Notes on Optimal Taxation
(Guest Lectures for Macro Analysis II)

Roozbeh Hosseini

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Contents

1 Ramsey Taxation - Primal Approach

Consider an economy with n types of consumption good that are produced using labor input:

$$F(c_1 + g_1, \dots, c_n + g_n, l) = 0 \tag{1}$$

c_i is private and g_i is public consumption of good i and l is the labor input. F is a constant return to scale technology. Consumers face the following maximization problem

$$\max_{c_1,\dots,c_n,l} U(c_1, \dots, c_n, l)$$

subject to

$$\sum_{i=1}^{n} p_i(1+\tau_i)c_i = l$$

in which τ_i is the taxed levied on consumption of good i (wage is normalized to 1).

There is a representative firm that produces goods using technology F:

$$\max_{x_1,\dots,x_n,l} \sum_{i=1}^{n} p_i x_i - l$$

$$F(x_1, \dots, x_n, l) = 0$$

Government has to finance its purchase $g = (g_1, \dots, g_n)$ using linear taxes τ_i

$$\sum_{i=1}^{n} p_i g_i = \sum_{i=1}^{n} p_i \tau_i c_i \tag{2}$$

Let's take government purchase as given. *A Competitive Equilibrium* is

- Consumers and producers allocations: (c, x, l)
- prices: $p = (p_1, \dots, p_n)$
- policy: $\pi = (\tau_1, \dots, \tau_n)$

such that

1. Given policy π and prices p, (c, l) solve consumers problem.
2. Given prices, p, (x, l) solves producers problem.

3. Government budget (equation (2)) holds

4. Allocations are feasible (or market clearing if you like!)

$$c_i + g_i = x_i \text{ for } i = 1, \dots, n \tag{3}$$

Proposition 1 *Any competitive equilibrium allocations must satisfy the resource feasibility constraint*

$$F(c_1 + g_1, \dots, c_n + g_n, l) = 0 \tag{4}$$

and an implementability constraint

$$\sum_{i=1}^{n} U_i c_i + U_l l = 0. \tag{5}$$

Furthermore, any allocations that satisfy (4) and (6) can be supported as a competitive equilibrium for appropriately constructed polices and prices.

Proof.

Suppose (c, x, l) is a competitive equilibrium allocation. Then the following FOC must hold

$$\frac{U_i}{U_l} = -(1 + \tau_i)p_i \text{ for } i = 1, \dots, n$$

together with the following budget constraint

$$\sum_{i=1}^{n} p_i(1 + \tau_i)c_i = l.$$

Replacing out for prices (and taxes) from FOC into budget constraint gives the implementability constraint. The feasibility follows by definition of equilibrium.

Now consider allocations (c, x, l) that are feasible (given vector of g) and satisfy (5). Construct prices from the FOC of the firm

$$p_i = -\frac{F_i}{F_l} \text{ for } i = 1, \dots, n$$

set policy as

$$1 + \tau_i = \frac{U_i}{U_l}\frac{F_l}{F_i} \text{ for } i = 1, \dots, n$$

You can verify that the policy and prices (as constructed above) together with the allocation (c, x, l) is a competitive equilibrium.

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We are interested in the problem of choosing the best policy π to maximize the welfare of consumers. One restriction on such a problem is that the resulting allocation be a competitive equilibrium allocation for each given policy. The timing is the following: First, government chooses a policy, Second, private agents makes decision. We are interested in finding the equilibrium of this game.

1.1 Ramsey problem

Suppose the set of feasible policy for government in Π.

Definition 1 *A Ramsey equilibrium is a policy* $\pi = (\tau_1, \cdots \tau_n) \in \Pi$, *allocation rules* $c(\cdot)$, $x(\cdot)$ *and* $l(\cdot)$ *and price function* $p(\cdot)$ *such that*

$$\pi \in \arg\max_{\pi' \in \Pi} U(c(\pi'), l(\pi'))$$

subject to

$$\sum_{i=1}^{n} p_i g_i = \sum_{i=1}^{n} p_i \tau_i c_i$$

and $(c(\pi'), x(\pi'), l(\pi'))$ *together with* $p(\pi')$ *is a competitive equilibrium for every* $\pi' \in \Pi$.

Suppose π, $(c(\cdot), x(\cdot), l(\cdot))$ and $p(\cdot)$ is a *Ramsey equilibrium.* Then we call $(c(\pi), x(\pi), l(\pi))$ a *Ramsey allocation.*

Proposition 2 *Suppose* c^* *and* l^* *are part of a Ramsey allocation. Then*

$$(c^*, l^*) \in \arg\max_{c,l} U(c, l)$$

subject to (5) *and* (4).

Proof.

Follows from the definition of Ramsey allocation.

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1.2 Elasticities and optimal taxes

Suppose $n = 2$. Consider the following Ramsey problem

$$\max_{c_1,c_2,l} U(c_1, c_2, l)$$

subject to

1. Implementability constraint

$$U_1c_1 + U_2c_2 + U_l l = 0 \tag{6}$$

2. Feasibility

$$F(c_1 + g_1, c_2 + g_2, l) = 0 \tag{7}$$

Let λ and γ be multipliers on implementability constraint (equation (6)) and feasibility (equation (7)). First order conditions are

$$U_i + \lambda (U_i + U_{1i}c_1 + U_{2i}c_2 + U_{li}l) = \gamma F_i \qquad i = 1, 2$$

$$U_l + \lambda (U_l + U_{1l}c_1 + U_{2l}c_2 + U_{ll}l) = \gamma F_l$$

We can write these equations as

$$\begin{aligned} 1 + \lambda - \lambda H_i &= \gamma \frac{F_i}{U_i} \qquad i = 1, 2 \\ 1 + \lambda - \lambda H_l &= \gamma \frac{F_l}{U_l} \end{aligned}$$

in which, $H_i = -\frac{(U_{1i}c_1+U_{2i}c_2+U_{li}l)}{U_i}$ and $H_l = -\frac{(U_{1l}c_1+U_{2l}c_2+U_{ll}l)}{U_l}$.

Note that from individual problem we have

$$1 + \tau_i = \frac{U_i}{U_l}\frac{F_l}{F_i}$$

in other words the optimal wedge must satisfy

$$1 + \tau_i = \frac{1 + \lambda - \lambda H_l}{1 + \lambda - \lambda H_i}$$

There you go! If $H_i > H_j$, then it is optimal to tax good i more than good j.

The problem is that, it is not very helpful. Unfortunately, without imposing assumption on U we cannot say much more. Next we consider some special (yet, interesting) cases.

1.2.1 Additive separable utility functions

Suppose U is of the form

$$U(c_1, c_2, l) = u_1(c_1) + u_2(c_2) - v(l)$$

then

$$H_i = -\frac{U_{ii}c_i}{U_i}$$

Our goal to relate H_i to income elasticity of demand for good i. In order to do that, suppose there is a non-wage income m, such that $p_1c_1 + p_2c_2 = l + m$. Consider FOC of consumer (notice that I have ignored taxes for this part)

$$U_i(c_i(p, m)) = p_i\phi(p, m)$$

in which $\phi(p, m)$ is the lagrange multiplier on budget constrain. Let's take derivative w.r.t m

$$U_{ii}\frac{\partial c_i}{\partial m} = p_i\frac{\partial \phi}{\partial m} = \frac{U_i}{\phi}\frac{\partial \phi}{\partial m}$$

or

$$\frac{U_{ii}c_i}{U_i}\frac{m}{c_i}\frac{\partial c_i}{\partial m} = \frac{m}{\phi}\frac{\partial \phi}{\partial m}.$$

Let $\eta_i = \frac{m}{c_i}\frac{\partial c_i}{\partial m}$. Then

$$H_i = -\frac{m}{\phi}\frac{\partial \phi}{\partial m}\frac{1}{\eta_i}$$

Therefore, $H_i > H_j$ if and only if $\eta_j > \eta_i$. Combine this with the above and we get the following:

Result 1 *If preferences are additive separable, necessities should be taxed more than luxuries.*

1.2.2 Quasi-linear utility function

Consider the utility function in the previous section and assume that $v(l) = l$. Then there is no income effect and using income elasticities for guiding us about optimal taxes is not useful. However, we use price elasticities. Consider again the FOC of consumer

$$U_i(c_i) = p_i \phi$$

Note that in this case $\phi = 1$ (independent of prices). Take derivative w.r.t p_i

$$U_{ii} \frac{\partial c_i}{\partial p_i} = \phi = \frac{U_i}{p_i}$$

and

$$H_i = \frac{1}{\epsilon_i}$$

Result 2 *If preferences are additive separable and quasi-linear, price-inelastic goods should be taxed more.*

1.2.3 Complementarity with leisure

Sandmo (1987) and Corlett and Hauge (1953-54) argue that goods that are more complement with leisure should be taxed more heavily.

1.3 Uniform commodity taxation

One of the most useful and interesting result in optimal taxation is the *uniform commodity taxation* result. Suppose the preferences are weakly separable in consumption and leisure

$$U(c_1, \ldots, c_n, l) = W(G(c_1, \ldots, c_n), l) \tag{8}$$

furthermore, $G(\cdot)$ is homothetic.

Proposition 3 *Suppose preferences satisfy* (8)*, then it is optimal to tax all goods at the same rate, i.e.* $\tau_i = \tau_j$ *for all* i *and* j*.*

Proof.

Note that the fact that $G(\cdot)$ is homothetic implies that

$$\frac{U_i(\alpha c, l)}{U_j(\alpha c, l)} = \frac{U_i(c, l)}{U_j(c, l)}$$

or

$$U_i(\alpha c, l) = \frac{U_i(c, l)}{U_j(c, l)} U_j(\alpha c, l).$$

Differentiate w.r.t α and set $\alpha = 1$ we get

$$\frac{\sum_{k=1}^n U_{ik} c_k}{U_i} = \frac{\sum_{k=1}^n U_{jk} c_k}{U_j}$$

Also, note that $U_l = W_l$, $U_{li} = W_{lg} G_i$ and $U_i = W_g G_i$. Therefore,

$$H_i = -\frac{\sum_{k=1}^n U_{ik} c_k}{U_i} - \frac{U_{il} l}{U_i} = -\frac{\sum_{k=1}^n U_{ik} c_k}{U_i} - \frac{W_{lg} l}{W_g} = H_j$$

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This can be generalized to utility functions of the form

$$u(c_1, \ldots, c_k, G(c_{k+1}, \ldots, c_n), l)$$

in which, $G(\cdot)$ is homothetic. Then the result is that commodities $(c_{k+1}, \ldots, c_n)$ should be taxed at uniform rate.

Exercise: Suppose consumer is endowed with y unit of good one that cannot be taxed away. Does the uniform commodity taxation still hold? what if the utility function is additive separable?

Exercise: Suppose government is restricted to setting taxed on c_1 to zero. How would modify the Ramsey problem? Does the uniform commodity taxation hold?

1.4 Intermediate good taxation

Another powerful and important result in Ramsey taxation is that intermediate good shall not be taxed.

Suppose there are two sectors. One sector produces commodity x_1 that is consumed by private agent, c_1 and by government, g. Commodity x_1 is produced using intermediate good z and labor l_1 as input according to the following production function

$$f(x_1, z, l_1) = 0.$$

The other sector, uses labor l_2 as input to produce good x_2 that can be used as input in production of good x_1 (that is z) or it can be consumed (c_2 and g_2). The technology is the following

$$h(x_2, l_2) = 0.$$

- *Private agents* solves

  $$\max_{c,l} U(c_1, c_2, l_1 + l_2)$$

  subject to

  $$p_1(1 + \tau_1)c_1 + p_2(1 + \tau_2)c_2 \leq l_1 + l_2.$$

- *Producer of good* x_1 solves

  $$\max_{x_1,z,l_1} p_1 x - l_1 - p_2(1 + \tau_z)z$$

  subject to

  $$f(x_1, z, l_1) = 0.$$

  The FOC for this problem implies

  $$\frac{f_z}{f_l} = p_2(1 + \tau_z).$$

- Producer of good x_2 solves

  $$\max_{x_2,l_2} p_2 x_2 - l_2$$

  subject to

  $$h(x_2, l_2) = 0.$$

  and FOC implies

  $$\frac{h_x}{h_l} = -p_2.$$

  Combining the FOC condition for two sector we get

  $$\frac{h_x}{h_l}(1 + \tau_z) = -\frac{f_z}{f_l}.$$

- Government budget constraint is

  $$\tau_1 p_1 c_1 + \tau_2 p_2 c_2 + \tau_z p_2 z = p_1 g_1 + p_2 g_2$$

- Finally, feasibility and market clearing

$$\begin{aligned} c_1 + g_1 &= x_1 \\ c_2 + g_2 + z &= x_2 \\ f(x_1, z, l_1) &= 0 \\ h(x_2, l_2) &= 0 \end{aligned}$$

The Ramsey problem is

$$\max U(c_1, c_2, l_1 + l_2)$$

subject to

$$\begin{aligned} U_1 c_1 + U_2 c_2 + U_l(l_1 + l_2) &= 0 \qquad \lambda \\ f(c_1 + g_1, z, l_1) &= 0 \qquad \phi_1 \\ h(c_2 + g_2 + z, l_2) &= 0 \qquad \phi_2 \end{aligned}$$

FOC w.r.t z

$$\phi_1 f_z = -\phi_2 h_x$$

FOC w.r.t l_1 and l_2

$$U_l + \lambda(U_{ll}(l_1 + l_2) + U_l + U_{cl}c) = f_l \phi_1$$

$$U_l + \lambda(U_{ll}(l_1 + l_2) + U_l + U_{cl}c) = h_l \phi_2$$

and therefore,

$$f_l \phi_1 = h_l \phi_2.$$

This implies that

$$\frac{h_x}{h_l} = -\frac{f_z}{f_l}$$

It means that it is optimal to set $\tau_z = 0$ and not distort production efficiency. For more on intermediate good taxation and production efficiency see Diamond and Mirrlees (1971).

2 Optimal Fiscal Policy-Dynamic Ramsey Taxation

The main focus of this section is the derivation of Chamley-Judd result (Chamley (1986) and Judd (1985)). We are only going to consider deterministic environment. See Chari and Kehoe (1994) and Chari and Kehoe (1998) for stochastic environment and optimal policy over business cycle.

The environment is the following. There are infinitely lived identical consumers. Government has to finance expenditure g_t every period and levies distortionary taxes (or subsidies) on consumption, investment, labor and capital income. It can also issue debt.

Consumer's problem: consumers are endowed with k_0 unit of capital and b_0 unit of government debt

$$\max_{c_t,l_t,x_t,k_{t+1},b_{t+1}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$\begin{aligned} (1+\tau_{ct})c_t + (1+\tau_{xt})x_t + b_{t+1} &\leq (1-\tau_{lt})w_t l_t + (1-\tau_{kt})r_t k_t + R_{bt} b_t \quad ;\lambda_t \\ k_{t+1} &\leq (1-\delta)k_t + x_t \\ -b_{t+1} &\leq M \end{aligned}$$

$$k_0, b_0 \text{ given}$$

in which M is some large positive number.

The FOC's are

$$\begin{aligned} \beta^t U_{ct} &= \lambda_t(1+\tau_{ct}) && (9) \\ -\beta^t U_{lt} &= \lambda_t w_t (1-\tau_{lt}) && (10) \\ (1+\tau_{xt})\lambda_t &= \lambda_{t+1}\left[(1-\tau_{xt+1})(1-\delta) + (1-\tau_{kt+1})r_{t+1}\right] && (11) \\ \lambda_t &= \lambda_{t+1} R_{bt+1} && (12) \end{aligned}$$

Government Budget:

$$g_t + R_{bt}b_t = b_{t+1} + \tau_{xt}x_t + \tau_{ct}c_t + \tau_{lt}w_t l_t + \tau_{kt}r_t k_t \quad (13)$$

Feasibility:

$$c_t + g_t + k_{t+1} = F(k_t, l_t) + (1-\delta)k_t \quad (14)$$

Competitive pricing implies that

$$r_t = F_k(k_t, l_t) \quad (15)$$
$$w_t = F_l(k_t, l_t)$$

A competitive equilibrium is: the sequence of allocations $x = \{c_t, l_t, b_{t+1}, k_{t+1}, x_t\}_{t=0}^{\infty}$, prices $\{r_t, w_t, R_{bt}\}_{t=0}^{\infty}$, policy $\pi = \{\tau_{ct}, \tau_{lt}, \tau_{xt}, \tau_{kt+1}\}_{t=0}^{\infty}$ such that, the allocations solve consumer problem, given prices and policy, prices are competitive, government budget holds and allocations are feasible.

A Ramsey Equilibrium is a policy π, an allocation rule $x(\cdot)$ and price rules $r(\cdot)$, $w(\cdot)$ and $R_b(\cdot)$ such that:

$$\pi \in \arg\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to 12 and $x(\pi)$ be a competitive equilibrium, and

for any policy π', allocation $x(\pi')$ and prices $(r(\pi'), w(\pi'), R_b(\pi'))$ be a competitive equilibrium.

We next derive the implementability condition. Note that if conditions of Ekeland and Scheinkman (1986) and/or Weitzman (1973) are satisfied, then the equilibrium allocations should also satisfy the following Transversality conditions

$$\lim_{t\to\infty} \lambda_t b_{t+1} = 0 \quad (16)$$
$$\lim_{t\to\infty} \lambda_t k_{t+1} = 0 \quad (17)$$

Now multiply consumer's budget constraint by λ_t and sum over t and use (16)-(17)

$$\sum_{t=0}^{\infty} \lambda_t \left[(1+\tau_{ct})c_t + (1+\tau_{xt})(k_{t+1} - (1-\delta)k_t) + b_{t+1}\right] = \sum_{t=0}^{\infty} \lambda_t \left[(1-\tau_{lt})w_t l_t + (1-\tau_{kt})r_t k_t + R_{bt} b_t\right].$$

Now use (9)-(12) and we get

$$\sum_{t=0}^{\infty} \lambda_t \left[(1+\tau_{ct})c_t - (1-\tau_{lt})w_t l_t\right] = \lambda_0 \left\{\left[(1+\tau_{x0})(1-\delta) + (1-\tau_{k_0})r_0\right] k_0 + R_{b0} b_0\right\}.$$

Now replace (9)-(10) and we arrive at the implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \left[U_{ct}c_t + U_{lt}l_t\right] = U_0 \left\{\left[(1+\tau_{x0})(1-\delta) + (1-\tau_{k_0})r_0\right]k_0 + R_{b0}b_0\right\} \tag{18}$$

Proposition 4 *A feasible allocation $x = \{c_t, l_t, b_{t+1}, k_{t+1}, x_t\}_{t=0}^{\infty}$ is a competitive equilibrium allocation if and only it satisfies the implementability constraint (18) (for some period zero policies).*

Proof.

Suppose x is the competitive equilibrium allocation, then following the steps outlines above we can show that it should satisfy the implementability constraint (18). Now suppose an allocation x^* is feasible and satisfy (18) for some period zero policies.

Note that in any competitive equilibrium, the bond holding must satisfy

$$b_{t+1} = \sum_{s=t+1}^{\infty} \beta^{t-s} \frac{\left[U_{cs}c_s + U_{ls}l_s\right]}{U_{ct}} - k_{t+1} \tag{19}$$

in other words, any sequence of c_t^*, l_t^* and k_{t+1}^* uniquely identifies a sequence of b_t that is a part of competitive equilibrium. Candidate wage and rate of return on capital is given by (15). Therefore, from the FOC (9)-(12) we have

$$\begin{aligned}
\frac{1-\tau_{lt}}{1+\tau_{ct}} &= -\frac{U_{lt}^*}{F_{lt}^* U_{ct}^*} \\
(1+\tau_{xt})\frac{U_{ct}^*}{1+\tau_{ct}} &= \beta\frac{U_{ct+1}^*}{1+\tau_{ct+1}}\left[(1-\tau_{xt+1})(1-\delta) + (1-\tau_{kt+1})F_{kt+1}^*\right] \\
\frac{U_{ct}^*}{1+\tau_{ct}} &= \beta\frac{U_{ct+1}^*}{1+\tau_{ct+1}}R_{bt+1}
\end{aligned} \tag{20}$$

any two of the four taxes can be chosen such that the above conditions hold.

■

2.1 Ramsey problem

The Ramsey problem is the following

$$\max_{c_t, k_{t+1}, l_t} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$\sum_{t=0}^{\infty} \beta^t \left[U_{ct}c_t + U_{lt}l_t\right] = U_0 \left\{\left[(1+\tau_{x0})(1-\delta) + (1-\tau_{k_0})r_0\right] k_0 + R_{b0}b_0\right\} \quad ; \lambda$$

$$c_t + g_t + k_{t+1} = F(k_t, l_t) + (1-\delta)k_t \quad ; \phi_t$$

Define function $W(\cdot, \cdot, \cdot)$ as

$$W(c, l, \lambda) = U(c, l) + \lambda \left[U_c c + U_l l\right].$$

Now we can rewrite the Ramsey problem as

$$\max_{c_t, k_{t+1}, l_t} \sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \lambda)$$

subject to

$$c_t + g_t + k_{t+1} = F(k_t, l_t) + (1-\delta)k_t \quad ; \phi_t$$

Take first order conditions

$$\frac{W_{lt}}{W_{ct}} = -F_{lt} \tag{21}$$

$$\frac{W_{ct}}{W_{ct+1}} = \beta(1-\delta+F_{kt}) \quad \text{for } t \geq 1 \tag{22}$$

2.2 Chamley-Judd result

Proposition 5 *If the solution to the Ramsey problem converges to a steady state, then at the steady state, the tax rate on capital income is zero.*

Proof.

In (22) at the steady state we have

$$\beta(1-\delta+F_{kt+1}) = 1.$$

This implies that at the steady state there is no inter-temporal distortion. Compare with (20) we have

$$\frac{(1+\tau_{xt})(1+\tau_{ct+1})}{(1+\tau_{ct})(1+\tau_{xt+1})} = \beta\left[1-\delta+\left(\frac{1-\tau_{kt+1}}{1+\tau_{xt+1}}\right)F_{kt+1}\right]$$

Note that any feasible allocation that satisfies (18) can be implemented by two of the four taxes (that is we only need two of the τ_c, τ_l,τ_x and τ_k to implement the same allocations). This in turn implies that

$$\begin{aligned} \tau_{kt} &= 0 \\ \frac{1+\tau_{ct}}{1+\tau_{xt}} &= \text{constant} \end{aligned}$$

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2.2.1 Heterogeneous consumers

Suppose there are two type of consumers $i = 1, 2$ with preferences

$$\sum_{t=0}^{\infty} \beta^t U^i(c_{it}, l_{it})$$

The resources constraint for the economy is

$$c_{1t} + c_{2t} + k_{t+1} = F(k_t, l_{1t}, l_{2t}) + (1-\delta)k_t \tag{23}$$

implementability constraint for consumer i is

$$\sum_{t=0}^{\infty} \beta^t \left[U^i_{ct} c_{it} + U^i_{lt} l_{it}\right] = U^i_0 \left\{\left[(1+\tau_{x0})(1-\delta) + (1-\tau_{k_0})r_0\right] k^i_0 + R_{b0} b^i_0\right\} \tag{24}$$

Suppose government puts welfare weights ω_i on consumers of type i. The Ramsey problem is

$$\max \omega_1 \sum_{t=0}^{\infty} \beta^t U^1(c_{1t}, l_{1t}) + \omega_2 \sum_{t=0}^{\infty} \beta^t U^2(c_{2t}, l_{2t})$$

subject to (23) and (24).

Attached multiplier λ_i to implementability constraint of type i and write

$$W(c_1, c_2, l_1, l_2, \lambda_1, \lambda_2) = \sum_{i=1,2} \left[\omega_i U^i(c_i, l_i) + \lambda_i \left(U^i_c c_i + U^i_l l_i\right)\right]$$

$$\max \sum_{t=0}^{\infty} \beta^t W(c_{1t}, c_{2t}, l_{1t}, l_{2t}, \lambda_1, \lambda_2)$$

subject to

$$c_{1t} + c_{2t} + k_{t+1} = F(k_t, l_{1t}, l_{2t}) + (1-\delta)k_t \quad ; \phi_t$$

where W^i is defined the obvious way.

First order conditions imply

$$W_{c_{it}} = \beta W_{c_{it+1}}(1 - \delta + F_{kt+1})$$

and in the steady state

$$1 = \beta(1 - \delta + F_{kt+1})$$

and, therefore, tax on capital should be zero in the steady state.

Capitalists vs Workers (Judd 1985)

Suppose consumer of type 1 does not hold any asset and cannot save, borrow or invest. We call these 'Worker'. Also, assume that all the capital is held by consumer 2 who do not supply any labor. We call these 'Capitalists'. The implementability constraint for 'Worker' is

$$U^1_{ct} c_{1t} + U^1_{lt} l_{1t} = 0 \quad \forall\ t$$

and for 'Capitalist'

$$\sum_{t=0}^{\infty} \beta^t \left[U^2_{ct} c_{2t} \right] = U^2_0 \left\{ \left[(1+\tau_{x0})(1-\delta) + (1-\tau_{k_0}) r_0 \right] k^2_0 + R_{b0} b^2_0 \right\} \tag{25}$$

Suppose the welfare weight on 'Worker' utility is 1 and on 'Capitalist' utility is zero.

$$\max \sum_{t=0}^{\infty} \beta^t U^1(c_{1t}, l_{1t})$$

subject to

$$U^1_{ct} c_{1t} + U^1_{lt} l_{1t} = 0 \quad \forall\ t$$

$$\sum_{t=0}^{\infty} \beta^t U^2_{ct} c_{2t} = U^2_0 \left\{ \left[(1+\tau_{x0})(1-\delta) + (1-\tau_{k_0}) r_0 \right] k^2_0 + R_{b0} b^2_0 \right\} \tag{26}$$

$$c_{1t} + c_{2t} + k_{t+1} = F(k_t, l_{1t}, l_{2t}) + (1-\delta)k_t \quad ; \phi_t$$

Define

$$W(c_1, c_2, l_1, l_2, \lambda_1, \lambda_2) = U^1(c_1, l_1) + \lambda_i \left(U_c^i c_i + U_l^i l_i\right)$$

First order conditions

$$\lambda \beta^t \left[U_{cct}^2 c_{2t} + U_{ct}^2\right] + \phi_t = 0$$

$$\phi_t = \phi_{t+1}(1 - \delta + F_{kt+1})$$

in steady state $\phi_{t+1} = \beta\phi_t$ and therefore

$$1 = \beta(1 - \delta + F_{kt+1})$$

and again, tax of capital is zero in the steady state.

Exercise: In the above set up we have implicitly assumed that government can levy different taxes on different consumer types. How would you add the following restrictions to the problem

1. Tax on capital income has to be uniform across different types. Does the result hold with this restriction? Under what assumptions?
2. Tax on labor income has to be uniform across different types. Does the result hold? Under what assumptions?
3. Tax on capital income cannot be more than 100 percent. Does the result hold? Under what assumptions?

Dividend Taxes?!!! (an interesting example)

Suppose we write the environment as in McGrattan and Prescott (2005) with corporate taxes and dividend taxes. Consumers can trade share of corporations, s_t, at price v_t. Let d_t be dividend and τ_{dt} be dividend tax. Consumers solve

$$\max_{c_t, s_{t+1}, l_t} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$\sum_{t=0}^{\infty} p_t \left[c_t + v_t(s_{t+1} - s_t)\right] \leq \sum_{t=0}^{\infty} p_t \left[(1 - \tau_{dt})d_t s_t + (1 - \tau_{lt})w_t l_t\right]$$

$$s_0 = 1$$

FOC implies

$$\frac{U_{ct}}{U_{lt}} = -(1-\tau_{lt})w_t$$

$$p_t v_t = p_{t+1}v_{t+1} + p_{t+1}(1-\tau_{dt+1})d_{t+1}$$

And therefore implementability constraint is

$$\sum_{t=0}^{\infty} \beta^t \left[U_{ct}c_t + U_{lt}l_t\right] = U_{c0}\left[v_0 + (1-\tau_{d0})d_0\right] s_0 \tag{27}$$

There is a corporation that maximizes the present discounted value of owners' dividends and pays taxes τ_t on corporate income.

$$\max \sum_{t=0}^{\infty} p_t(1-\tau_{dt})d_t$$

subject to

$$d_t = f(k_t,l_t) - x_t - w_t l_t - \tau_t\left(f(k_t,l_t) - \delta k_t - w_t l_t\right)$$

$$k_{t+1} = (1-\delta)k_t + x_t$$

First order conditions for the corporation is

$$f_{lt} = w_t$$

$$\frac{p_t(1-\tau_{dt})}{p_{t+1}(1-\tau_{dt+1})} = 1 - (1-\tau_{t+1})(f_{kt+1} - \delta)$$

For this economy the feasibility is

$$\begin{aligned} c_t + k_{t+1} + g_t &= f(k_t,l_t) + (1-\delta)k_t \\ s_t &= 1 \end{aligned}$$

and there is also a government budget constraint

$$\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} p_t\left[\tau_{dt}d_t s_t + \tau_t\left(f(k_t,l_t) - \delta k_t - w_t l_t\right) + \tau_{lt}w_t l_t\right]$$

Question: What is the appropriate implementability constraint? Is constraint (27) sufficient? In other words, is it true that any feasible allocation that satisfy (27) can be supported in a competitive equilibrium? If not, what other constraints should be added?

2.2.2 Non-Steady State

Proposition 6 *Suppose the utility function is of the form*

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - v(l),$$

Then Ramsey taxes on capital income is zero for $t \geq 2$.

Proof.

Do it as an exercise.

■

Exercise: Can you establish any connection between this result and uniform commodity taxation?

2.2.3 Werning (QJE, 2007)

Werning (2007) studies a dynamic environment in which individuals are heterogeneous in their skills. Instead of ruling out lump-sum taxation, he allows them. However, he does not allow government to condition the lump-sum tax on individual skill. Instead he allows for a distortionary labor income (and capital income) tax that government can use to redistribute income across people with different skill. In some sense, it is one step away from traditional Ramsey setup, towards rationalizing distortionary taxes.

The environment is the following: let c be consumption and l be the hours worked. Individual with skill θ who works l hours produce $y = \theta l$ efficiency labor unit. If period utility over hours worked and consumption is $U(c, l)$, then we can write it in terms of consumption and efficiency labor unit as $U^i(c, y) = U(c, y/\theta^i)$.

Suppose there are $\theta \in \Theta = \left\{\theta^i, \ldots, \theta^N\right\}$. We call the individual of type θ^i, *person* i or *type* i. The fraction of type i is π^i. Assume $\sum_i \pi^i \theta^i = 1$.

Aggregate state of economy is $s_t \in S$ (finite set) and is publicly observable. Denote the history of aggregate shocks by $s^t = (s_0, \ldots, s_t)$. Probability of history s^t is $\Pr(s^t)$.

Consumer problem

Individual of type i solves

$$\max \sum_{t,s^t} \beta^t \Pr(s^t) U^i(c_t(s^t), y(s^t)) \tag{28}$$

sub. to

$$\sum_{t,s^t} p(s^t)\left[c(s^t)+k(s^t)\right] \le \sum_{t,s^t} p(s^t)\left[w_t(s^t)(1-\tau(s^t))y(s^t)+R(s^t)k(s^{t-1})\right]-T$$

$$k^i(s_0)=k^i_0 \text{ is given}$$

in which $R(s^t) = 1+(1-\kappa(s^t))(r_t(s^t)-\delta)$ and $T = \sum_{t,s^t} p(s^t)T(s^t)$ is present value of lump-sum taxes. Note that there is heterogeneity in skills θ^i as well as initial capital holding $k^i(s_0)$.

Feasibility

Let $L(s^t) = \sum_i \pi^i y^i(s^t)$, $C(s^t) = \sum_i \pi^i c^i(s^t)$, $K(s^t) = \sum_i \pi^i k^i(s^t)$. Then feasibility is

$$C(s^t)+K(s^t)+g(s^t) = F(K(s^{t-1}), L(s^t), s^t, t) + (1-\delta)K(s^{t-1}) \tag{29}$$

Govern met

Government has exogenously given sequence of expenditure $g(s^t)$ to finance. It can levy linear tax on capital income $\kappa(s^t)$. It can also levy the following tax on income

$$\tau(s^t)w_t(s^t)y^i(s^t)+T(s^t)$$

Government budget constraint is

$$\sum_{t,s^t} p(s^t)g(s^t) \le T + \sum_{t,s^t} p(s^t)\left[\tau(s^t)w_t(s^t)L(s^t)+\kappa(s^t)(r_t(s^t)-\delta)K(s^{t-1})\right] \tag{30}$$

Firms

As usual the firm's problem is static and implies marginal product pricing

$$\begin{aligned} r_t(s^t) &= F_k(K(s^{t-1}), L(s^t), s^t, t) \\ w_t(s^t) &= F_L(K(s^{t-1}), L(s^t), s^t, t) \end{aligned} \tag{31}$$

Equilibrium is defined the usual way.

Next we derive the implementability constraints. Werning (2007) develops a methodology that incorporates the fact that labor income taxes are uniform across types (so no extra constraint needs to be added to the optimal taxation problem). Also, he shows implementability constraints can be written only in terms of aggregates.

First observe that in any equilibrium

$$\begin{aligned}
\frac{U_y^i(s^t)}{U_c^i(s^t)} &= \frac{U_y^j(s^t)}{U_c^j(s^t)} = -w(s^t)(1-\tau(s^t)) \\
\frac{U_c^i(s^t)}{U_c^i(s_0)} &= \frac{U_c^j(s^t)}{U_c^j(s_0)} = \frac{p(s^t)}{\beta^t \Pr(s^t) p(s_0)} \quad \forall\, i,j
\end{aligned} \tag{32}$$

Therefore, given the aggregate consumption and labor output $(C(s^t), L(s^t))$, the assignment of allocation of consumption and labor output $\{c^i(s^t), y^i(s^t)\}$ are efficient. In other words, given any sequence of aggregate output $(C(s^t), L(s^t))$, there are weights $\varphi = \left\{\varphi^1, \ldots, \varphi^N\right\}$ such that $\sum_i \pi^i \varphi^i = 1$ and $\{c^i(s^t), y^i(s^t)\}$ is the solution to

$$U^m(C(s^t), L(s^t); \varphi) \equiv \max_{\{c^i, y^i\}} \sum \pi^i \varphi^i U^i(c^i, y^i)$$

sub. to

$$\sum_i \pi^i c^i = C(s^t), \quad \sum_i \pi^i y^i = L(s^t)$$

Denote the solution by

$$c^i = h_c^i(C, L; \varphi), \quad y^i = h_y^i(C, L; \varphi) \tag{33}$$

therefore

$$(c^i(s^t), y^i(s^t)) = h^i(C, L; \varphi)$$

in which $h^i = (h_c^i, h_y^i)$.

Note also that

$$\begin{aligned}
U_C^m(C(s^t), L(s^t); \varphi) &= \varphi^i U_c^i(c^i, y^i) \\
U_L^m(C(s^t), L(s^t); \varphi) &= \varphi^i U_y^i(c^i, y^i)
\end{aligned} \tag{34}$$

and therefore, in any equilibrium

$$\begin{aligned}
\frac{U_L^m(s^t)}{U_C^m(s^t)} &= -w(s^t)(1-\tau(s^t)) \\
\frac{U_c^m(s^t)}{U_c^m(s_0)} &= \frac{p(s^t)}{\beta^t \Pr(s^t) p(s_0)} \quad \forall\, i,j
\end{aligned} \tag{35}$$

Now let's look at individual i's implementability constraint

$$\sum_{t,s^t} \beta^t \left[U_c^i(c^i(s^t), y^i(s^t)) c^i(s^t) + U_y^i(c^i(s^t), y^i(s^t)) y^i(s^t)\right] = U_c^i(c^i(s_0), y^i(s_0)) \left[R_0 k_0^i - T\right]$$

Now we can replace individual i's allocations in terms of aggregate allocations using (33) and (34)

$$\sum_{t,s^t} \beta^t \left[U_C^m(C(s^t), L(s^t); \varphi) h_c^i(C(s^t), L(s^t); \varphi) \right. = \tag{36}$$

$$\left. + U_L^m(C(s^t), L(s^t); \varphi) h_y^i(C(s^t), L(s^t); \varphi) \right] = U_c^i(C(s_0), L(s_0); \varphi) \left[R_0 k_0^i - T \right] \forall i \tag{37}$$

Note that (36) is expressed entirely in terms of aggregate allocations, weights φ and initial endowments.

Proposition 7 *Given initial wealth $R_0 k_0^i$, an aggregate allocation $\{C(s^t), L(s^t), K(s^t)\}$ can be implemented in a competitive equilibrium if and only if*

1. *It is feasible*
2. *There exists weights φ and lump-sum T such that implementability constraint (36) holds for all $i = 1, \ldots, N$*

Proof.

Any equilibrium allocation is feasible and we just showed that it satisfy (36) . Suppose there is a feasible aggregate allocation that satisfies (36) for sum weights and lump-sum taxes. Then individual allocations and prices can be constructed using (33) and (35). Then it is immediate that (32) (consumer optimality) holds. The individual allocations constructed as such are also feasible since they satisfy (36) .

■

A Panning Problem

Suppose λ^i is planer's weight on type i. $\sum_i \pi^i \lambda^i = 1$. Consider the following planning problem

$$\max \sum_{t,s^t,i} \lambda^i \pi^i \beta^t \Pr(s^t) U^i(h^i(C(s^t), L(s^t); \varphi))$$

sub. to

$$\sum_{t,s^t} \beta^t \left[U_C^m(C(s^t), L(s^t); \varphi) h_c^i(C(s^t), L(s^t); \varphi) \right.$$

$$\left. + U_L^m(C(s^t), L(s^t); \varphi) h_y^i(C(s^t), L(s^t); \varphi) \right] = U_c^i(C(s_0), L(s_0); \varphi) \left[R_0 k_0^i - T \right] \forall i \quad ; \mu^i \pi^i$$

$$C(s^t) + K(s^t) + g(s^t) = F(K(s^{t-1}), L(s^t), s^t, t) + (1-\delta)K(s^{t-1})$$

Make our usual change of variable

$$\begin{aligned} W(C, L; \varphi, \mu, \lambda) \equiv & \sum_i \pi^i \left(\lambda^i U^i(h^i(C, L; \varphi))\right. \\ & \left. + \mu^i \left[U_C^m(C, L; \varphi) h_c^i(C, L; \varphi) + U_L^m(C, L; \varphi) h_y^i(C, L; \varphi)\right]\right. \end{aligned}$$

and rewrite the problem as

$$\max \sum_{t,s^t,i} \lambda^i \pi^i \beta^t \Pr(s^t) W(C(s^t), L(s^t); \varphi, \mu, \lambda) - U_c^i(C(s_0), L(s_0); \varphi) \sum_i \pi^i \mu^i \left[R_0 k_0^i - T\right]$$

sub. to

$$C(s^t) + K(s^t) + g(s^t) = F(K(s^{t-1}), L(s^t), s^t, t) + (1-\delta)K(s^{t-1})$$

First order conditions are

$$F_L(K(s^{t-1}), L(s^t), s^t, t) = -\frac{W_C(C(s^t), L(s^t); \varphi, \mu, \lambda)}{W_L(C(s^t), L(s^t); \varphi, \mu, \lambda)}$$

$$W_C(C(s^t), L(s^t); \varphi, \mu, \lambda) = \beta \sum_{s^{t+1}|s^t} W_C(C(s^{t+1}), L(s^{t+1}); \varphi, \mu, \lambda) R^*(s^{t+1}) \Pr(s^{t+1})$$

in which $R^*(s^{t+1}) = 1 + \delta + F_K(K(s^t), L(s^{t+1}), s^{t+1}, t+1)$.

FOC with respect to tax on initial capital

$$\sum_i \mu^i \pi^i k_0^i = 0 \ \text{ or } \ R_0 = 0$$

Optimal Taxes

$$\tau^*(s^t) = 1 - \frac{U_L^m(C, L; \varphi)}{W_L(C, L; \varphi, \mu, \lambda)} \frac{W_C(C, L; \varphi, \mu, \lambda)}{U_C^m(C, L; \varphi)}$$

Consumer inter-temporal optimality in equilibrium implies

$$U_C^m(C(s^t), L(s^t); \varphi) = \beta \sum_{s^{t+1}|s^t} U_C^m(C(s^{t+1}), L(s^{t+1}); \varphi) R(s^{t+1}) \Pr(s^{t+1})$$

One way to get this is to set the capital income taxes such that

$$R(s^{t+1}) = R^*(s^{t+1}) \frac{U_C^m(C(s^t), L(s^t); \varphi)}{W_C(C(s^t), L(s^t); \varphi, \mu, \lambda)} \frac{W_C(C(s^{t+1}), L(s^{t+1}); \varphi, \mu, \lambda)}{U_C^m(C(s^{t+1}), L(s^{t+1}); \varphi)}$$

Note that FOC with respect to initial capital implies

$$\sum_{i=1}^{N} \mu^i k_0^i \pi^i = 0$$

Example: Consider the following preferences

$$U^i(c, y) = \frac{c^{1-\sigma}}{1-\sigma} - \alpha \frac{(y/\theta^i)^\gamma}{\gamma}$$

Note that we have $h_c^i(C, L; \varphi) = \omega_c^i C$ and $h_y^i(C, L; \varphi) = \omega_y^i L$, with

$$\omega_c^i = \frac{(\varphi^i)^{1/\sigma}}{\sum_i \pi_i (\varphi^i)^{1/\sigma}} \text{ and } \omega_y^i = \frac{(\theta^i)^{\frac{\gamma}{\gamma-1}} (\varphi^i)^{\frac{-1}{\gamma-1}}}{\sum_i \pi_i (\theta^i)^{\frac{\gamma}{\gamma-1}} (\varphi^i)^{\frac{-1}{\gamma-1}}}$$

and therefore

$$U^m = \Phi_u^m \frac{c^{1-\sigma}}{1-\sigma} - \Phi_v^m \alpha \frac{(y/\theta^i)^\gamma}{\gamma} \text{ and } W = \Phi_u^W \frac{c^{1-\sigma}}{1-\sigma} - \Phi_v^W \alpha \frac{(y/\theta^i)^\gamma}{\gamma}$$

in which Φ_u^m, Φ_v^m, Φ_u^W and Φ_v^W are some constant. Note that this implies

$$\tau^*(C, L) = 1 - \frac{\Phi_v^m}{\Phi_u^m} \frac{\Phi_u^W}{\Phi_v^W}$$

Note also that

$$\frac{U_C^m(C(s^t), L(s^t); \varphi)}{W_C(C(s^t), L(s^t); \varphi, \mu, \lambda)} \frac{W_C(C(s^{t+1}), L(s^{t+1}); \varphi, \mu, \lambda)}{U_C^m(C(s^{t+1}), L(s^{t+1}); \varphi)} = 1$$

and therefore

$$R(s^{t+1}) = R^*(s^{t+1})$$

which implies

$$\kappa(s^t) = 0 \quad \text{for all } t \geq 1$$

This implies that the result for optimal taxes on capital income holds from date zero (not just for $t \geq 1$ as it was the case before). When $k_0^i = k_0$ for all i, taxing initial capital is like a lump-sum tax. But since lump-tax is allowed here, it is not necessary. However, when individuals are heterogeneous in their initial wealth, then taxing wealth for redistribution is desirable.

Example: Now consider the following preferences

$$U^i(c, y) = \alpha \log(c) + (1 - \alpha) \log(1 - \frac{y}{\theta^i})$$

then $h_c^i(C, L; \varphi) = \omega^i C$ and $h_y^i(C, L; \varphi) = \theta^i - \omega^i(1 - L)$ and

$$\omega^i = \frac{\varphi^i}{\sum_i \pi^i \varphi^i}$$

therefore,

$$U^m(C, L; \phi) = \alpha \log(C) + (1 - \alpha) \log(1 - L) + \sum_i \left[\alpha \log\left(\omega^i\right) + (1 - \alpha) \log\left(\omega^i/\theta^i\right)\right].$$

Also we can we can verify that

$$W(C, L) = \Phi_U^W \left(\alpha \log(C) + (1 - \alpha) \log(1 - L)\right) + \Phi_{U_L}^W \frac{(1 - \alpha)}{1 - L}$$

and therefore

$$\tau^*(L) = \frac{1}{(1 - L)\Phi_U^W / \Phi_{U_L}^W + 1}$$

also

$$\kappa(s^t) = 0 \quad \text{for all } t \geq 1$$

2.3 Taxing Capital in Life Cycle Economies (Erosa and Gervais (2002))

Here, I present a 2 period version of Erosa and Gervais (2002). Individuals live 2 periods (born at age 0, die at age 1). Each generation is indexed by its date of birth. For example in period t, the generations alive are $t - 1, t$. Assume no population growth.

Each individual is endowed with one unit of time at each age j and can transform one unit of time into z_j unit of efficient labor. Let $c_{t,j}$ be the consumption of generation t at age j. Other variables follow the same notation.

Consumer's problem is the following (for generation $t > 0$)

$$\max U(c_{t,0}, l_{t,0}) + \beta U(c_{t,1}, l_{t,1})$$

subject to

$$\begin{aligned}(1-\tau^c_{t,0})c_{t,0} + a_{t,1} &\leq (1-\tau^l_{t,0})w_t z_0 l_{t,0} \\ (1-\tau^c_{t,1})c_{t,1} &\leq (1-\tau^l_{t,1})w_t z_1 l_{t,1} + (1 + (1-\tau^k_{t,1})(r_t - \delta))a_{t,1}\end{aligned}$$

There is a constant return to scale technology and

$$\begin{aligned} r_t &= f_k(k_t, l_t) \\ w_t &= f_l(k_t, l_t)\end{aligned}$$

and feasibility requires that

$$\begin{aligned} c_t + k_{t+1} &= f(k_t, l_t) + (1-\delta)k_t \\ c_t &= c_{t,0} + c_{t-1,1} \\ l_t &= l_{t,0} + l_{t-1,1} \\ k_t &= a_{t-1,1}\end{aligned}$$

Government budget constrain is

$$\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} p_t \left[\sum_{j=0,1} \tau^c_{t-j,j} c_{t-j,j} + \sum_{j=0,1} \tau^l_{t-j,j} w_t z_j l_{t-j,j} + \tau^k_{t-1,1}(r_t - \delta)a_{t-1,1} \right]$$

Let $U^t = U(c_{t,0}, l_{t,0}) + \beta U(c_{t,1}, l_{t,1})$ be the lifetime utility of generation t for a given sequence of consumption and leisure and let $0 < \gamma < 1$ be government's discount factor across generations. Government objective is to maximize

$$\sum_{t=0}^{\infty} \gamma^t U^t$$

Exercise: Show that, in this environment, implementability constraint for generation t is the following

$$U_{c_{t,0}} c_{t,0} + U_{l_{t,0}} l_{t,0} + \beta \left(U_{c_{t,1}} c_{t,1} + U_{l_{t,1}} l_{t,1} \right) = 0 \tag{38}$$

Exercise: Show that a feasible allocation is implementable if and only if it satisfy (38).

Ramsey problem

Ramsey problem is the following

$$\max \sum_{t=0}^{\infty} \gamma^t \left[U(c_{t,0}, l_{t,0}) + \beta U(c_{t,1}, l_{t,1}) \right]$$

subject to

$$U_{c_{t,0}} c_{t,0} + U_{l_{t,0}} l_{t,0} + \beta \left(U_{c_{t,1}} c_{t,1} + U_{l_{t,1}} l_{t,1} \right) = 0 \quad ; \gamma^t \lambda_t$$

$$\begin{aligned} c_t + k_{t+1} &= f(k_t, l_t) + (1-\delta) k_t \quad ; \gamma^t \phi_t \\ c_t &= c_{t,0} + c_{t-1,1} \\ l_t &= l_{t,0} + l_{t-1,1} \\ k_t &= a_{t-1,1} \end{aligned}$$

First order conditions are

$$\begin{aligned} \gamma^t U_{c_{t,0}} + \gamma^t \lambda_t \left(U_{c_{t,0}} + U_{cc_{t,0}} c_{t,0} + U_{lc_{t,0}} l_{t,0} \right) &= \gamma^t \phi_t \\ \gamma^t \beta U_{c_{t,1}} + \gamma^t \beta \lambda_t \left(U_{c_{t,1}} + U_{cc_{t,1}} c_{t,1} + U_{lc_{t,1}} l_{t,1} \right) &= \gamma^{t+1} \phi_{t+1} \end{aligned} \tag{39}$$

$$\begin{aligned} \gamma^t U_{l_{t,0}} + \gamma^t \lambda_t \left(U_{l_{t,0}} + U_{ll_{t,0}} l_{t,0} + U_{lc_{t,0}} c_{t,0} \right) &= \gamma^t \phi_t f_{lt} \\ \gamma^t \beta U_{l_{t,0}} + \gamma^t \beta \lambda_t \left(U_{l_{t,1}} + U_{ll_{t,1}} l_{t,1} + U_{cl_{t,1}} c_{t,1} \right) &= \gamma^{t+1} \phi_{t+1} f_{lt+1} \end{aligned} \tag{40}$$

$$\gamma^t \phi_t = \gamma^{t+1} \phi_{t+1} \left(1 - \delta + f_{kt+1} \right) \tag{41}$$

Combine (39) and (41)

$$\frac{U_{c_{t,0}} + \lambda_t \left(U_{c_{t,0}} + U_{cc_{t,0}} c_{t,0} + U_{lc_{t,0}} l_{t,0} \right)}{U_{c_{t,1}} + \lambda_t \left(U_{c_{t,1}} + U_{cc_{t,1}} c_{t,1} + U_{lc_{t,1}} l_{t,1} \right)} = \beta \left(1 - \delta + f_{kt+1} \right) \tag{42}$$

Steady State: In the steady state $(c_{t,0}, c_{t,1}, l_{t,0}, l_{t,1}, a_{t,1}) = (c_0, c_1, l_0, l_1, a_1)$ and $\lambda_t = \lambda$. Therefore,

$$\frac{U_{c_0} + \lambda \left(U_{c_0} + U_{cc_0} c_0 + U_{lc_0} l_0\right)}{\beta U_{c_1} + \lambda \left(U_{c_1} + U_{cc_1} c_1 + U_{lc_1} l_1\right)} = \beta \left(1 - \delta + f_{kt+1}\right)$$

Note that this in general does not imply zero tax on capital. When profile of labor productivity, z_j, is not flat over lifetime, in general consumption and leisure allocations over lifetime is not flat.

Question: Intuitively, why is it optimal to distort inter-temporal decision in this environment?

We can impose assumptions on preferences (both for government and individuals) to arrive at zero capital taxation result again.

Proposition 8 *Suppose period utility function is of the following form*

$$u(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - v(l)$$

Then the Ramsey problem prescribes no inter-temporal distortions for periods $t \geq 1$*, provided that labor income taxes can be age-dependent.*

Proof.

Note that equation (42) becomes

$$\frac{U_{c_{0,t}}}{U_{c_{1,t}}} = \beta \left(1 - \delta + f_{kt+1}\right)$$

Individual problems Euler equation is

$$\frac{U_{c_{0,t}}}{U_{c_{1,t}}} = \beta \left(1 + (1 - \tau_{t,1})(f_{kt+1} - \delta)\right)$$

■

This result should be viewed as a consequence of uniform commodity taxation.

Question: Note that we get this result independent of γ. Isn't that surprising? Why is that?

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