

# Advanced Game Theory

## 1. Introduction to the course

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# Dissolving a partnership: Texas shoot out



# Dissolving a partnership: Texas shoot out

- You name a price.
- Your partner can either **buy your share of the company** for that price or **sell you his share of the company** for that price
- You think the company is worth 1M USD

What price do you propose?

**First Predicament of Game theory:**

in a strategic situation you have to put yourself in your opponent's shoes!

# Dissolving a partnership: Auction

- Both you and your partner simultaneously announce a valuation for the company.
- The person with the highest valuation keeps the company and pays half of the valuation to the other partner.

What valuation do you announce?

# Dissolving a partnership: Auction

- Your bid depends on what you think your partner will bid.
- Your partner's bid depends on what he thinks your bid will be.



- Your bid depends on what *your partner thinks you will bid*.
- Your partner's bid depends on what *he thinks you will bid*.



- Your bid depends on what *your partner thinks you think he will bid*.
- Your partner's bid depends on what *you think he thinks you will bid*.



- ...

## **Second Predicament of Game theory:**

putting yourself in your opponent shoes becomes very complicated when your opponent is doing the same!

## Example: today's game

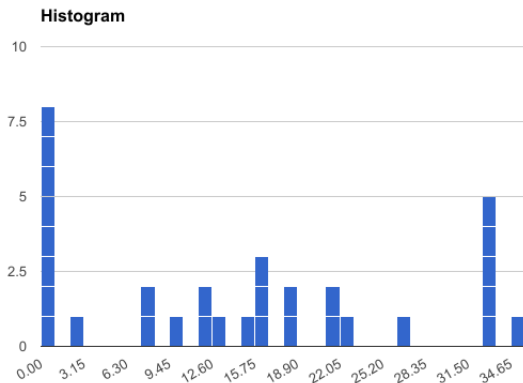
### Beauty contest (guess $2/3$ of the average)

Each of the students in this course have to choose an integer between 0 and 100 in order to guess "2/3 of the average of the responses given by all students in the course". Each student who guesses  $2/3$  of the average of all responses rounded up to the nearest integer will have a **coffee paid by the teacher after the class**.

- How did you approach it?
- Did you try to anticipate what your classmates would do?
- Did you anticipate that your classmates would do the same?



# Beauty context (guess 2/3 of the average)



- Average: 14.93
- 2/3 of the average: 9.95
- and the winner is: Joao!

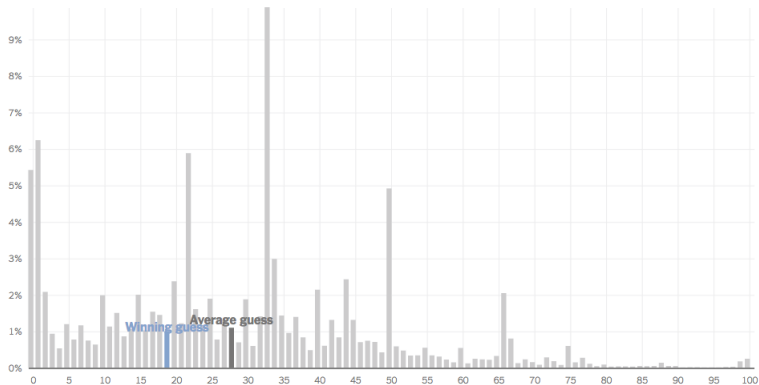
## Beauty context (guess $2/3$ of the average)

Would you have played differently knowing that:

- Your classmates were all drunk?
- Your classmates are all PhDs in game theory?
- You were playing not against your classmates, but against a random person?
- You were playing not against your classmates, but against a very experienced consultant?
- You would play it a second time?

# Beauty context (guess 2/3 of the average): NYT readers

PERCENT OF READERS PICKING EACH NUMBER:

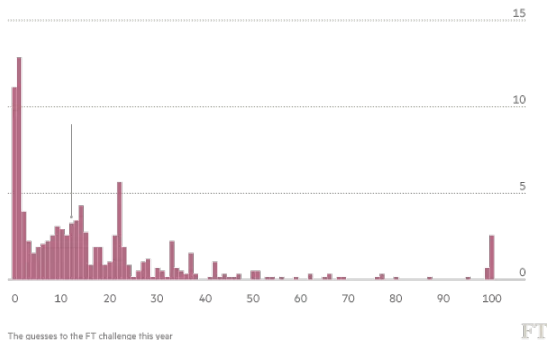


Average guess: 28.5; Winning guess: 19.

# Beauty context (guess 2/3 of the average): FT readers

## The Thaler challenge

% of total (583)



Average guess: 17.3; Winning guess: 12.

## Extra: why “Beauty Contest”?

“...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.” Keynes (1936, p. 156)

## **This course**

# Game theory: definition

Game theory studies **strategic interactions**: situations in which a player's payoff depends not only on his/her actions but also on other player's actions.

- **Key problem**: each player needs to anticipate what the other players will do, knowing that every other player is doing the same.
- **It uses math**: some things are much easier said in math than in words:
  - ▶ *In words*: firm *a*'s behavior depends on its expectation regarding firm *b*'s behavior, which depends on *b*'s expectation regarding *a*'s expectation, which depends on ...
  - ▶ *In math*: Nash equilibrium (coming up).

# Overview of the topics that we will discuss

- 1 Review of basic concepts.
- 2 Network externalities: Nash equilibrium with many agents.
- 3 Organizational structure and vertical integration: bargaining with endogenous BATNA.
- 4 Games of incomplete information.
- 5 Adverse selection and market unraveling.
- 6 Signaling and screening.
- 7 Auctions.
- 8 Herding.



# What is a game?

# What are the elements of a game?

- The **players**.
- Their **strategies** (i.e. what they can do)
- Their possible **payoffs**.
- The **rules** of the game.
  - ▶ order of plays, timing, what each player knows at each stage, ...
  - ▶ the rules of the games allow to go from strategies to payoffs.

Not always easy to extract these elements from a real-life strategic situation.

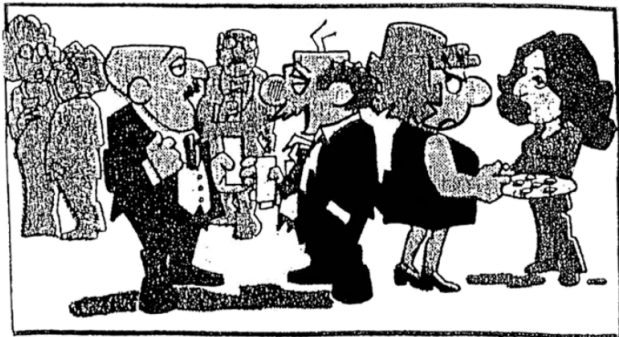
# The fundamental assumptions of game theory:

- **No outside communication:** players communicate only through actions.
- **Full knowledge of the rules of the game.**
- **Rationality:** every player maximizes his/her payoff.
- **Common knowledge of rationality:**
  - ▶ everybody knows that everybody else is rational
  - ▶ everybody knows that everybody else knows that they are rational
  - ▶ ...

## Equilibrium: how to “solve” a game

# Equilibrium

*Intuitively:* a strategy for each player such that each player is happy with his/her strategy **given what the other players are doing.**



"LORETTA'S DRIVING BECAUSE I'M DRINKING,  
AND I'M DRINKING BECAUSE SHE'S DRIVING."

Figure: An example of equilibrium (from *The Lockhorns*)

# Different types of equilibrium

- 1 equilibrium in dominant strategies
- 2 iterated elimination of dominant strategies
- 3 Nash equilibrium

# Equilibrium in dominant strategies: prisoner's dilemma

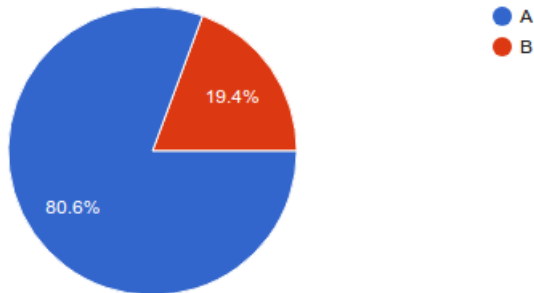
- A strategy is **dominated** if a player should *never* play this strategy, under any expectation regarding his/her opponent's strategy
- Example: find the dominated strategies in the following game:

		Player 2	
		A	B
Player 1	[1] A	5,5	10,4
	[2] B	4,10	7,7

A game has an **equilibrium in dominant strategies** if, after, eliminating all the dominated strategies, there is a unique prediction left.



# Prisoner's dilemma: how did you do?

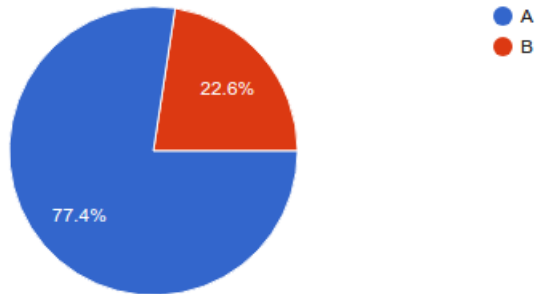


## Solutions in iterated elimination of dominated strategies: relying on the other's rationality

		Player 2	
		A	B
Player 1	[1] A	5,5	-100,4
	[2] B	0,1	0,0

- Any strategy that is dominated?
- Suppose player 1 understands that player 2 is rational.
- Can you make a further prediction regarding how player 1 will play?
- This game has a solution only if **player 1 understands that player 2 is rational!**

# Relying on the other's rationality: how did you do?

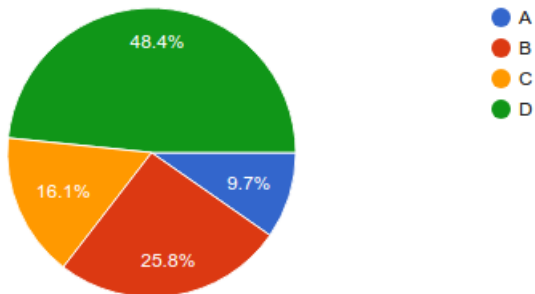


# Solutions in iterated elimination of dominated strategies: successive elimination

		Player 2			
		A	B	C	D
Player 1	[1] A	5,2	2,6	1,4	0,4
	[2] B	0,0	3,2	2,1	1,1
	[3] C	7,0	2,2	1,5	5,1
	[4] D	9,5	1,3	0,2	4,8

- Any dominated strategy?
- Suppose player 1 understands that player 2 is rational. Can you predict how player 1 should play/not play?
- Suppose player 2:
  - ▶ understands that player 1 is rational,
  - ▶ knows that player 1 understands that he is rational.
  - ▶ Can you predict how player 1 will play?
- Can you predict the outcome of the game?

# Successive Elimination: how did you do



# Nash equilibrium

- Requires common knowledge of rationality.
- Each player is “best responding” to each other.
- How to find a NE:
  - 1 Consider one player, and check this player’s best response to every possible action from his/her opponent.
  - 2 Do the same for the other player,
  - 3 An equilibrium is an action profile in which each player is best responding to his/her opponent.
- Example: guess  $2/3$  of the average.

# Summary:

*“put yourself in your opponent shoes!”*

- The key elements of a game,
- An example of simultaneous move games,
- Different ways to solve a game:
  - ▶ some are more restrictive than others in terms of the expected sophistication of the players.





# Administration

# About me

- PhD in Economics from Boston University,
- My research activities:
  - ▶ Innovation, organizations and contracts,
  - ▶ Development economics,
- My non-academic activities:
  - ▶ I advise startups & work with French accelerators / incubators,
  - ▶ I manage a small fund for angel investments in startups.

# Administrative matters

- Before class: play the games, read the readings.
- During class:
  - ▶ doors close at the start of class, no entry after the *lecture has started*,
  - ▶ no laptop or cellphone use,
  - ▶ no leaving classroom during lecture,
  - ▶ bring your name tag,
  - ▶ active participation,
    - ★ all questions are good questions,
    - ★ share your business experience when related to the discussion.
- After class:
  - ▶ problem sets,
  - ▶ office hours,
  - ▶ grades
- Theory vs applications.

## Other random (but important!) points

- Don't fall behind.
- Please, devote **quality time** to AGT.
- Please, speak up:
  - ▶ speed: too slow? too fast?
  - ▶ difficulty: too hard? too easy?
- Only one unexcused absence is allowed.