Problem Set 8

1. **(20 points) Equivalent Tax Systems:** Consider and economy in which government has to finance a given sequence of expenditure $\{g_t\}_{t=0}^{\infty}$.

   (a) (5 points) Define competitive equilibrium in the special case in which the only taxes present are on capital income, τ_{kt} and labor income τ_{lt}.

   (b) (5 points) Define competitive equilibrium in the special case in which the only taxes present are on consumption, $\tilde{\tau}_{ct}$ and labor income $\tilde{\tau}_{lt}$.

   (c) (5 points) Show that given $\{\tau_{kt}\}_{t=0}^{\infty}$ and $\{\tau_{lt}\}_{t=0}^{\infty}$, there exist $\{\tilde{\tau}_{ct}\}_{t=0}^{\infty}$ and $\{\tilde{\tau}_{lt}\}_{t=0}^{\infty}$ so that the equilibrium allocations in economy (a) and (b) are the same. Find $\{\tilde{\tau}_{ct}\}_{t=0}^{\infty}$ and $\{\tilde{\tau}_{lt}\}_{t=0}^{\infty}$ in terms of $\{\tau_{kt}\}_{t=0}^{\infty}$ and $\{\tau_{lt}\}_{t=0}^{\infty}$ and equilibrium allocations. (Hint: take the allocation in economy (a), use the first order conditions, feasibility and budget constant in economy (b) to construct $\{\tilde{\tau}_{ct}\}_{t=0}^{\infty}$ and $\{\tilde{\tau}_{lt}\}_{t=0}^{\infty}$.)

   (d) (5 points) Suppose in the economy defined in (a): $\tau_{kt} = \bar{\tau}_k > 0$. What is $\lim_{t\to\infty} \tilde{\tau}_{ct}$ in the equivalent economy in part (b). (Hint: write the Euler equations in both economies and stair at them! remember that allocations are the same in both economies, by construction of $\{\tilde{\tau}_{ct}\}_{t=0}^{\infty}$ and $\{\tilde{\tau}_{lt}\}_{t=0}^{\infty}$.)

2. **(20 points) Taxing necessities:** Find the Ramsey taxes for the following economy. Which consumption good should be taxed at a higher rate? Why?

$$\max \log(c_1) + \log(c_2 - \bar{c}) - v(l)$$

   subject to

$$(1+\tau_1)\, p_1 c_1 + (1+\tau_2)\, p_2 c_2 = l$$

   Technology

$$F(x_1, x_2, l) = 0$$

   Feasibility

$$\begin{aligned} c_1 + g_1 &= x_1 \\ c_2 + g_2 &= x_2 \end{aligned}$$

3. **(20 points) Complementarity with leisure:** Find the Ramsey taxes for the following economy. Which consumption good should be taxed at higher rate? Why?

$$\max c_1^{\alpha} + c_2^{\alpha}(1-l)^{\beta}$$

   subject to

$$(1+\tau_1)\, p_1 c_1 + (1+\tau_2)\, p_2 c_2 = wl$$

   Technology

$$F(x_1, x_2, l) = 0$$

   Feasibility

$$\begin{aligned} c_1 + g_1 &= x_1 \\ c_2 + g_2 &= x_2 \end{aligned}$$

4. **(20 points) Consumption tax vs. capital income tax:** Consider the following economy

$$\max_{c_t, l_t, x_t, k_{t+1}, b_{t+1}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$\begin{aligned}(1+\tau_{ct})c_t + x_t + b_{t+1} &\leq (1-\tau_{lt})w_t l_t + r_t k_t + R_{bt} b_t \quad ;\lambda_t \\ k_{t+1} &\leq (1-\delta)k_t + x_t \\ -b_{t+1} &\leq M\end{aligned}$$

$$k_0, b_0 \text{ given}$$

Government Budget:

$$g_t + R_{bt} b_t = b_{t+1} + \tau_{ct} c_t + \tau_{lt} w_t l_t$$

Feasibility:

$$c_t + g_t + k_{t+1} = F(k_t, l_t) + (1-\delta)k_t$$

Competitive pricing:

$$\begin{aligned}r_t &= F_k(k_t, l_t) \\ w_t &= F_l(k_t, l_t)\end{aligned}$$

What can you say about the optimal (Ramsey) consumption taxes in the steady state?

5. **(20 points) Non-steady State:** In the problem above assume the utility function is of the form

$$U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - v(l),$$

what can you say about optimal taxes in any period $t \geq 2$ (and not necessarily steady state).