Teaching Notes for
Dragon Lesson Plan 5A: Sampling Estimates
Curriculum Level 5

Statistics Learning Centre

Overview

You can order Dragon Cards and download other free lessons at: shop.StatsLC.com

Achievement Objectives

S5-1: … comparing sample distributions visually, using measures of centre, spread, and proportion …

Purpose

This lesson is designed to illustrate one important statistical concept, that sampling can be used to estimate population parameters. This lesson can be followed by the sampling variation and comparing differences lessons to cover other big ideas of statistics introduced at curriculum level 5.

Specific Learning Outcomes

- Appreciate that a sample can be used to estimate a population parameter.

Outline

- Shorter dragons [Full class, 5 min]
- Calculating the population parameter - [Full class, 10-15 min]
- Sampling to estimate the population parameter - [Groups, 20 min]
- Results and conclusions - [Groups, 10 min]
- Discussion - [Full class, 10 min]

Equipment

- Two or more class packs of dragon cards (more cards are better)
- Some way for students to record their results

Key Vocabulary

Inference, mean, population, population parameter, random sample, sample, sample parameter, sample size

Teacher Notes

Timings are approximate.

The statistics strand of the New Zealand mathematics curriculum is made up of three threads: Statistical Investigation, Statistical Literacy and Probability at all curriculum levels. This lesson is a part of the Statistical Investigation thread. The Statistical Enquiry Cycle underlies that thread. While it is used to structure this lesson it is not the main focus of the lesson. Instead the clear focus of the lesson is to introduce students to sampling for estimating population parameters. Often sampling is the only way to find population parameters.

Shorter dragons

The story is intended to provide context and purpose.

It is important to have a context to give the statistical investigation its purpose. This will inform the statistical questions to ask, the appropriate analyses and give meaning to the conclusions. In a statistical investigation (or lesson) it should be easy to answer the "why are we doing this" questions.

Calculating the population parameter

The main purpose of this part of the lesson is to give the students an appreciation as to why we might choose not to determine a population parameter. The more cards used as a population here, the clearer it will be that determining the population parameter is just not practical.

By finding the population parameter the students will be able to definitively answer the investigative question. That is, either the population mean height is less than 4.5m, proving the claim, or it is 4.5m or more, disproving the claim.

Focus the discussion on an efficient way to determine the population mean. This will require both summing dragon heights and counting the number of cards. All students should be actively involved in the process chosen as it is important for them all to experience the 'difficulty'.

One way to do this is to partition the cards into subsets with a group of students responsible for each subset. The students will need to determine the height sum and the number of cards for their subset. If subset i has n_i cards and height sum H_i, the mean for the population is then:

$$\text{mean} = \frac{H_1 + H_2 + H_3 + \cdots}{n_1 + n_2 + n_3 + \cdots}$$

Another option is to sort the cards into subsets where all dragons have the same height. In this case the height sums can be found by multiplying the height by the number of cards.

Ensure a good fraction of the class have become aware that determining the population parameter is not practical. If needed the process can be slowed by checking that students know what they should be doing and how it contributes to calculating the mean. You might also get students to recalculate values to check they have not made a mistake.

Once most students are beginning to ask if there isn't an easier way to do find the mean height or question why they are doing this, it is time to introduce the concept of sampling. If this hasn't happened after about 10 minutes, prompt students to ask whether there might be an easier way.

Sampling to estimate the population parameter

Make it clear that sampling will only provide a good estimate, not an exact value, but that sampling is a trade-off between time (or cost) and accuracy. So long as the sample is a random sample and the sample is big enough, the mean of the sample will be a good estimate of the population mean.

A random sample is one in which every card has the same chance to be chosen. By thoroughly mixing the cards you are ensuring this.

The sample size suggested for this lesson is 30 cards. This is a small sample and the estimates found can vary a lot. The sample size was chosen as a balance between being large enough to produce a sufficiently good estimate and few enough to not take too long. It is best not to use samples smaller than this.

It is also important that the whole population is not used for the samples. It is best if there is about half of the population left after the samples are taken. Given the population size and number of students you may need to adjust the group size or the sample size. However, if the sample size gets too small the results are unlikely to best illustrate the usefulness of sampling.

Results and conclusions

This could be the first opportunity the students have to apply sampling and to make statements about the results and conclusions they draw from sampling. Use this as an opportunity to point out how they should be careful about both what they say and which group they refer to. That is, statements need to correctly refer to the sample and the population as appropriate. The statements also need draw appropriate conclusions, in particular not stronger than sampling error would allow.

The mean of a group's sample is definite. The mean of the population is estimated based on the sample mean. The best estimate a group has of the population mean is their sample mean.

Discussion

The teacher should make clear that each group has done a statistical analysis called inference. They have estimated (or inferred) a population parameter from a sample. Importantly, in such an analysis it is not possible to know the population parameter exactly. Re-emphasise that the estimate will be a reasonable estimate of the population mean if the sample was big enough because it is a random sample.

Since this is a lesson and the whole population is available, the class might be able to find a way to satisfy themselves that their sample estimates are reasonable estimates of the population mean. The teacher should emphasise that this is not a part of statistical analysis but, instead, a part of learning about statistics.

One way to do this is to complete the process of finding the population mean, from the start of the lesson. If there is time, this process could be completed. Data from the samples could be used as a part of this process.

Another option could be to find the mean of all of the sample means. To see that this should provide a better estimate note that some of the sample means will be too high and others too low. We could expect the high means to more or less cancel out the low ones and give a better estimate.

You might finish the discussion by telling the students that they do not need to check this themselves. Statisticians and academics have done analyses like the ones discussed, as well as more sophisticated analyses, in order to show that sample parameters do indeed estimate population parameters.

Further activities and resources are provided on http://shop.StatsLC.com
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