A Competitive Model of Annuity and Life Insurance with Nonexclusive Contracts*

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Abstract

I study a two period economy in which altruistic individuals have uncertain lifetime and are privately informed about their survival probabilities. Individuals buy insurance by participating in insurance pools. In this environment competitive equilibrium with life and annuity insurance exists despite the presence of adverse selection. This environment has three key features: 1) In any equilibrium there is at most one insurance market in which trade occurs. 2) Increase in social security tax and benefits leads to increase in price of annuity and crowds out trade activities in annuity market. 3) In this environment ex ante efficient allocations are independent of private information and can be optimally implemented using simple tax-transfer instruments. Under optimal policy both annuity and life insurance markets are endogenously closed.

JEL Classification: D82, G22, H21, H55.
Keywords: Adverse Selection, Annuities, Life Insurance, Non-exclusive contracts.

1 Introduction

In this paper I study a two period economy in which agents are altruistic and privately informed about their survival probabilities. Consumers can purchase annuity and life insurance by participating in insurance pools. I adopt a competitive equilibrium notion similar to Bisin and Gottardi (1999), Bisin and Gottardi (2003) and Dubey and Geanakoplos (2002) for insurance markets with non-exclusive contracts. I argue that when insurance companies

*I would like to thank Laurence Ales, Alberto Bisin, V.V. Chari, Larry Jones, Pricila Maziero and Chris Phelan for helpful comments and suggestions. All remaining errors are mine.

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cannot monitor consumers' trade, and hence do not have access to exclusive contracts, there is always a competitive equilibrium provided that consumers can only buy (and not sell) annuity and life insurance contracts. Furthermore, I show that there is at most one insurance market (annuity or life insurance) in which trade occurs.

Annuities are considered to be an important means by which consumers can insure themselves against uncertain lifetime. At the same time Hosseini (forthcoming); Pashchenko (2013); Johnson et al. (2004) document that only 5 percent of the retirees own private annuities in their own name in the United States. Many studies have attributed these observations to market failure and incompleteness in the market due to asymmetric information. For example Friedman and Warshawsky (1990) and Mitchell et al. (1999) have pointed out that private annuities are priced at a rate higher than the one implied by average population mortality rate. This suggests some degree of selection in the purchase of annuity insurance. People who expect to live longer buy more annuities and therefore force the insurance provider to charge prices so high that people with shorter life expectancy choose to stay out of the market. Finkelstein and Poterba (2004) have tested the implication of ex ante asymmetric information in annuity markets using UK data and confirmed the existence of adverse selection in the UK annuity market.

At the same time, the market for life insurance is huge. Cawley and Philipson (1999) call it "the largest market for private insurance in the world". They cite American Council of Life Insurance (1994) and Swiss Reinsurance Corporation (1997) and state that out of $2.1 trillion paid in insurance premia of any type in 1995, 58 percent was paid for life insurance. In the United States, life insurance premia is roughly 3.6 percent of GDP. Hong and Rios-Rull (2006) also look at *Life Insurance Factbook* and report total face value of $13.2 trillion in 1997 for life insurance.

The important role of annuities in insuring the uncertain lifetime and widespread evidence of market failure in providing adequate insurance is the motivation of many studies such as Diamond (1977) in suggesting the evaluation of social security program and in general the need for public provision of annuity insurance.[1] It is argued that because of the failure of the private markets in efficiently providing annuity insurance, public provision is necessary. However, optimal size and welfare benefit of such program have been the subject of much debate. There is a very large empirical literature on evaluating the welfare benefit of social security program or exploring the optimal social security taxes and estimating the welfare

[1] See also Congressional Budget Office (1998)

gain of switching from the current system to an optimal system. The range of findings in this literature is also very wide. For example İmrohoroğlu et al. (1995) study the optimal social security replacement ratio in a general equilibrium overlapping generation model with uncertain lifetime. They assumed no market for annuity without modeling the reason for market modeling (there is no asymmetric information in their model). Their finding is that the optimal social security replacement ratio is 30% and the steady state gain from switching to the optimal level is roughly 2% of GNP. On the other hand Hong and Rios-Rull (2006) have studied a similar environment except that they have included bequest motives and calibrated the model with agents differing in age, sex and marital status. They consider various market structure for annuity and life insurance (without modeling the forces behind those market structure) and found no support for social security.

The theoretical model presented in this paper has few interesting implications. First, the model predicts that there is at most one active insurance market. In other words, if there is trade in life insurance market, the market for annuity closes. This prediction is qualitatively consistent with the empirical regularities cited above, although it is a bit extreme since after all the trade in the annuity market in the data is not zero. The second important result is that the size of the social security program has a direct effect on the structure of the market for annuity and life insurance. In Theorem 2 and 3, I prove that the larger the social security taxes are the lower is the demand for annuity. At the same time the size of the life insurance market grows with the social security taxes. In this model the consumers enjoy leaving bequest. Facing with high social security benefit and therefore lower marginal utility for consumption, they choose higher level of life insurance to increase the bequest and reduce their marginal utility from leaving bequest.

In the environment studied in this paper efficient allocations are constant across types. This makes these allocations implementable by simple social security tax and transfer available to the government in the model. At the optimal social security policy both annuity and life insurance market are endogenously closed (inactive). Social security provides annuity to consumers at a rate that is always lower than the one in the private market. Therefore increasing social security can increase welfare. However, at the same time social security crowds out lower survival types from annuity market. This leads to higher prices in the market and can potentially lower consumers' welfare. I show that in fact the former effect is dominating. Therefore, to achieve efficiency government levies large enough tax and transfer enough benefit so that no consumer purchase annuity at high market prices.

In all of the empirical investigations of the social security programs the structure of the insurance market is taken as given and more importantly the economy does not feature any asymmetric information friction. However, the rationale for the assumed incomplete insurance market is generally stated to be adverse selection. The model studied in this paper provides a framework that enables us to study the interaction between social security policy and the extent of adverse selection and the size of the annuity and life insurance market. Whether this produces a quantitatively sizable effect is the subject of another research project. However, the theoretical results in this paper can potentially challenge a widely held assumption in the empirical investigation of the optimal social security literature; exogenous market incompleteness.

The studies closest to the current paper are Villeneuve (2003), Abel (1986) and Brugiavini (1993). Villeneuve (2003) uses a similar environment and shows the crowding out effect of social security on annuity insurance market. Abel (1986) also studies a similar setup but with no market for life insurance in a two period overlapping generation model. Both these papers have a heuristic treatment of equilibrium notion and existence and lack formal statement of equilibrium conditions.Brugiavini (1993) studies the case where agents enter in the private insurance contract before the uncertainty about their expectation of lifetime is realized and hence in his model there is no adverse selection. He finds that once a contract is signed before the agent knows about his lifetime expectancy, there will be no more trade after the agent learns his type. None of the papers above discuss welfare and optimal policy.

The rest of the paper is organized as follows. The model is introduced in the section 2 and equilibrium is defined. In Section 3 I first discuss some properties of the same environment under full information and then establish properties of the equilibrium under asymmetric information. In section 4 I discuss policy and effect of social security on annuity and life insurance markets. Section 5 concludes and contains discussion and direction of future research.

2 The Economy

I study the following environment. There is a two period economy with single consumption good that can be consumed in period 1 and 2 or can be left as bequest. There are infinitely many consumers in the economy indexed by their survival probability in the second period $\pi \in [\underline{\pi}, \overline{\pi}]$. I refer to this survival probability as the consumer's type. There is a continuum of unit measure of consumers of each type. There are also large number of intermediaries who trade in the market.

Information. There is no aggregate uncertainty in the economy. Consumers face an idiosyncratic uncertainty in lifetime. Each consumer of type π lives through the second period with probability π or dies at the beginning of the second period with probability $1-\pi$. The survival probability is private information and it is known only by the consumer. Consumers are informed about their survival probability at the beginning of period 1 before they make any trade. Types are independent across consumers and are drawn from a distribution μ which is commonly known by all agents in the economy. When the uncertainty about death is realized, it is observed by everyone (there is no ambiguity about who is dead and who is alive in the second period).

Consumers. Consumers are endowed with e unit of consumption good in the first period. They have preferences over first period consumption, c_1, second period consumption (if they are alive in the second period), c_2, and the amount of bequest they leave if they die, b. Their lifetime expected utility is

$$u(c_1) + \beta \pi U(c_2) + \beta(1-\pi)v(b)$$

where u, U and v are utility functions over consumption and bequest respectively and they satisfy the following assumption

Assumption 1 *$u(\cdot)$, $U(\cdot)$ and $v(\cdot)$ are twice continuously differentiable, strictly increasing, strictly concave and satisfy INADA conditions ($\lim_{c\to 0} u'(c) = \infty$, etc).*

Consumers discount second period utility at rate $0 < \beta < 1$.

Contracts There are two types of insurance contracts; *annuity* and *life insurance* contracts. One unit of annuity contract pays a unit of consumption good if the consumer is alive and one unit of life insurance contract pays a unit of consumption good when consumer dies[2]. Consumers can only buy (and cannot sell) either of these contracts. By buying each of these contracts, consumers in effect make contributions to a pool in the first period. In the second period, contingent on whether they are alive or not, they receive payments from the pool that is proportional to their contribution. Therefore, buying a contract is in fact buying a share in these pool.

[2]Note that in the background we assume that this consumption good is received by consumer's heirs, although they are not present in the model.

Technology. There is a saving technology with return $A > 1$ that is available to everyone.

Intermediaries. There are large number of intermediaries. Each intermediary holds two pools. One for annuity and one for life insurance. Given their expectation about the per capita contributions and per capita payouts in each pool, they choose the size of the pool, i.e. the fraction of population that can join the pool. Intermediaries collect all the contributions to the pools in the first period and invest them in the saving technology.

Prices. Annuity and life insurance contracts are traded at prices p^a and p^l respectively. Note that I assume intermediaries cannot monitor consumers trading activities. Therefore, the insurance contract cannot be contingent on the volume of trade that each consumer makes. It also cannot be contingent on the consumer's asset holdings. Hence, although different types of consumers make different purchases of insurance contracts, intermediaries are not able to screen them. Therefore, they charge a single market determined price for any type of purchase independent of the volume and portfolio.

In this environment contracts are linear, i.e. one unit of annuity can be purchased at a constant price p^a independent of the volume. This linearity of contract is assumed (through modeling the insurance company as a pool that collects contributions and make payments) rather than being driven from a detailed strategic model. In this paper I do not take any stand on the optimality of such linear contracts. In other words there is no claim (nor a conjecture) that if intermediaries were more sophisticated and could compete over non-exclusive contracts, such linear contract would emerge.[3].

Social Security. There is a Social Security system that collects taxes from every consumer at rate τ and lump-sum rebate it equally to anyone who is alive in the second period. So it can be viewed as a partial annuity insurance that is publicly provided at the economy wide actuarially fair rate. Government transfers the social security receipts to the second period using the saving technology $A > 1$ that is available in the economy.

Consumer's choice problem is stated formally as the following. Let $a(\pi; p^a, p^l)$, $l(\pi; p^a, p^l)$ and $s(\pi; p^a, p^l)$ denote the holdings of annuity, life insurance and saving for type π consumer

[3] Ales and Maziero (2011) and Attar et al. (2014) prove that almost linear contracts emerge as the result of non-exclusivity in a static model with adverse selection. Cawley and Philipson (1999) provide evidence on almost linear prices for life insurance market. Cannon and Tonks (2008) and Finkelstein and Poterba (2004) provide evidence of linear prices for annuity market in the UK.

respectively at prices (p^a, p^l). Then the choice problem faced by this consumer is

$$\max_{c_1,c_2,b,a,l,s} u(c_1) + \pi\beta U(c_2) + (1-\pi)\beta v(b) \tag{1}$$

subject to

$$\begin{aligned} c_1 + p^a a + p^l l + s &\leq e(1-\tau) \\ c_2 &\leq As + a + T \\ b &\leq As + l \\ c_1, c_2, b, a, l, s &\geq 0 \end{aligned}$$

Where T is the social security transfer.

I follow Bisin and Gottardi (2003) and Dubey and Geanakoplos (2002) in modeling intermediaries' behavior.[4] Each intermediary forms expectation about per capita contributions and payouts in each pool (annuity and life insurance). Taking those expectations as given the intermediary decides about the size of each pool. Let R^a and Q^a the per capita contributions and payouts in annuity pool, and R^l and Q^l be the per capita contributions and payouts in life insurance pool. Intermediary chooses the fraction of population that can join either of these pools, n^a and n^l.

$$\max_{0\leq n^a, n^l \leq 1, k\in\mathbb{R}_+} n^a R^a + n^l R^l - k \tag{2}$$

subject to

$$n^a Q^a + n^l Q^l \leq Ak$$

Remark 1: Since intermediaries are identical and have constant return to scale technology, there is no loss in generality in assuming only one representative intermediary.

Remark 2: In equilibrium, I require the expected per capita contributions and payouts to be consistent with consumer behavior and market price of contracts.

Definition 1 *A Competitive Equilibrium with Asymmetric Information is: consumer choices* $\{c_1^*(\pi), c_2^*(\pi), b^*(\pi), a^*(\pi), l^*(\pi), s^*(\pi)\}_{\pi\in[\underline{\pi},\bar{\pi}]}$, *intermediaries choices* $\{(n^{j*})_{j=a,l}, k^*\}$, *contract*

[4] There is however two important difference between my environment and the one studied in Dubey and Geanakoplos (2002). One is that they consider exclusive participations in pools and the other is that pools have capacity limits that varies across intermediaries. In my environment all the pools are identical and there is no limit.

prices (p^{a*}, p^{l*})*, anticipated contributions and payouts in each pool* $\{(R^{j*}, Q^{j*})_{j=a,l}\}$*, and social security policy* (τ, T) *such that*

1. *Consumers optimize: Given prices* (p^{a*}, p^{l*})*, and policy* (τ, T)*, consumer choices* $\{c_1^*(\pi), c_2^*(\pi), b^*(\pi), a$ *solve the problem (1) for every* $\pi \in [\underline{\pi}, \bar{\pi}]$

2. *Intermediaries maximize profit: Given anticipated contributions and payouts in each pool* $(R^{j*}, Q^{j*})_{j=a,l}$*,* $(n^{j*})_{j=a,l}$ *and* k^* *solves problem (2)*

3. $\{(R^{j*}, Q^{j*})_{j=a,l}\}$ *satisfy the following consistency conditions*

$$\begin{aligned} R^{a*} &= \int_{\underline{\pi}}^{\bar{\pi}} p^{a*} a^*(\pi) d\mu(\pi) \\ R^{l*} &= \int_{\underline{\pi}}^{\bar{\pi}} p^{l*} l^*(\pi) d\mu(\pi) \end{aligned} \tag{3}$$

   and

$$\begin{aligned} Q^{a*} &= \int_{\underline{\pi}}^{\bar{\pi}} \pi a^*(\pi) d\mu(\pi) \\ Q^{l*} &= \int_{\underline{\pi}}^{\bar{\pi}} (1-\pi) l^*(\pi) d\mu(\pi) \end{aligned} \tag{4}$$

4. *Social security budget:*

$$A\tau e = T\mathbf{E}[\pi]$$

   (where $\mathbf{E}[\pi] = \int_{\underline{\pi}}^{\bar{\pi}} \pi d\mu(\pi)$ *)*

5. *Market for contract clears:*

$$n^{j*} = 1 \quad j = a, l$$

6. *Goods market clear*

$$\int_{\underline{\pi}}^{\bar{\pi}} [\pi c_2^*(\pi) + (1-\pi) b^*(\pi)] d\mu(\pi) = A\left(e - \int_{\underline{\pi}}^{\bar{\pi}} c_1^*(\pi) d\mu(\pi)\right)$$

3 Equilibrium Properties and Existence

3.1 Full Information Economy

Before I discuss the existence and properties of the equilibrium just defined it is useful to study the properties of the equilibrium under full information. In this environment agents' type is public information and insurance pools are type specific. Therefore, each consumer faces a fair price. It is useful to study the consumer behavior when they are faced with such fair prices, i.e. for consumer of type π , $p^a(\pi) = \frac{\pi}{A}$ and $p^l(\pi) = \frac{1-\pi}{A}$. Then, the problem of consumer of type π is

$$\max_{c_1,c_2,b,a,l,s} u(c_1) + \pi\beta U(c_2) + (1-\pi)\beta v(b)$$

subject to

$$\begin{aligned} c_1 + \frac{\pi}{A}a + \frac{1-\pi}{A}l + s &\leq e(1-\tau) \\ c_2 &\leq As + a + T \\ b &\leq As + l \\ c_1, c_2, b, a, l, s &\geq 0 \end{aligned}$$

It is a straight forward application of maximum theorem to show that solutions of the above problem are continuous functions of π. Solution to this problem is characterized by the following first order conditions

$$\frac{u'(c_1(\pi))}{A\beta} = U'(c_2(\pi)) = v'(b(\pi)) \tag{5}$$

if $a(\pi) > 0$ and $l(\pi) > 0$ and

$$\frac{u'(c_1(\pi))}{A\beta} > U'(c_2(\pi)) \text{ and } \frac{u'(c_1(\pi))}{A\beta} = v'(b) \tag{6}$$

if $a(\pi) = s(\pi) = 0$.[5]

In both cases the sign of $a(\pi) - l(\pi)$, which I call *net annuity purchase*, has a special property

Proposition 1 *In any equilibrium with full information, the sign of net annuity purchase is the same for all types. In other words if $a(\pi) - l(\pi)$ is positive (negative or equal zero) for some type π, it is positive (negative or equal zero) for all $\pi \in [\underline{\pi}, \bar{\pi}]$*

Proof.

First I show that if one type chooses $a(\pi) = l(\pi)$, then all types must do so. Suppose type π chooses $a(\pi) = l(\pi)$. Then the allocation that he chooses must satisfy first order conditions

[5]Because of INADA condition on $v(\cdot)$, s and l cannot be both equal to zero.

(5) or (6) together with budget constraint, which after imposing $a(\pi) = l(\pi)$ becomes

$$\begin{aligned} c_1(\pi) + \frac{1}{A}(l(\pi) + As(\pi)) &\leq e(1-\tau) \\ c_2(\pi) &\leq Tr + l(\pi) + As(\pi) \\ b(\pi) &\leq l(\pi) + As(\pi) \end{aligned}$$

Note that these equations do not depend on π. Therefore, they must hold for all types $\pi \in [\underline{\pi}, \bar{\pi}]$.

Now suppose for some type $a(\pi) - l(\pi) > 0$ and there is some other type such that $a(\pi') - l(\pi') < 0$. Then by continuity there must exists a $\tilde{\pi} \in [\underline{\pi}, \bar{\pi}]$ such that $a(\tilde{\pi}) - l(\tilde{\pi}) = 0$. This is a contradiction. ■

The result I just proved will be useful in proving a related result in asymmetric information economy where I show that all types purchase the same kind of insurance (or do not any insurance at all). Next, I show that there is a unique level of social security benefit such that all households choose not to buy any kind of insurance. For any level of social security below this critical level all households purchase more annuity than life insurance and for levels of social security above it all household buy more life insurance.

Proposition 2 *In the full information economy there exists a level of social security tax τ^* such that for any $\tau < \tau^*$ all consumers purchase positive net annuity and for all $\tau > \tau^*$ all consumers purchase negative net annuity. At the value $\tau = \tau^*$ purchase of net annuity is zero.*

Proof.

Consider the allocation that satisfies the following conditions.

$$u'(c_1^*) = U'(c_2^*) = v'(b^*)$$

$$c_1^* + \frac{1}{A}\mathbf{E}[\pi]c_2^* + \frac{1}{A}(1 - \mathbf{E}[\pi])b^* = e$$

where $\mathbf{E}[\pi]$ is average survival probability in the economy. Let τ^* be such that

$$c_1^* + \frac{1}{A}b^* = e(1 - \tau^*)$$

and therefore, $T^* = c_2^* - b^* = \frac{e\tau^*}{\mathbf{E}[\pi]}$. It is easy to check that these allocations can only be achieved by choosing $b^* = l^* + As^*$ and $l^* = a^*$. Furthermore, they are independent of type and hence are the same for every type $\pi \in [\underline{\pi}, \bar{\pi}]$.

The claim is that for tax and transfer (τ^*, T^*) no consumer purchase any type of insurance and all types choose the same allocation (the ex-ante efficient allocation). But this is trivial since this is the unique allocation that satisfy the first order condition and budget constraint of all types and is independent of π.

Now consider the choice of the type $\pi' = \mathbf{E}[\pi]$. This consumer's consumption and bequest is independent of τ and T and it always equals the star allocation defined above (easy to show). Consider this type's budget constraint for $T < T^*$

$$a(\pi') - l(\pi') = c_2^* - b^* - T > c_1^* - b^* - T^* = 0$$

Since one type purchases positive net annuity, all the types must do so. Now consider the case where $T > T^*$. The budget constraint of type $\pi' = \mathbf{E}[\pi]$ is

$$\begin{aligned} c_1^* + \frac{1}{A}\pi' a(\pi') + \frac{1}{A}(1-\pi')l(\pi') + s(\pi') &= e(1-\tau) \\ c_2^* &= T + a(\pi') + As(\pi') \\ b^* &= As(\pi') + l(\pi') \end{aligned}$$

replace for $s(\pi')$ in the second equation

$$a(\pi') - l(\pi') = c_2^* - b^* - T < c_1^* - b^* - T^* = 0$$

■

I will show in the next section that the same result holds in the asymmetric information economy. At social security tax $\tau^* = \frac{e\tau^*}{\mathbf{E}[\pi]}$ both life insurance and annuity market is crowded out. Furthermore, I will also show that τ^* will implement efficient allocations in the economy.

3.2 Asymmetric Information Economy

Consider intermediary's problem (problem (2)). In the equilibrium when intermediary has correct expectations about contributions and payouts in each pool and makes optimal decision, the following equations will characterize equilibrium prices

$$Ap^a \int_{\underline{\pi}}^{\bar{\pi}} a(\pi) d\mu(\pi) = \int_{\underline{\pi}}^{\bar{\pi}} \pi a(\pi) d\mu(\pi) \tag{7}$$

and

$$Ap^l \int_{\underline{\pi}}^{\bar{\pi}} l(\pi) d\mu(\pi) = \int_{\underline{\pi}}^{\bar{\pi}} (1-\pi) l(\pi) d\mu(\pi) \tag{8}$$

Note however that when $\int_{\underline{\pi}}^{\bar{\pi}} a(\pi)d\mu(\pi) = 0$ or $\int_{\underline{\pi}}^{\bar{\pi}} l(\pi)d\mu(\pi) = 0$ the price is indeterminate by these equations. In other words any price will satisfy these equation when aggregate demand for insurance is zero (since we get zero on both sides of the equations). For now, in such cases I choose the prices to be the following

$$p^a = \frac{\bar{\pi}}{A} \text{ whenever } \int_{\underline{\pi}}^{\bar{\pi}} a(\pi) d\mu(\pi) = 0 \tag{9}$$

and

$$p^l = \frac{1-\underline{\pi}}{A} \text{ whenever } \int_{\underline{\pi}}^{\bar{\pi}} l(\pi) d\mu(\pi) = 0$$

Later (in the proof of existence) I will justify this selection and show that they are in fact consistent with equilibrium behavior of consumers.

Next lemma is a technical result that helps us establish properties of equilibrium prices and consumer behavior. This lemma is useful in establishing that if there is adverse selection in a market (in the sense that higher risk types purchase more insurance) then the equilibrium price is higher that the one implied by average risk in the population.

Lemma 1 *Consider a function f on $[\underline{\pi}, \bar{\pi}]$ such that $f(\pi) \lesseqqgtr 0$ as $\pi \lesseqqgtr \pi^*$ and $\int_{\underline{\pi}}^{\bar{\pi}} f(\pi)d\mu(\pi) = 0$. Let $g(\pi)$ be an increasing function with $g(\bar{\pi}) > g(\underline{\pi})$. Then $\int_{\underline{\pi}}^{\bar{\pi}} f(\pi)g(\pi)d\mu(\pi) > 0$*

Proof.

$$\begin{aligned} \int_{\underline{\pi}}^{\bar{\pi}} f(\pi)g(\pi)d\mu(\pi) &= \int_{\underline{\pi}}^{\pi^*} f(\pi)g(\pi)d\mu(\pi) + \int_{\pi^*}^{\bar{\pi}} f(\pi)g(\pi)d\mu(\pi) \\ &> g(\pi^*)\left[\int_{\underline{\pi}}^{\bar{\pi}} f(\pi)d\mu(\pi)\right] = 0 \end{aligned}$$

■

Now we are ready to study consumer behavior. First, I show in two steps that in any equilibrium prices must satisfy $A(p^a + p^l) > 1$.

Lemma 2 *Equilibrium prices satisfy $A(p^a + p^l) \geq 1$*

Proof.

Suppose $A(p^a + p^l) < 1$ and consider the first order conditions of the consumer.

$$u(c_1) \geq \pi A\beta U'(c_2) + (1-\pi)\beta A v'(b) \tag{10}$$

$$p^a u'(c_1) \geq \pi\beta U'(c_2) \tag{11}$$

$$p^l u'(c_1) \geq (1-\pi)\beta v'(b) \tag{12}$$

Under these prices $s(\pi)$ cannot be positive for any type. Simply because of no arbitrage condition (it can be easily checked by setting (10) at equality and sum the other two inequalities). Therefore we must have $s(\pi)$. Then assumption 2 implies $l(\pi) = b(\pi) > 0$ for all π. $a(\pi)$ can be either positive of zero. I first show that $l(\pi)$ is strictly decreasing function of π in both cases. Suppose $a(\pi) = 0$, then solution to consumers problem is characterized by

$$p^l u'(c_1(\pi)) = \beta(1-\pi)v'(b(\pi))$$

$$c_1(\pi) + p^l b(\pi) = e(1-\tau) \text{ and } c_2(\pi) = T$$

Now consider problem of a type $\pi' > \pi$. Consider the first order condition for this type at the $c(\pi)$ and $l(\pi)$ allocations.

$$p^l u'(c_1(\pi)) > \beta(1-\pi')v'(b(\pi))$$

Suppose $a(\pi') = 0$. It is obvious that both $b(\pi)$ and $c_1(\pi)$ cannot increase or decrease at the same time. Then the only way to restore equality is to have $c_1(\pi') > c_1(\pi)$ and $b(\pi') < b(\pi)$. If $a(\pi') > 0$, then $c_2(\pi') > c_2(\pi)$. Also, c_1 and b cannot increase at the same time (since $c_1 + p^l b$ must decrease). The only possibility that doesn't have $b(\pi') < b(\pi)$ is the case where $b(\pi') > b(\pi)$ and $c(\pi') < c(\pi)$. But this cannot restore equality in first order condition (which is required for $b(\pi') > 0$). Therefore, the claim is established that $b(\pi)$ is strictly increasing function of π. Now consider the price equation (8). We can rewrite it as (since $l(\pi) = b(\pi) > 0$ for all π)

$$Ap^l = \frac{\int_{\underline{\pi}}^{\bar{\pi}}(1-\pi)l(\pi)d\mu(\pi)}{\int_{\underline{\pi}}^{\bar{\pi}} l(\pi)d\mu(\pi)}$$

Using result of lemma 1 we can show that $Ap^l > (1 - \mathbf{E}[\pi])$. Now if $a(\pi) = 0$ for all π, we know that $Ap^a = \bar{\pi}$. If $a(\pi) > 0$ for some π using similar argument as above we can show that it must be strictly increasing function of π. Here is how the argument works. Consider choices of type π

$$p^l u'(c_1(\pi)) = \beta(1-\pi)v'(b(\pi))$$

$$
p^a u'(c_1(\pi)) \;=\; \beta\pi U'(c_2(\pi))
$$

$$
c_1(\pi) + p^l b(\pi) + p^a a(\pi) = e(1-\tau) \text{ and } c_2(\pi) = T + a(\pi)
$$

Now suppose $\pi' > \pi$. Evaluate the first order conditions of type π' at $c_1(\pi)$, $c_2(\pi)$ and $b(\pi)$

$$
\begin{aligned}
p^l u'(c_1(\pi)) &> \beta(1-\pi')v'(b(\pi)) \\
p^a u'(c_1(\pi)) &< \beta\pi' U'(c_2(\pi))
\end{aligned}
$$

and note that $c_1(\pi)+p^l b(\pi)+p^a a(\pi) = e(1-\tau)$. The only possible way to restore the equality without violating budget constraint is to have $c_2(\pi') > c_2(\pi)$ (and therefore $a(\pi') > a(\pi)$), $c_1(\pi') < c_1(\pi)$ and $b(\pi') < b(\pi)$. Again, applying lemma 1 to equation (7) we can show that $Ap^a > \mathbf{E}[\pi]$. In both cases we get $A(p^a + p^l) > 1$. A contradiction. ■

The above lemma restrict equilibrium prices to be $A(p^a + p^l) \geq 1$. The next lemma shows that the equality ($A(p^a + p^l) = 1$) is also incompatible with any equilibrium.

Lemma 3 *In equilibrium $A(p^a + p^l) = 1$ cannot hold.*

Proof.
Again consider the consumer problem (replace for budget constraint for $a(\pi)$ and $l(\pi)$)

$$
\begin{aligned}
p^l u'(c_1(\pi)) &= \beta(1-\pi)v'(b(\pi)) \\
p^a u'(c_1(\pi)) &= \beta\pi U'(c_2(\pi))
\end{aligned}
$$

$$
c_1(\pi) + p^a c_1(\pi) + p^l b(\pi) = e(1-\tau) + p^a T
$$

Note also that when prices are such that $A(p^a + p^l) = 1$ the first order conditions hold with equality and portfolio composition of consumer is indeterminate (consumer is indifferent between any combination of a, l and s). First I show that $c_2(\pi)$ (respectively $b(\pi)$) is strictly increasing (respectively decreasing) in π. The argument work as the one used in previous lemma. Consider the first order condition of type π

$$
\begin{aligned}
p^l u'(c_1(\pi)) &= \beta(1-\pi)v'(b(\pi)) \\
p^a u'(c_1(\pi)) &= \beta\pi U'(c_2(\pi))
\end{aligned}
$$

now suppose $\pi' > \pi$ and evaluate this types first order condition at $c_1(\pi)$, $c_2(\pi)$ and $b(\pi)$

$$\begin{aligned} p^l u'(c_1(\pi)) &> \beta(1-\pi')v'(b(\pi)) \\ p^a u'(c_1(\pi)) &< \beta\pi' U'(c_2(\pi)) \end{aligned}$$

then the only way to restore equality without violating budget constraint is to have $c_1(\pi') < c_1(\pi)$, $c_2(\pi') > c_2(\pi)$ and $b(\pi') < b(\pi)$. This also implies that net annuity purchase, $a(\pi) - l(\pi) = c_2(\pi) - b(\pi) - T$, is strictly increasing.
Next step is to show that $\int_{\underline{\pi}}^{\bar{\pi}}(a(\pi)-l(\pi))d\mu(\pi) \neq 0$. Suppose otherwise, then since $a(\pi)-l(\pi)$ is monotone and continuous, there must exists a π^* such that $a(\pi^*) - l(\pi^*) = 0$. Now lets recall pricing equations (7) and (8) and replace $Ap^l = 1 - Ap^a$

$$Ap^a \int_{\underline{\pi}}^{\bar{\pi}} a(\pi)d\mu(\pi) = \int_{\underline{\pi}}^{\bar{\pi}} \pi a(\pi)d\mu(\pi)$$

$$(1 - Ap^a)\int_{\underline{\pi}}^{\bar{\pi}} l(\pi)d\mu(\pi) = \int_{\underline{\pi}}^{\bar{\pi}} (1-\pi)l(\pi)d\mu(\pi)$$

add the above equations we get

$$Ap^a \int_{\underline{\pi}}^{\bar{\pi}} (a(\pi) - l(\pi))d\mu(\pi) = \int_{\underline{\pi}}^{\bar{\pi}} \pi(a(\pi) - l(\pi)d\mu(\pi) > 0$$

where the last inequality is just an application of lemma 1. This is contradiction. Therefore, $\int_{\underline{\pi}}^{\bar{\pi}}(a(\pi) - l(\pi))d\mu(\pi) \neq 0$. Now consider the same pricing equations as above

$$Ap^a = \frac{\int_{\underline{\pi}}^{\bar{\pi}} \pi(a(\pi) - l(\pi)d\mu(\pi)}{\int_{\underline{\pi}}^{\bar{\pi}}(a(\pi) - l(\pi))d\mu(\pi)} > \mathbf{E}[\pi]$$

Similarly we can show that $Ap^l > 1 - \mathbf{E}[\pi]$. This is a contradiction with $A(p^a + p^l) = 1$. ■

So far I have established that any equilibrium price must satisfy the following inequality

$$A(p^a + p^l) > 1 \tag{13}$$

Next, I will focus on characterizing the consumer behavior under the admissible prices that satisfy inequality (13). Important implication of this property is that purchasing both annuity and life insurance is more expensive than saving. Therefore, each consumer prefers to purchase at most one of them (and possibly hold some savings). This is formally shown in the next lemma.

Lemma 4 *For any set of prices such that $A(p^a + p^l) > 1$ (and not just the the equilibrium prices) and any tax and transfer (τ, T)*

1. *Consumer choices, $\{c_1(\pi), c_2(\pi), b(\pi), a(\pi), l(\pi), s(\pi)\}$ are continuous in π*
2. *$a(\pi)$ (respectively $l(\pi)$) is monotone increasing (respectively decreasing) function of π*
3. *There are survival probabilities $\underline{\pi}_a$ and $\overline{\pi}_l$ such that $\underline{\pi}_a > \overline{\pi}_l$ and*
   if $\pi < \overline{\pi}_l$ consumer purchases life insurance and not annuity
   if $\pi \in [\overline{\pi}_l, \underline{\pi}_a]$ consumer is not insured
   if $\pi > \underline{\pi}_a$ consumer purchase annuity and not life insurance
   (Note that this allows for the possibility that $\overline{\pi}_l < \underline{\pi}$ or $\underline{\pi}_a > \overline{\pi}$ or both.)

Proof.
Proof of 1) First part is a direct application of Maximum Theorem.

Proof of 2) For the second part first note that it is immediate from the assumption on prices that holdings of annuity and life insurance cannot be positive at the same time (just add the first order conditions for a and l can compare the result with first order condition fro savings). So each consumer either buys annuity or life insurance or none of them. Suppose that $a(\pi) > 0$ and consider $a(\pi')$ where $\pi' < \pi$. Note that as I just argued, this implies $l(\pi) = 0$. Then either $a(\pi') = 0$, in which case the proof is done, or $a(\pi') > 0$ (and hence $l(\pi') = 0$). When $l(\pi)$ and $(l(\pi'))$ are zero, by the INADA condition $s(\pi) = b(\pi)$ (and $s(\pi') = b(\pi')$) has to be positive. First order conditions for π are

$$\begin{aligned} u(c_1(\pi)) &= \pi A\beta U'(c_2(\pi)) + (1-\pi)\beta A v'(b(\pi)) \\ p^a u'(c_1(\pi)) &= \pi\beta U'(c_2(\pi)) \\ p^l u'(c_1(\pi)) &> (1-\pi)\beta v'(b(\pi)) \end{aligned}$$

From top two equations we get

$$\begin{aligned} p^a u'(c_1(\pi)) &= \beta\pi U'(c_2(\pi)) \\ (1-Ap^a)u'(c_2(\pi)) &= (1-\pi)v'(b(\pi)) \end{aligned}$$

Also the budget constraint must be satisfied for all types (hence I suppress the π argument)

$$c_1 + p^a c_2 + (1-Ap^a)b = e(1-\tau) + p^a T$$

Now consider type $\pi' < \pi$ and evaluate its first order conditions at allocations chosen by type π are

$$\begin{aligned} p^a u'(c_1) &> \beta\pi' U'(c_2) \\ (1 - Ap^a)u'(c_2) &< (1 - \pi)v'(b) \end{aligned}$$

The only possible allocations that could restore equality in above equations and be consistent with must satisfy

$$c_1(\pi') > c_1(\pi), c_2(\pi') < c_2(\pi), b(\pi') > b(\pi)$$

Note that $b = s$ for both type π and π' immediately implies that we must have $a(\pi) > a(\pi')$, since

$$a(\pi) = c_2(\pi) - b(\pi) - T > c_2(\pi') - b(\pi') - T = a(\pi')$$

A similar argument can be used to prove $l(\pi)$ is decreasing in π.

Proof of 3) Restriction on prices (inequality (13)) implies that consumers of each type do not demand annuity and life insurance at the same time. For $\pi = 0$ there is no demand for annuity, $a(0) = 0$. If $a(1) = 0$ the claim is established, otherwise the existence of $\underline{\pi}_a$ is established by continuity (and monotonicity) of $a(\pi)$. The existence of $\bar{\pi}_l$ is also similar. We only need to prove that $\underline{\pi}_a > \bar{\pi}_l$. Let $\lambda^a(\pi)$ and $\lambda^l(\pi)$ be the multipliers on non-negativity constraints for $a(\pi)$ and $l(\pi)$ in consumer's problem. Then λ^a and λ^l are also continuous functions of π. $\lambda^a(\pi) = 0$ for all $\pi > \underline{\pi}_a$ and $\lambda^a(\pi) > 0$ for all $\pi < \underline{\pi}_a$. By continuity then we must have $\lambda^a(\underline{\pi}_a) = 0$. That is consumer of type $\underline{\pi}_a$ is indifferent between buying annuity or not. Similarly, it must be true that $\lambda^l(\bar{\pi}_l) = 0$. If, $\underline{\pi}_a = \bar{\pi}_l$, then $\lambda^a(\underline{\pi}_a) = \lambda^l(\bar{\pi}_l) = 0$. This means that first order conditions for a and l must hold with equality. But I argued in the proof of second part that this cannot happen when prices satisfy inequality (13). Therefore, we must have $\underline{\pi}_a > \bar{\pi}_l$ (note that $\underline{\pi}_a < \bar{\pi}_l$ is already ruled out by the fact that each consumer -at most- buys only one type of insurance). ■

Lemma 4 highlights two key characteristics of consumer's behavior. One is the fact that in each insurance market the consumer that has higher risk buys more insurance at any given price. This is the adverse selection aspect of the consumers behavior. The second point is established in the third part of lemma 2. At any given price each consumer buys either life insurance or annuity or chooses self insurance through savings. In fact the cut-off probabilities $\underline{\pi}_a$ and $\bar{\pi}_l$ determine the size of the market for each type of insurance contract.

They also determine the risk in each pool. For example the higher the is cut-off $\underline{\pi}_a$ is, the higher is the average survival of participant in annuity pool. This is one force that increases the risk of the pool. Another one comes from the fact that among those who participate in the annuity pool, consumers with higher survival probabilities contribute more and receive payouts with higher probabilities (obviously similar effects are present in life insurance pools).

The cut-off probabilities are found by studying the choices of a type that is indifferent (on the margin) between buying an insurance contract or self insure. Since the consumer choices are continuous, these cut-offs are also continuous function of prices. Also, cut-off probability for annuity (life insurance) increases monotonically with price of annuity (life insurance). This is simply because at higher prices, lower survival types choose not to purchase annuity. Therefore, as price of annuity increases (and only then) the cut-off types gets closer and closer to the boundary $\bar{\pi}$ (and therefore, aggregate demand becomes smaller and smaller). This loose argument suggests that the price selection (9) for inactive market is consistent with consumer behavior. This ideas is formally expressed and used in the existence proof that comes next. Existence is established by using fixed point theorem on equations (7) and (8) and follows straight forward and standard arguments.

Proposition 3 *A competitive equilibrium exists.*

Proof. We need to prove that there are prices p^{a*} and p^{l*} that solve the equations (7) and (8). The strategy is to write these equations as a fixed point problem and use the *Brouwer's Fixed Point Theorem* to prove the existence of equilibrium prices. Define the following functions

$$h_a(p^a, p^l) = \begin{cases} \frac{\int_{\underline{\pi}_a(p^a,p^l)}^{\bar{\pi}} \pi a(\pi; p^a, p^l) d\mu(\pi)}{A \int_{\underline{\pi}_a(p^a,p^l)}^{\bar{\pi}} a(\pi; p^a, p^l) d\mu(\pi)} & \text{if } \underline{\pi}_a(p^a, p^l) < \bar{\pi} \\ \frac{\bar{\pi}}{A} & \text{if } \underline{\pi}_a(p^a, p^l) = \bar{\pi} \end{cases}$$

and

$$h_l(p^a, p^l) = \begin{cases} \frac{\int_{\underline{\pi}}^{\bar{\pi}_l(p^a,p^l)} (1-\pi) l(\pi; p^a, p^l) d\mu(\pi)}{A \int_{\underline{\pi}}^{\bar{\pi}_l(p^a,p^l)} l(\pi; p^a, p^l) d\mu(\pi)} & \text{if } \bar{\pi}_l(p^a, p^l) > \underline{\pi} \\ \frac{(1-\underline{\pi})}{A} & \text{if } \bar{\pi}_l(p^a, p^l) = \underline{\pi} \end{cases}$$

Where $\bar{\pi}_l(p^a, p^l)$ and $\underline{\pi}_a(p^a, p^l)$ are cut-off probabilities in consumer's problem at prices (p^a, p^l). First note that $\underline{\pi}_a(p^a, p^l)$ solves the following system of equations

$$\begin{aligned} u'(e(1-\tau) - q\widetilde{s}) &= \widetilde{\pi} A \beta u'(Tr + \widetilde{s}) + (1-\widetilde{\pi}) A \beta v'(\widetilde{s}) \\ p^a u'(e(1-\tau) - q\widetilde{s}) &= \widetilde{\pi} \beta u'(Tr + \widetilde{s}) \end{aligned}$$

Where the unknowns are $\widetilde{\pi}$ and $\widetilde{s}$, the cut-off probability and optimal holding of risk free security at the cut-off probability (for given prices)[6]. It is clear that $\underline{\pi}_a(p^a, p^l)$ is a continuous function of prices (direct application of implicit function theorem to the above equations). The same is true for $\bar{\pi}_l(p^a, p^l)$.

Now define the fixed point function

$$H(p^a, p^l) = (h_a(p^a, p^l), h_l(p^a, p^l))$$

And note that $\underline{\pi} \leq Ah_a(p^a, p^l) \leq \bar{\pi}$ and $1 - \bar{\pi} \leq Ah_l(p^a, p^l) \leq 1 - \underline{\pi}$.

The claim is that function $H(p^a, p^l)$ has a fixed point in $[\frac{1}{A}\underline{\pi}, \frac{1}{A}\bar{\pi}] \times [\frac{1}{A}(1-\bar{\pi}), \frac{1}{A}(1-\underline{\pi})]$. Aggregate demand for annuity and life insurance are continuous in prices for standard reasons. We only need to prove the continuity of $H(p^a, p^l)$ at those prices such that $\underline{\pi}_a(p^a, p^l) = \bar{\pi}$ or $\bar{\pi}_l(p^a, p^l) = \underline{\pi}$ (where aggregate demand is zero).

Suppose $\bar{p} = (\bar{p^a}, \bar{p^l})$ be a vector of price such that $\underline{\pi}_a(\bar{p}) = \bar{\pi}$. Take a sequence p_n converging to $\bar{p}$ and notice that

$$\bar{\pi} \geq Ah_a(p_n) \geq \underline{\pi}_a(p_n)$$

Now take the limit $p_n \to \bar{p}$ and by continuity of $\underline{\pi}_a(p)$ we get $\lim_{p_n \to \bar{p}} \underline{\pi}_a(p_n) = \bar{\pi}$ and therefore $A \lim_{p_n \to \bar{p}} h_a(p_n) = \bar{\pi}$ and $h_a(p_n)$ is continuous. The same is true for $h_l(p)$ and by *Brouwer's Fixed Point Theorem* there is a vector of prices $(p^{a*}, p^{l*}) \in [\frac{1}{A}\underline{\pi}, \frac{1}{A}\bar{\pi}] \times [\frac{1}{A}(1-\bar{\pi}), \frac{1}{A}(1-\underline{\pi})]$ such that $H(p^{a*}, p^{l*}) = (p^{a*}, p^{l*})$. This establishes the existence of equilibrium. ■

It is important to note that the short sale constraints on holdings of annuity and life insurance by consumers is crucial in obtaining this existence result. To better understand the importance of this assumption consider the simple case where there are only two types and consumers are equally likely to be of either types. Suppose we relax the short sale constraint on annuity contract and now a consumer can buy and sell annuities. To help providing the intuition consider the extreme case where the high survival type buys and low survival type sells that contract. Then it may be possible that the aggregate position on annuity contract is zero. Then equation (7) implies that the aggregate payout to be zero as well. But these

[6] If $\tilde{s}$ turns out to be negative, we can set it equal zero and then $\tilde{\pi}$ is just the solution to the second equation with $\tilde{s} = 0$

two conditions fail to hold at the same time and equilibrium unravels.[7]

As it is pointed out in Bisin and Gottardi (1999) the root of the problem here is that unlike the case of symmetric information, contracts are different commodities for different types. Consumers of different types have different valuation for a contract and their choices affect the return of the contract. As a result, the aggregate return on the contract is not simply a linear function of aggregate positions. This poses a problem for the feasibility of such trade. In other words, there are not enough prices to separate the traders in the market. Imposing a short sale constraint provides a way to separate traders to buyers and sellers (consumers always buy and the only sellers are intermediaries). This degree of separation is enough for market to be cleared by linear prices.[8]

This existence result is also in contrast with a rather large literature on the non-existence of competitive equilibrium in economies with adverse selection that followed Rothschild and Stiglitz (1976). In that environment insurance providers are sophisticated players that offer menus of contracts to consumers. They show that in their environment the only contracts that are purchased in equilibrium are separating contracts where each type self-selects in buying contract specific for his type. This equilibrium set of contracts can be attacked by other contracts that are not in equilibrium set. In other words, there can be contracts outside equilibrium set that if offered will attract some types and earn positive profit. In the environment studied in this paper, I do not allow pools to differ in capacity. I also do not allow for monitoring. Therefore, I rule out the type of entry that is considered in Rothschild and Stiglitz (1976). By assuming identical pools, I impose identical decisions on all intermediaries.[9]

Although equilibrium always exist, all insurance pools are not active. An insurance pool is active if aggregate contributions in that type of pool is positive. Next theorem shows that there is always at most one type of insurance pool active. In other words if there are survival types that purchase annuity, that means no one is buying life insurance .

In Proposition 1 I showed that under full information, when each type faces fair prices for

[7] In fact using Lemma 1 one can prove that if aggregate contribution to a pool is zero while some types hold short and some types hold long positions in annuity, the total payouts must be bigger than zero.

[8] Bisin and Gottardi (1999) propose another mechanism through which the traders can be separated and that is through a non-linear price scheme that has a bid-ask spread structure.

[9] Dubey and Geanakoplos (2002) consider finite type of different pools that differ in capacity. They show that the Rothschild and Stiglitz (1976) always exist for reasons similar to the one discussed here.

annuity and life insurance, we cannot have one type buy more annuity than life insurance and another type does the opposite. There is nothing in the asymmetric information environment that changes this result. To gain intuition for this result suppose annuity market is active. Then there must be one (and only one) survival type that faces fair price of annuity. Consumers of this type choose same allocations as the one they choose under full information. They pick zero life insurance and choose annuity purchase equal to net annuity that they would have chosen under full information. Now suppose life insurance market is also active. Then there must be one (and only one) type that faces fair life insurance prices. Again these consumers choose the same allocations as the one they choose under full information. In particular they would choose zero annuity and pick life insurance equal to negative net annuity that they would have chosen under full information. But this implies that these two types must choose net annuity of different sign under full information. As it was shown in Proposition 1 this cannot happen.

Theorem 1 *In any equilibrium there is at most one active insurance market.*

Proof.
Suppose that annuity market is active, i.e, $\int_{\underline{\pi}}^{\bar{\pi}} a(\pi)d\mu(\pi) \neq 0$ Then from the equation (7) we have

$$\bar{\pi} > Ap^{a*} > \underline{\pi}_a$$

and therefore, there is a type $\pi = Ap^{a*}$ that faces fair annuity price. All consumers of this type can achieve full insurance and their allocation satisfy

$$u'(c_1(\pi)) = \frac{\beta\pi}{p^{a*}}U'(c_2(\pi)) = \frac{\beta(1-\pi)}{(\frac{1}{A} - p^{a*})}v'(b(\pi))$$

together with the following budget constraint (which is derived by replacing for $a(\pi)$

$$c_1(\pi) + p^{a*}(c_2(\pi) - T) + (\frac{1}{A} - p^{a*})b(\pi) = e(1-\tau)$$

These equations are simplified to

$$u'(c_1(\pi)) = A\beta U'(c_2(\pi)) = A\beta v'(b(\pi))$$

$$c_1(\pi) + \frac{\pi}{A}(c_2(\pi) - T) + \frac{(1-\pi)}{A}b(\pi) = e(1-\tau)$$

and since the annuity purchase by this type is positive we have $l(\pi) = 0$ and $b(\pi) = s(\pi)$ and

$$c_2(\pi) - Tr - b(\pi) = a(\pi) > 0$$

Now suppose to the contrary that the market for life insurance is also active. Then

$$1 - \underline{\pi} > Ap^{l*} > 1 - \bar{\pi}_l$$

and there is a type $\tilde{\pi} = 1 - Ap^{l*} < \pi$ that faces fair life insurance price and chooses allocations that satisfy

$$u'(c_1(\tilde{\pi})) = A\beta U'(c_2(\tilde{\pi})) = A\beta v'(b(\tilde{\pi}))$$

$$c_1(\tilde{\pi}) + \frac{\tilde{\pi}}{A}(c_2(\tilde{\pi}) - T) + \frac{(1-\tilde{\pi})}{A} b(\tilde{\pi}) = e(1-\tau)$$

and since the consumers of this type purchases life insurance we have $a(\tilde{\pi}) = 0$ and $c_2(\tilde{\pi}) = T + As(\tilde{\pi})$. Also $b(\tilde{\pi}) = l(\tilde{\pi}) + As(\tilde{\pi})$. By subtracting these two we get

$$c_2(\tilde{\pi}) - T - b(\tilde{\pi}) = -l(\tilde{\pi}) < 0$$

But recall form lemma 1 that we showed that when consumers are facing fair prices they choose either positive net annuity or negative (or zero). Here, consumer type π faces fair annuity prices and chooses positive (net) annuity purchase. At the same time consumer type $\tilde{\pi}$ faces fair life insurance prices and chooses positive life insurance (negative net annuity) purchase. This as we showed in lemma 1 is a contradiction. ■

4 Policy

4.1 Ex Ante Efficient Allocations

Before, I discuss optimal policy it is useful to define and characterize efficient allocation

Definition 2 *An allocation is Ex-ante Pareto Efficient if it is the solution to the following problem*

$$\max \mathbf{E}_\pi \left[u_1(c_1(\pi)) + \pi U_2(c_2(\pi)) + (1-\pi)v(b(\pi))\right]$$

subject to

$$\mathbf{E}_\pi \left[c_1(\pi) + \frac{1}{A}\pi c_2(\pi) + \frac{1}{A}(1-\pi)b(\pi)\right] = e$$

First order conditions imply that efficient allocations must satisfy

$$u'(c_1(\pi)) = A\beta U'(c_2(\pi)) = A\beta v'(\pi)$$

and feasibility

$$\mathbf{E}_{\pi}\left[c_1(\pi)+\frac{1}{A}\pi c_2(\pi)+\frac{1}{A}(1-\pi)b(\pi)\right]=e$$

Note that this immediately implies that the efficient allocations are constant across types. The fact that efficient allocations do not depend on private information about types makes them easy to implement. In fact they can be implemented by lump sum transfer that is assumed to be available in the form of social security in this environment. Next proposition shows that these allocations can be implemented by choosing a level of social security tax and transfer that completely crowds out activities in both annuity and life insurance market.

Theorem 2 *There exists a level of social security tax τ^* such that for any $\tau < \tau^*$ all consumers purchase annuity and for all $\tau > \tau^*$ all consumers purchase life insurance. At the value $\tau = \tau^*$ consumers do not purchase any kind of insurance (this τ^* is the same as the one we found under symmetric information)*

Proof.

Consider the τ^* that is constructed in the proof of lemma 2 (which is constructed using ex ante efficient allocations). The claim is that every consumer optimally choose the ex ante efficient allocation and purchase no insurance in private market. These allocation by construction satisfy every type's budget constraint since τ^* is such that

$$c_1^* + \frac{1}{A}b^* = (1-\tau^*)e$$

and

$$T^* = c_2^* - b^*$$

and $a^* = l^* = 0$ and $s^* = \frac{1}{A}b^*$. Where star allocations are efficient allocations. Now, it only left to show that these allocations satisfy optimality for all types. Suppose otherwise. Suppose positive measure of types purchase annuity (and by theorem 1 no life insurance) and therefore the price of annuity is $\bar{\pi} > Ap^a > \underline{\pi}$. Then there must be a type $\pi = Ap^a$ that face fair price. It is obvious that this type chooses the star allocation (since it satisfies budget constraint and first order condition for this consumer type). But this implies that all types with lower survival probability than $\pi = Ap^a$, also would choose zero annuity purchase. This in turn implies that the market price in fact must be higher than $\frac{1}{A}\pi$. A contradiction. Therefore annuity purchase must be zero for all and $Ap^{a*} = \bar{\pi}$. Similarly it can be shown that there is no activity in life insurance market and $1 - Ap^{l*} = \underline{\pi}$.

For $\tau < \tau^*$, there is one type that faces fair prices (this type can be $\bar{\pi}$). As it was shown in lemma 2 a consumer facing fair prices in this case chooses positive net annuity purchase. This means that this type will choose positive annuity (and zero life insurance). If this type's survival probability is equal $\bar{\pi}$, then by continuity of $a(\pi)$ there is exist neighborhood around $\bar{\pi}$ such that demand for annuity is positive. But this implies that the type that faces fair price must be lower than $\bar{\pi}$. If this type's survival probability is strictly less than $\bar{\pi}$, the claim is established and annuity market is active. Then argument can be used to show that for $\tau > \tau^*$ the life insurance market is active. ■

Social security benefit is a substitute for annuity income. Therefore, it is not surprising that increase in social security crowds out activities in annuity market. In fact it does so for two reasons. One is that as the level of social security transfer increases lower survival type, who have lower demand for annuity, leave the market. This leave the market with higher survival types and as it is shown in the next section that it increases the price in the market. This increase in price leads to even further reduction in demand for annuities.

Increase in social security lowers income in the first period and increases consumption in survival state in the second period. Therefore, lowers the marginal utility of consumption in survival state. This leads to lowering the savings and increase the purchase of life insurance to smooth consumption over the state of survival and death (bequest).

It was shown in the proof of last proposition and it is stated formally next that there is an optimal social security tax and transfer that implements the efficient allocation. Furthermore, under this optimal policy annuity and life insurance markets endogenously closed.

Proposition 4 *Optimal size of social security is* (τ^*, T^*) *and the resulting allocations are ex-ante efficient.*

Proof.
The proof is immediate by construction of τ^*. ■

4.2 Effect of Social Security on Prices

Although social security can implement the efficient allocation and in that sense it is beneficial, it also has effect on insurance prices in private market. Social security can increase price in annuity market. This is because it provides a substitute for annuity. The rate of return on annuity provided by social security is better than the one in the private market. Therefore, increasing social security's tax and benefit lowers demand for annuity for all types.

Social security affects the choice of lower survival types buy larger amount. Increase in tax and transfer has an income effect and a substitution effect. The substitution effect is negative and is the same for all types. However, the income effect is positive. Higher survival types spend bigger share of their income on purchasing annuity so the positive income effect is larger for them. Therefore, for higher survival types the overall effect is smaller. This makes the risk in annuity pool worse. As social security tax and transfer increases higher survival types are more represented in the pools and this in turn increases the prices of annuity. Next proposition offers a formal proof of this argument for homothetic preferences.

Proposition 5 *Suppose the preferences are homothetic, i.e, let u be homothetic, $U = \alpha u$ and $v = \gamma u$ for some constants α and γ. Let p^{a*} and p^{l*} be equilibrium annuity and life insurance prices. Then*

$$\frac{\partial p^{a*}}{\partial \tau} > 0 \text{ and } \frac{\partial p^{l*}}{\partial \tau} = 0 \text{ for } \tau < \tau^*$$

$$\frac{\partial p^{l*}}{\partial \tau} < 0 \text{ and } \frac{\partial p^{a*}}{\partial \tau} = 0 \text{ for } \tau > \tau^*$$

Proof.

Suppose $\tau < \tau^*$. Then we know there will be no demand for life insurance and $p^{l*} = \frac{1}{A} - \underline{\pi}$. Therefore $\frac{\partial p^{l*}}{\partial \tau} = 0$. Now consider the consumer problem (and we know $l(\pi) = 0$)

$$\max u(c_1) + \beta \pi U(c_2) + \beta(1-\pi)v(b)$$

subject to

$$\begin{aligned} c_1 + p^a a + s &\leq e(1-\tau) \\ c_2 &\leq T + a + As \\ b &\leq As \end{aligned}$$

replace for a and s in budget constraint and we get

$$c_1 + p^a c_2 + \left(\frac{1}{A} - p^a\right) b \leq (1-\tau)e + p^a T$$

Then because of homotheticity the solution will have the following form (if $\pi \geq \underline{\pi}_a$)

$$\begin{aligned} c_1 &= \phi_1(p^a, \pi)((1-\tau)e + p^a T) \\ c_2 &= \phi_2(p^a, \pi)((1-\tau)e + p^a T) \\ b &= \phi_b(p^a, \pi)((1-\tau)e + p^a T) \end{aligned}$$

where $\phi_1(p^a, \pi)$, $\phi_2(p^a, \pi)$ and $\phi_b(p^a, \pi)$ are between zero and one. Then we can find the demand for annuity

$$a(p^a, \pi, \tau) = \begin{cases} \phi_a(p^a, \pi)((1-\tau)e + p^a T) - T & \text{if } \pi \geq \underline{\pi}_a \\ 0 & \text{Otherwise} \end{cases}$$

where $\phi_a(p^a, \pi) = \phi_1(p^a, \pi) - \phi_b(p^a, \pi)$. I have already showed that annuity purchase is an increasing function of type, therefore

$$\frac{\partial \phi_a}{\partial \pi} > 0$$

also using $T = \frac{A\tau e}{\mathbf{E}[\pi]}$ we can show

$$\begin{aligned} \frac{\partial a}{\partial \tau} &= \phi_a(p^a, \pi)(-e + \frac{Ap^a}{\mathbf{E}[\pi]}e) - \frac{Ae}{\mathbf{E}[\pi]} \\ &= -\phi_a(p^a, \pi)e - \frac{Ae}{\mathbf{E}[\pi]}(1 - p^a \phi_a(p^a, \pi)) < 0 \end{aligned}$$

Also we can determine the sing of the following cross derivative

$$\frac{\partial^2 a}{\partial \tau \partial \pi} = \frac{\partial \phi_a}{\partial \pi} \left(\frac{p^a A e}{\mathbf{E}[\pi]} - e \right) > 0$$

where the last inequality follow form $Ap^a > \mathbf{E}[\pi]$ which was established before.
Next consider the price equation (7). Let's define the function $h(p^a, \tau)$ as

$$h(p^a, \tau) = \frac{\mathrm{E}_\pi[\pi a(p^a, \pi, \tau) | \pi \geq \underline{\pi}_a(p^a, \tau)]}{A\mathrm{E}_\pi[a(p^a, \pi, \tau) | \pi \geq \underline{\pi}_a(p^a, \tau)]}$$

Note that since we assume $\tau < \tau^*$ the aggregate demand for annuity is positive and the above expression is well defined. The goal is to show that $\frac{\partial h(p^{a*}, \tau)}{\partial \tau} > 0$. But note first that

$$\begin{aligned} \frac{\partial \mathrm{E}_\pi[a(p^a, \pi, \tau) | \pi \geq \underline{\pi}_a(p^a, \tau)]}{\partial \tau} &= - \frac{\partial \underline{\pi}_a(p^a, \tau)}{\partial \tau} a(p^a, \underline{\pi}_a(p^a, \tau), \tau) + \mathrm{E}_\pi[\frac{\partial a}{\partial \tau} | \pi \geq \underline{\pi}_a(p^a, \tau)] \\ &= \mathrm{E}_\pi[\frac{\partial a}{\partial \tau} | \pi \geq \underline{\pi}_a(p^a, \tau)] \end{aligned}$$

where the last equality is true because by definition $a(p^a, \underline{\pi}_a(p^a,\tau),\tau) = 0$. Similarly we have

$$\frac{\partial \mathrm{E}_\pi[\pi a(p^a,\pi,\tau)|\pi \geq \underline{\pi}_a(p^a,\tau)]}{\partial \tau} = \mathrm{E}_\pi[\pi \frac{\partial a}{\partial \tau}|\pi \geq \underline{\pi}_a(p^a,\tau)]$$

Now we can find the sing of derivative

$$\begin{aligned}
\frac{\partial h(p^{a*},\tau)}{\partial \tau} &= \frac{\mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[\pi \frac{\partial a}{\partial \tau}]\mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[a(p^a,\pi,\tau)] - \mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[\frac{\partial a}{\partial \tau}]\mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[\pi a(p^a,\pi,\tau)]}{A(\mathrm{E}_\pi[a(p^a,\pi,\tau)|\pi \geq \underline{\pi}_a(p^a,\tau)])^2} \\
&= \frac{\mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[\pi \frac{\partial a}{\partial \tau}] - \mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[\frac{\partial a}{\partial \tau}]Ah(p^{a*},\tau)}{A\mathrm{E}_\pi[a(p^a,\pi,\tau)|\pi \geq \underline{\pi}_a(p^a,\tau)]} \\
&= \frac{\mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[(\pi - Ah(p^{a*},\tau))\frac{\partial a}{\partial \tau}]}{A\mathrm{E}_\pi[a(p^a,\pi,\tau)|\pi \geq \underline{\pi}_a(p^a,\tau)]} \\
&= \frac{\mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[\frac{\partial a}{\partial \tau}]\mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[\pi - Ah(p^{a*},\tau)] + \mathrm{Cov}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[(\pi - Ah(p^{a*},\tau))\frac{\partial a}{\partial \tau}]}{A\mathrm{E}_\pi[a(p^a,\pi,\tau)|\pi \geq \underline{\pi}_a(p^a,\tau)]} > 0
\end{aligned}$$

Where the last inequality is true because the this first term in denominator is multiplication of two negative terms ($\frac{\partial a}{\partial \tau} < 0$ is shown above and $\mathrm{E}_{\pi \geq \underline{\pi}_a(p^a,\tau)}[\pi - Ah(p^{a*},\tau)] < 0$ is an application of lemma 1 and it was shown in the proof of Theorem 1). The covariance term is negative because $(\pi - Ah(p^{a*},\tau))$ is positively correlated with π and since $\frac{\partial^2 a}{\partial \tau \partial \pi} > 0$, the two terms are positively correlated.

Therefore, $h(p^{a*},\tau)$ is an increasing function of τ at equilibrium price. Therefore, the equilibrium must increase with τ. ■

5 Concluding Remarks

In this paper I have studied a competitive model of annuity and life insurance in an stylized two period economy where there is asymmetric information about survival probabilities. I made two important assumption to be able to establish the existence of competitive equilibrium. The first assumption is that insurance providers cannot monitor the trade activities of consumers and therefore cannot offer exclusive contracts. There ore two motivations for this assumption. The first one is technical. There is a large literature, that started with Rothschild and Stiglitz (1976), on robust instances of non-existence of competitive equilibrium in insurance economies with adverse selection when insurers have access to exclusive contracts. The exclusivity of contracts gives the insurer the ability to directly pick the consumption of the consumer at each state. This leads to equilibria where prices are type specific. It also makes it possible for insurers to make offers that attract specific risk types at given market prices and earn positive profit. This (as it is shown by Rothschild and Stiglitz (1976)) will unravel the equilibrium. With non-exclusivity assumption, insurers cannot affect the composition of types that buy insurance from them and they have to take the type distribution of buyers as given. Another motivation is due to the nature of the insurance contracts that I am considering in this paper. The annuity and life insurance are financial contracts. Asset trades and financial contract are difficult and costly to monitor.

The second important assumption that I made is the existence of short sale constraint on holdings of life and annuity insurance. This assumption is a way to separate buyers and sellers of insurance in the economy. Without this assumption a single linear price is not enough to clear the market and we need some non-linearity in prices (See Bisin and Gottardi (1999) and Bisin and Gottardi (2003)).

The competitive equilibrium has two interesting features. One is that there is at most one insurance market active. This result is qualitatively consistent with the observations on very large life insurance market and very small private annuity market. The second result is that social security tax and transfer can increase the price in annuity market (and lower price of life insurance) and lowers the aggregate purchase of annuity in the market.

Another feature of the environment considered here is that ex ante efficient allocations are the same across types and therefore can be easily implemented using lump-sum transfers. In fact the optimal social security tax and transfer can implement the efficient allocations. Furthermore, under this optimal policy both annuity and life insurance market are endogenously closed. This result, although interesting, makes the solution to the problem of optimal policy trivial. An important future work is finding ways to break this optimality of lump-sum taxes.

In this paper a very restrictive structure was imposed on the nature of contracts in insurance markets. One way to proceed in future works is to study a richer environment where insurers are more sophisticated decision makers. It is interesting to investigate whether the linear contracts assumed in this paper can arise from a strategic interaction among insurers in the market.

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