Topological defect states in coupled resonator optical waveguides

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1. Motivation for studying topology in open systems
   - Non-Hermitian, Open systems
2. CROWs: What are they?
   - What is a coupled resonator?
   - Asymmetric backscattering
   - Chains and the wave equation
   - Dispersion
3. Defect States
   - A-B phase diagram
   - Petermann Factor
4. Checking Robustness
   - Disorder
   - topological invariant

What do we already know?

- In 1997 Altland and Zirnbauer finalised the '10-fold way', a complete set of symmetry classes, based off of the combinations of the three fundamental symmetries:
- $\mathcal{T}$- Time reversal symmetry,
- $\mathcal{PH}$- Particle-hole symmetry,
- $\mathcal{C}$- Chirality (also known as sublattice symmetry as this is the natural realisation).

- With respect to the different combinations of symmetries, the topological invariants (Q) for each class are well known.
- These systems have one thing in common: $H = H^\dagger$.

- So what happens when $H \neq H^\dagger$?

Non-Hermitian systems

- Do not conserve energy/occupation of modes (i.e open system with leakage to the environment).
- Eigenvalues don't have to be real.
- Basis is biorthogonal.
- It is not known which non-Hermitian systems should have topological phases.
- New symmetries such as $\mathcal{PT}$-symmetry (Parity-Time) can appear.
  $\mathcal{PT}H\mathcal{PT} = H$

Biorthogonal basis and consequences of $\mathcal{PT}$-symmetry

- $H|\psi_n\rangle = E|\psi_n\rangle \quad \langle\psi_n|H^\dagger = E\langle\psi_n|$,
- $H^\dagger|\phi_n\rangle = E|\phi_n\rangle \quad \langle\phi_n|H = E\langle\phi_n|$,
- $\langle\phi_n|\psi_m\rangle = \delta_{nm}$

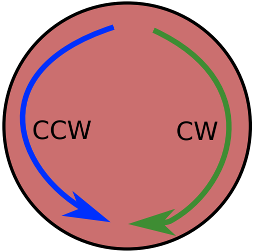
- When the system is $\mathcal{PT}$ symmetric, breaking of this $\mathcal{PT}$ symmetry results in so-called exceptional points.
- Occur as: Singularities in the complex energy plane of scattering amplitudes, or as square-root branch points which appear in the complex plane (previously avoided due to level repulsion).

Aims:

- To show that we can create defect states for a system with $\mathcal{PT}$-symmetry and $\mathcal{C}$-symmetry.
- To show that these defect states are topologically protected by showing that they are robust to disorder.

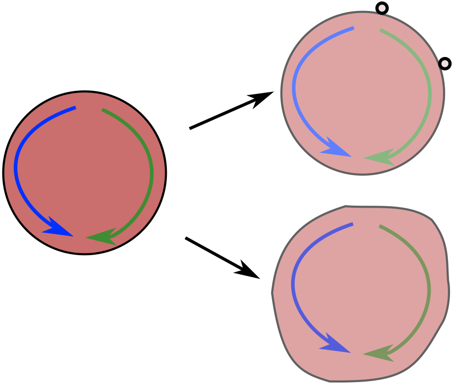
CROWs: What are they?

- Coupled Resonator optical waveguides are microcavities (discs) with two ‘whispering gallery’ modes.
- Propagate via total internal reflection.
- Come in two varieties, Clockwise (CW) and Counter-Clockwise (CCW), with frequency Ω_A and Ω_B respectively.
- CW $\rightarrow$ CCW and CCW $\rightarrow$ CW via backscattering.
- Asymmetric backscattering?

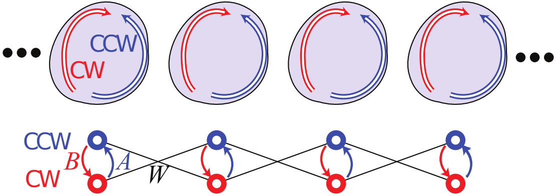


$$H = \begin{pmatrix} \Omega_A & A \\ B & \Omega_B \end{pmatrix}$$

- Asymmetrical backscattering is considered and can be realised by nanoparticle perturbations or by deforming the microdisc.
- Hence $A \neq B^*$



- Joining these resonators together we can form resonator chains:



$$T = \begin{pmatrix} 0 & W \\ W & 0 \end{pmatrix}$$

- Where the coupling between neighbouring resonators (W) is dominated by CW-CCW coupling.

- Putting all these bits together, we can construct a wave equation for the system

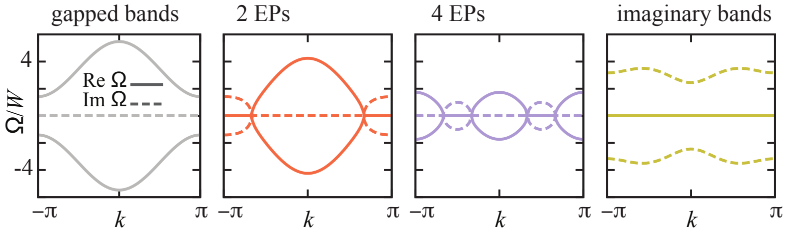
$$\omega\psi_n = H\psi_n + T(\psi_{n+1} + \psi_{n-1}) \qquad H = \begin{pmatrix} \Omega_A & A \\ B & \Omega_B \end{pmatrix},\ T = \begin{pmatrix} 0 & W \\ W & 0 \end{pmatrix}$$

- The Bloch Hamiltonian for this system is then given by

$$H = \begin{pmatrix} \Omega & A + 2W\cos k \\ B + 2W\cos k & \Omega \end{pmatrix}, \text{ where } \omega = \Omega + \Omega_\pm$$

$$\Omega_\pm = \pm\sqrt{(A + 2W\cos k)(B + 2W\cos k)}$$

- W=1, $\Omega_{\pm} = \pm\sqrt{(A + 2\cos k)(B + 2\cos k)}$

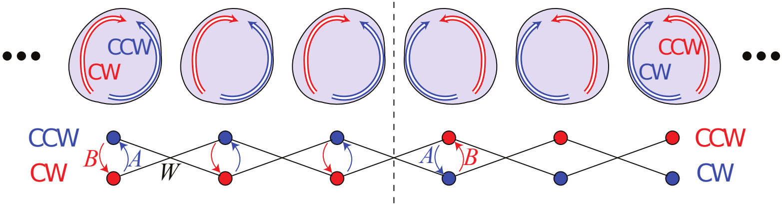


- Three conditions: Is $|A/2| > 1$, $|B/2| > 1$ and $AB > 0$?

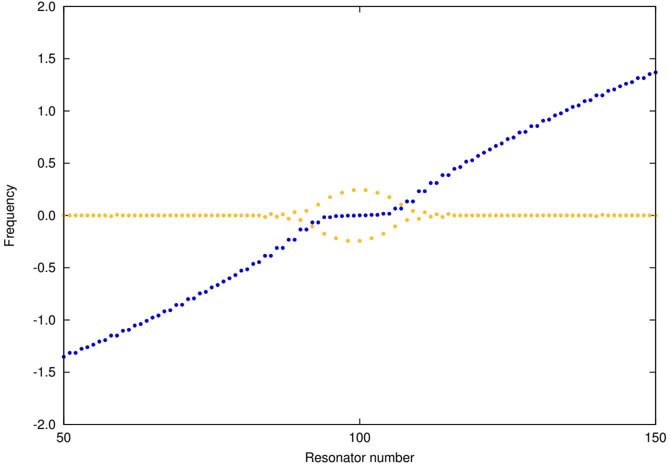
Defect States

How do we check if this system is capable of supporting a topological state?

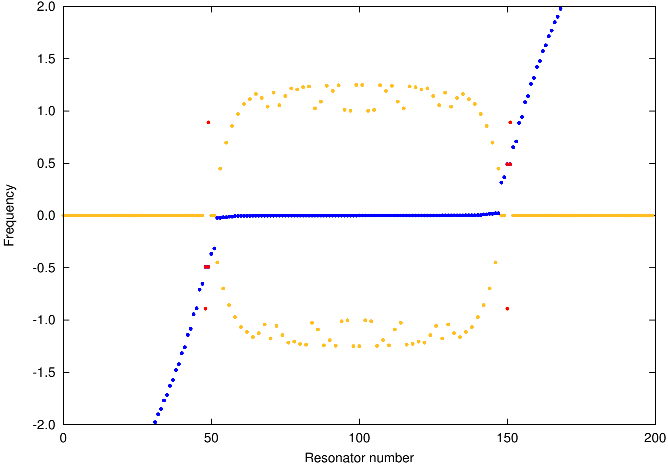
- Inspiration: SSH Chain
- Introduce defect into the chain.
- What does a defect in a CROW chain look like?


CCW
CW
CCW
CW
B
A
W
A
B
CCW
CW
CCW
CW

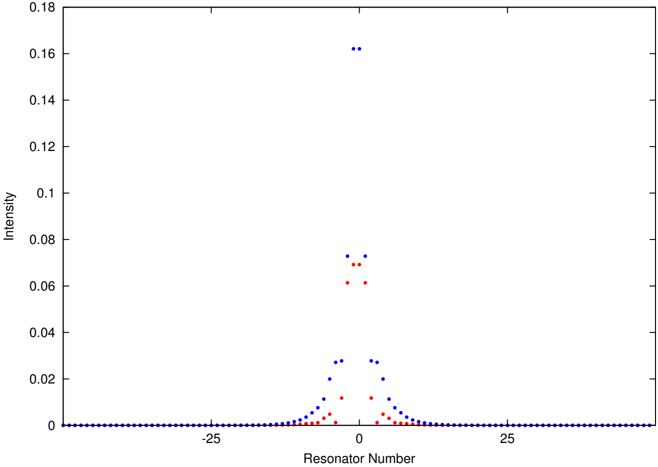
- Numerical dispersion for 100 resonators, where $A = 1 = W$ and $B = 0.5$



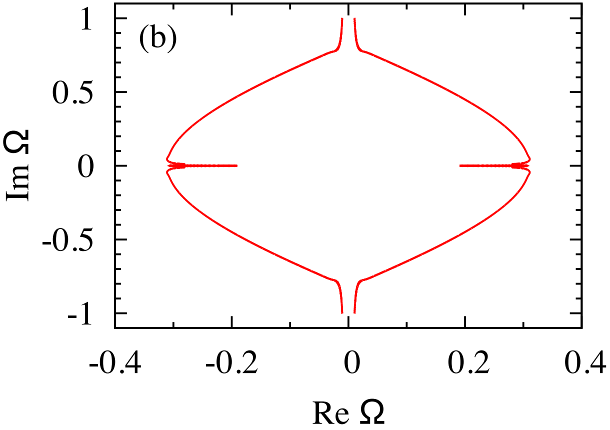
- $A = 1 = W$ and $B = 2.5$


2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
Frequency
0
50
100
150
200
Resonator number

- $A = 1 = W$ and $B = 2.5$


0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Intensity
-25
0
25
Resonator Number

- Trace of the four defect states for $A = W = 1$ and $1 > B > 4$.



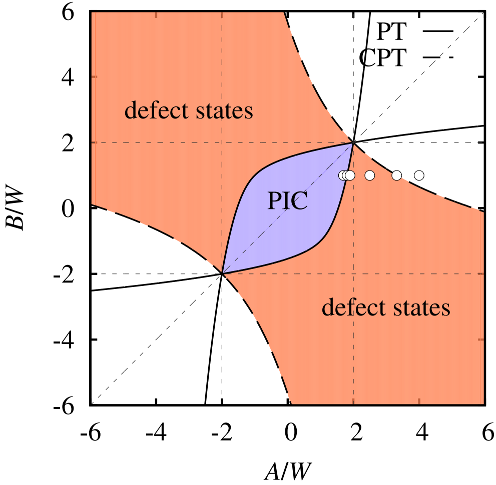
- So how do we describe these defect states?
- For what backscattering amplitudes (A and B) do they exist for?
- What identifies the phase transition into a region containing defect states?

A Localised state is an exponentially decaying wave function which decays away from the defect.

- Can rearrange the dispersion in terms of exponential factors $\lambda_{\pm} = e^{ik_{\pm}}$
- $\psi_n^L = A_+^L(\lambda_+)^{n+1} + A_-^L(\lambda_-)^{n+1} \quad (n \leq -1)$
- $\psi_n^R = A_+^R(\lambda_+)^{-n} + A_-^R(\lambda_-)^{-n} \quad (n > 0)$
- Solving the wave equation at $n = 0$ and $n = -1$ we find two conditions:

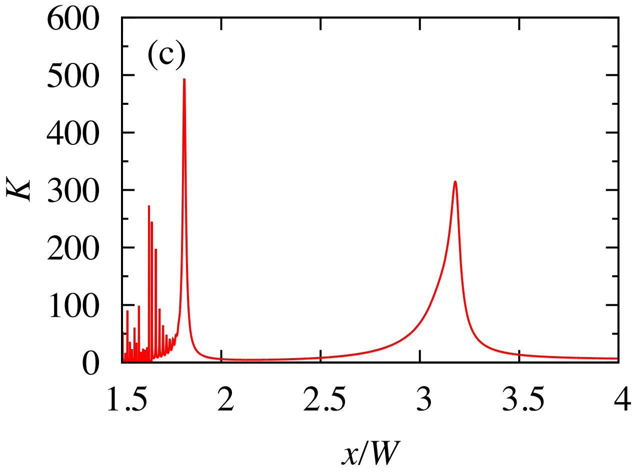
  $\psi_0^L = \psi_0^R$ and $\psi_{-1}^L = \psi_{-1}^R$
- This gives two conditions on the amplitudes, either the defect states are symmetric ($A_{\pm}^L = A_{\pm}^R$) or antisymmetric ($A_{\pm}^L = -A_{\pm}^R$).

- Condition for defect state with symmetric wavefunction:
  $(\Omega - 2W)^2\Omega - AB\Omega = \frac{W}{2}(A - B)^2$
- Defect state with antisymmetric wavefunction:
  $-(\Omega + 2W)^2\Omega - AB\Omega = \frac{W}{2}(A - B)^2$
- Solutions to this are either real or coming in complex conjugate pairs.



- $\mathcal{PT}$: $27(A+B)^4 = 16A^2B^2(1+\frac{AB}{W^2}) + 8(8W^2+9AB)(A+B)^2$
- $\mathcal{CPT}$: $A^2 + 6AB + B^2 = 32W^2$.

- Let $A = 1$ and $B = x$ and $W = 1$.

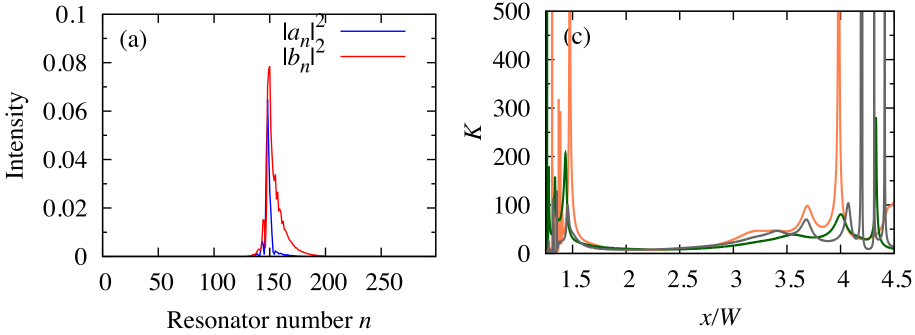


$$K = \frac{\langle\phi|\phi\rangle\langle\psi|\psi\rangle}{|\langle\phi|\psi\rangle|^2}.$$

Checking Robustness

- To check the robustness of these defect states we introduce a random disorder to a chain of 300 resonators.
- To achieve this we randomly draw a perturbation strength from a box distribution $y \in [-0.1W, 0.1W]$, which is added to the values of the backscattering strength A and B independently in each resonator.
- We additionally break the left right symmetry of the chain.

- Let $W = 1.0$, Left half: $A = 0.5$, $B = 2.5$ Right half: $A = 0.5 + y$ and $B = 1.0 + y$.



- Topological invariant?
- Winding number?

Future research: Berry Phase and exceptional points.

Thank you for listening!