,.,.)}{\partial d_2} = -\phi_2 A_1$$
$$\frac{\partial b_2(.,.,.,.)}{\partial o_1} = \phi_2 A_2$$
$$\frac{\partial b_2(.,.,.,.)}{\partial d_1} = -\phi_1 A_2$$

where

$$A_1 = -\frac{1}{\frac{\partial^2 f(0,0)}{\partial b_1^2}(S - \phi_1 - \phi_2)^2} \geq 0.$$
$$A_2 = \frac{1}{\frac{\partial^2 f(0,0)}{\partial b_2^2}(S - \phi_1 - \phi_2)^2} \geq 0.$$

Note that the mediator can achieve any value $A_1 \geq 0$ and $A_2 \geq 0$ by manipulating the function $f(.,.)$.

Hence, using the players' FOCs, investments will be zero whenever

$$\gamma\phi_2 - \phi_2 A_1(1-\alpha) \leq c_o.$$
$$(1-\gamma)\phi_1 + \phi_1 A_1(1-\alpha) \leq c_d$$
$$(1-\gamma)\phi_1 - \phi_1 A_2(1-\alpha) \leq c_o,$$
$$\gamma\phi_2 + \phi_2 A_2(1-\alpha) \leq c_d.$$

where $\gamma = f(0,0)$ is the surplus share implemented in equilibrium.

These four conditions together are equivalent to

$$\begin{aligned} 1 - \min\left\{\frac{c_d}{\phi_1} - (1-\alpha)A_1, \frac{c_o}{\phi_1} + (1-\alpha)A_2\right\} \leq \\ \gamma \leq \min\left\{\frac{c_o}{\phi_2} + (1-\alpha)A_1, \frac{c_d}{\phi_2} - (1-\alpha)A_2\right\} \end{aligned} \tag{18}$$

Hence, there is a contest with zero investments and zero transfers if and only if $\alpha < 1$ and there is $A_1 \geq 0$ and $A_2 \geq 0$ that satisfies the above condition.

□

References

Aghion, P., M. Dewatripont, and P. Rey (1994). Renegotiation design with unverifiable information. *Econometrica: Journal of the Econometric Society*, 257–282.

Anbarci, N., S. Skaperdas, and C. Syropoulos (2002). Comparing bargaining solutions in the shadow of conflict: how norms against threats can have real effects. *Journal of Economic Theory 106*(1), 1–16.

Avery, C. and P. B. Zemsky (1994). Money burning and multiple equilibria in bargaining. *Games and Economic Behavior 7*(2), 154–168.

Balzer, B. and J. Schneider (2015). Managing a conflict alternative dispute resolution in contests. *Working paper*.

Basak, D. (2017). Transparency and delay in bargaining. *Working paper*.

Bercovitch, J. (1997). Mediation in international conflict: An overview of theory, a review of practice. In I. W. Zartman and J. L. Rasmussen (Eds.), *Peacemaking in International Conflict: Methods and Techniques*, pp. 125–154. Washington DC: United States Institute of Peace Press.

Binmore, K. (1987). Nash bargaining and incomplete information. In K. Binmore and P. Dasgupta (Eds.), *The Economics of Bargaining*, Chapter 8, pp. 155–192. Oxford: Basil Blackwell.

Chatterjee, K. and L. Samuelson (1990). Perfect equilibria in simultaneous-offers bargaining. *International Journal of Game Theory 19*(3), 237–267.

Esteban, J., M. Morelli, and D. Rohner (2015). Strategic mass killings. *Journal of Political Economy 123*(5), 1087–1132.

Fanning, J. (2016). Mediation in reputational bargaining. *Working paper*.

Fisher, R. J. (2012). Methods of third-party intervention. In M. Fischer, H. Geismann, and B. Schmelzle (Eds.), *Berghof Handbook for Conflict Transformation II*, pp. 157–182. Berlin.

Garfinkel, M. R., M. McBride, and S. Skaperdas (2012). Governance and norms as determinants of arming. *Revue d'économie politique 122*(2), 197–212.

Goltsman, M., J. Hörner, G. Pavlov, and F. Squintani (2009). Arbitration, mediation and cheap talk. *Journal of Economic Theory 144*(4), 1397–1420.

Grossman, S. J. and O. D. Hart (1986). The costs and benefits of ownership: A theory of vertical and lateral integration. *The Journal of Political Economy*, 691–719.

Hoffman, D. A. (2011). Mediation and the art of shuttle diplomacy. *Negotiation Journal 27*(3), 263–309.

Hörner, J., M. Morelli, and F. Squintani (2015). Mediation and peace. *The Review of Economic Studies 82*(4), 1483–1501.

Jackson, M. O. and M. Morelli (2009). Strategic militarization, deterrence and wars. *Quarterly Journal of Political Science 4*(4), 279–313.

Jackson, M. O. and M. Morelli (2011). The reasons for wars: an updated survey. In C. J. Coyne and R. L. Mathers (Eds.), *The handbook on the political economy of war*, pp. 34–57. Edward Elgar Publishing.

Kambe, S. (1999). When is there a unique equilibrium in less structured bargaining? *The Japanese economic review 50*(3), 321–342.

Kydd, A. (2000). Arms races and arms control: Modeling the hawk perspective. *American Journal of Political Science*, 228–244.

Kydd, A. (2003). Which side are you on? bias, credibility, and mediation. *American Journal of Political Science 47*(4), 597–611.

Luce, R. D. and H. Raiffa (1957). *Games and decisions: Introduction and critical surveys*. Wiley New York.

Meirowitz, A., M. Morelli, K. W. Ramsay, and F. Squintani (2015). Dispute resolution institutions and strategic militarization. *Working paper*.

Meirowitz, A. and A. E. Sartori (2008). Strategic uncertainty as a cause of war. *Quarterly Journal of Political Science 3*(4), 327–352.

Nash, J. (1953). Two-person cooperative games. *Econometrica: Journal of the Econometric Society*, 128–140.

Perry, M. and P. J. Reny (1993). A non-cooperative bargaining model with strategically timed offers. *Journal of Economic Theory 59*(1), 50–77.

Ponsatí, C. and J. Sákovics (1998). Rubinstein bargaining with two-sided outside options. *Economic Theory 11*(3), 667–672.

Powell, R. (1993). Guns, butter, and anarchy. *American Political Science Review 87*(01), 115–132.

Raiffa, H. (1982). *The art and science of negotiation.* Harvard University Press.

Sákovics, J. (1993). Delay in bargaining games with complete information. *Journal of Economic Theory 59*(1), 78–95.

Slantchev, B. L. (2005). Military coercion in interstate crises. *American Political Science Review 99*(04), 533–547.

Svensson, I. (2014). *International Mediation Bias and Peacemaking: Taking Sides in Civil Wars.* Routledge.