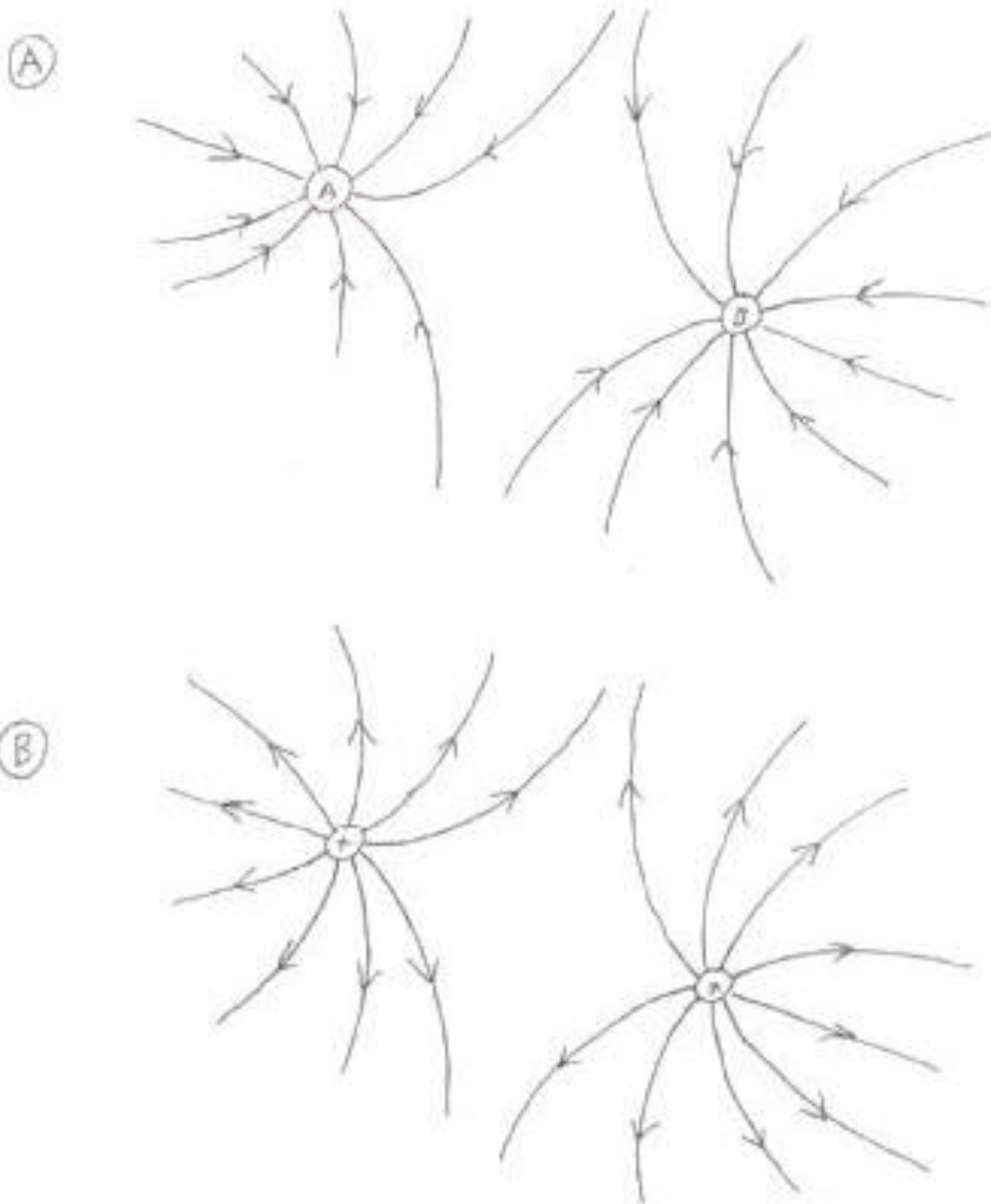


SOLUTIONS: PROBLEM SET 1

ELECTRIC FORCE AND ELECTRIC FIELD

PART A: CONCEPTUAL QUESTIONS



C. Charges in the uncharged sphere respond to the electric field produced by the charged sphere. The two *halves* of the uncharged sphere become equally and oppositely charged, with the side close to the charged sphere having the opposite charge to the charged sphere. The side closer to the charged sphere gets attracted and the side further gets repelled, but the attractive force is greater ($F \propto 1/r^2$, r smaller for the closer side). Therefore, the **net** force is attractive.

The instant the sphere touch, charges flow from one sphere into the other (they are metal and conduct charges easily), making them both charged similarly. Like charges repel, therefore they get pushed apart.

We know that afterward both spheres are charged with the same charge, but we have no way of knowing if they are positive or negative. If the charged sphere had been positive, then electrons would have flowed into it from the uncharged sphere, making it less positive and the other sphere positive. If the charged sphere had initially been negative, electrons would have flowed from it into the uncharged sphere, making it less negative and the other sphere negative as well.

D.

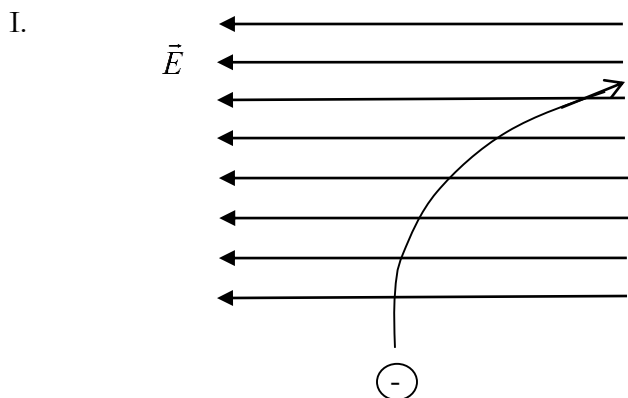
a) Different clothing fabrics have different affinities for electric charges. When rubbed together, some clothing items tend to *grab* electrons from others, making some negatively charged and some positively charged. As a result, the oppositely charged items attract each other. By the way, since charged objects will also be attracted to neutral objects (see question C), charged clothing will also be attracted to human legs, arms, etc. (Oh, the heartbreak of static cling!)

b) The effect would be much smaller, since there would be less tendency for charges to be transferred if all the material had the same affinity for electrons. (Try this at home!)

c) For the same reason discussed in a), bits of clothing become charged as they rub together. If the charge build-up is sufficient, there might be a spark as the electrons *jump* back across space. (This is referred to as *dielectric breakdown*, when the electric field magnitude becomes stronger than the *dielectric strength* of the medium between the charges. In this case that medium is air; dry air has a dielectric strength of 3×10^6 N/C. Descriptively, what happens is this: the electric field is so strong that electrons are pulled free from the negatively charged piece of material. These electrons crash into molecules of air, ionizing them and liberating even more electrons. Essentially, we end up with an avalanche of charge, and the air conducts electrons from the negative to the positive material. This happens with amazing quickness. The air is suddenly heated and expands, causing the sound. As the electrons recombine with the ionized atoms in the air, light is emitted. If this description sounds a lot like thunder and lightning, that's essentially what it is, on a tiny scale.)

d) Water molecules are polar. Warm, humid air contains a large number of these polar molecules. The polar molecules tend to *steal* excess charge from a charged object, making it difficult for any material to accumulate a significant amount of excess charge.

- E. a) F. b) G. d) H. c)

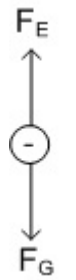


PART B: NUMERICAL QUESTIONS

QUESTION 1

a) $q = -3e = -4.8 \times 10^{-19} \text{ C}$.

Since the charge is at equilibrium, the electric force and gravitational force are equal and opposite.



The diagram shows a central circle with a minus sign inside. An upward-pointing arrow is labeled F_E and a downward-pointing arrow is labeled F_G .

$$F_E = F_G = mg$$
$$= 3.92 \times 10^{-13} \text{ N}$$
$$\vec{E} = \frac{\vec{F}_E}{q}$$
$$= \frac{3.92 \times 10^{-13} \text{ N}}{-4.8 \times 10^{-19} \text{ C}} = \boxed{-8.17 \times 10^5 \hat{j} \text{ N/C}}$$

b) $\boxed{-9.81 \hat{j} \text{ N/kg}}$ down

QUESTION 2

a) The magnitude of the electric force is given by:

$$\begin{aligned} |\vec{F}_E| &= \left| \frac{kq_1q_2}{r^2} \right| \\ &= \left| \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(10^{-10})^2} \right| = \boxed{2.3 \times 10^{-8} \text{ N}} \end{aligned} \quad (\text{attraction})$$

b) the magnitude of the gravitational force is given by:

$$\begin{aligned} |\vec{F}_G| &= \left| \frac{Gm_1m_2}{r^2} \right| \\ &= \left| \frac{(6.67 \times 10^{-11})(9.1 \times 10^{-31})(1.67 \times 10^{-27})}{(10^{-10})^2} \right| = \boxed{1 \times 10^{-47} \text{ N}} \end{aligned}$$

Note that $F_E \gg F_G$

c) Halving the distance r increases the force $|\vec{F}|$ by a factor of 4.

Doubling one charge q , doubles the electric force

Therefore, the new electric force:

$$\begin{aligned} F_{E_{new}} &= 8F_E \\ &= 1.8 \times 10^{-7} \text{ N} \end{aligned}$$

Doubling one mass (i.e 2 protons) doubles the force.

$$\begin{aligned} F_{G_{new}} &= 8F_G \\ &= 8 \times 10^{-47} \text{ N} \end{aligned}$$

A helium nucleus (2 protons and 2 neutrons) has 4 times the original mass.

$$\begin{aligned} F_{G_{new}} &= 16F_G \\ &= 1.6 \times 10^{-46} \text{ N} \end{aligned}$$

QUESTION 3

a) If $q_1 = q$ then $q_2 = 5 \times 10^{-5} - q$

We can isolate the unknown charge q in the force equation.

$$F = \frac{kq_1q_2}{r^2} = \frac{kq(5 \times 10^{-5} - q)}{r^2}$$

$$Fr^2 = 5 \times 10^{-5}(k)(q) - (k)q^2 \Rightarrow 9 \times 10^9 q^2 - 4.5 \times 10^5 q + 4 = 0$$

$$q = \frac{4.5 \times 10^5 \pm \sqrt{(4.5 \times 10^5)^2 - 4(9 \times 10^9)(4)}}{1.8 \times 10^{10}}$$

$$\boxed{q = 3.85 \times 10^{-5} C \quad \text{or} \quad q = 1.15 \times 10^{-5} C}$$

q_1 is either of these; q_2 is the other ($q_2 = 5 \times 10^{-5} - q$)

Parts b) and c)

For the sphere with $3.85 \times 10^{-5} C$:

$$\text{Number of electrons removed: } \frac{3.85 \times 10^{-5} C}{1.6 \times 10^{-19} C/e} = 2.47 \times 10^{14} e$$

$$\text{Mass of these electrons: } 2.47 \times 10^{14} (9.11 \times 10^{-31}) = 2.25 \times 10^{-16} \text{ kg}$$

$$\% \text{ of mass: } \frac{2.25 \times 10^{-16}}{3 \times 10^{-3}} \times 100 = 7.50 \times 10^{-12} \%$$

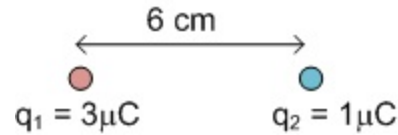
For the sphere with $1.15 \times 10^{-5} C$

$$\text{Number of electrons removed: } \frac{1.15 \times 10^{-5} C}{1.6 \times 10^{-19} C/e} = 7.19 \times 10^{13} e$$

$$\text{Mass of these electrons: } 7.19 \times 10^{13} (9.11 \times 10^{-31}) = 6.55 \times 10^{-17} \text{ kg}$$

$$\% \text{ of mass: } \frac{6.55 \times 10^{-17}}{3 \times 10^{-3}} \times 100 = 2.18 \times 10^{-12} \%$$

QUESTION 4



For the net force to be zero, q_3 ($6\mu\text{C}$) must be placed on line passing through q_1 and q_2 .

$$\vec{F}_{net} = \vec{F}_{1on3} + \vec{F}_{2on3} = 0 \Rightarrow \vec{F}_{1on3} = -\vec{F}_{2on3} \text{ (equal and opposite)}$$

Three situations to consider:

- Situation 1: q_3 at the left of q_1 :



- Situation 2: q_3 between q_1 and q_2 :

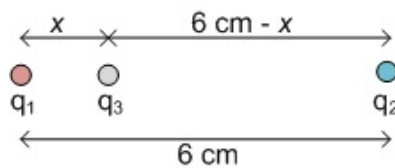


- Situation 3: q_3 at the right of q_2 :



In situations 1 and 3, the net force is not equal to zero. So q_3 must be placed somewhere between q_1 and q_2 . Note that this would not be the case if q_1 and q_2 were oppositely charged (try it!).

According to situation 2:



$$|\vec{F}_{1on3}| = |\vec{F}_{2on3}|$$

$$\frac{kq_1q_3}{x^2} = \frac{kq_2q_3}{(6-x)^2}$$

$$q_1(6-x)^2 = q_2x^2$$

$$(q_1 - q_2)x^2 - 12q_1x + 36q_1 = 0$$

$$2 \times 10^{-6}x^2 - 3.6 \times 10^{-5}x + 1.08 \times 10^{-4} = 0$$

Using the quadratic equation (again... it might be a good idea for you to remember it!)

$$x = \frac{3.6 \times 10^{-5} \pm \sqrt{(3.6 \times 10^{-5})^2 - 4(2 \times 10^{-6})(1.08 \times 10^{-4})}}{4 \times 10^{-6}}$$

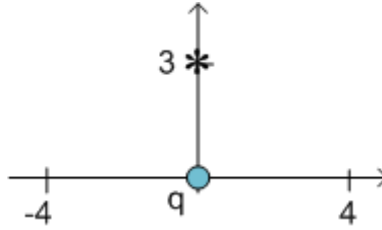
$$\boxed{x = 3.80 \text{ cm} \quad \text{or} \quad x = 14.2 \text{ cm}}$$

We'll choose the $x = 3.80 \text{ cm}$ because the other one ($x = 14.2 \text{ cm}$) puts q_3 to the right of q_2 . Which of course makes no sense! The reason it came about is that the equation doesn't consider the fact that \vec{F}_{2on3} changes direction as x gets bigger than 6 cm.

So the charge q_3 must be placed between the other charges, 3.80 cm from the $3 \mu\text{C}$ charge.

N.B. This problem could have been solved equivalently by finding the point P at which the electric field due to q_1 and q_2 is zero ($\vec{E}_{1P} + \vec{E}_{2P} = 0$). Why?? Try it!

QUESTION 5



The electric field of a point charged is given by:

$$\vec{E} = \frac{kq}{r^2} \hat{r}$$

a) For the point located at -0.03 m on the x-axis:

$$\vec{E} = \left| \frac{k(4 \times 10^{-6})}{(0.03)^2} \right| (-\hat{i}) = -4.00 \times 10^7 \hat{i} \text{ N/C}$$

b) the point located at 0.03 m on the y-axis:

$$\vec{E} = \left| \frac{k(4 \times 10^{-6})}{(0.03)^2} \right| \hat{j} = 4.00 \times 10^7 \hat{j} \text{ N/C}$$

c) for the point located at (0.04; 0.03)m:

the distance at which this point is located from the charge: $|\vec{r}| = \sqrt{0.03^2 + 0.04^2} = 0.05 \text{ m}$

The difference between the point P and the charge is $\vec{r} = (0.04\hat{i} + 0.03\hat{j}) \text{ m}$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{0.04\hat{i} + 0.03\hat{j}}{0.05} = (0.8\hat{i} + 0.6\hat{j}) \quad \text{or find the angle} \quad \theta = \tan^{-1}\left(\frac{0.03}{0.04}\right) = 36.8^\circ$$

$$\vec{E} = \left| \frac{k(4 \times 10^{-6})}{(0.05)^2} \right| (0.8\hat{i} + 0.6\hat{j}) \quad \text{or with the angle:} \quad \vec{E} = \left| \frac{k(4 \times 10^{-6})}{(0.05)^2} \right| (\cos \theta \hat{i} + \sin \theta \hat{j})$$

$$\vec{E} = (1.15 \times 10^7 \hat{i} + 8.64 \times 10^6 \hat{j}) \text{ N/C}$$

Before you go: are the directions of these vectors what you expected? What would happen if the charge is negative? Try it!

QUESTION 6

The force and the electric field are related together by the following equation:

$$\vec{F} = q\vec{E}$$

a) $\vec{F} = 2 \times 10^{-8} (4.00 \times 10^7 \hat{j}) = +0.800 \hat{j} N$

b) $\vec{F} = 2 \times 10^{-8} (-4.00 \times 10^7 \hat{i}) = -0.800 \hat{i} N$

c) $\vec{F} = -2 \times 10^{-8} (1.15 \times 10^7 \hat{i} + 8.64 \times 10^6 \hat{j}) = (-0.230 \hat{i} - 0.173 \hat{j}) N$

Before you go: are the directions of these vectors what you expected?

QUESTION 7

a) According to Newton's second law (yes, physics NYA is still valid!!) : $\vec{F} = m\vec{a}$

That is equal to the electric force experience by a charged in an electric field:

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

$$\vec{a} = \frac{(-1.6 \times 10^{-19})(0\hat{i} + 2 \times 10^2 \hat{j})}{9.11 \times 10^{-31}} = (0\hat{i} - 3.51 \times 10^{13} \hat{j}) m/s^2$$

b) Remember the projectile motion in physics NYA....

x-direction	y-direction
$v_{ox} = 2 \times 10^6 m/s$ (constant)	$v_{oy} = 3 \times 10^6 m/s$
$\Delta t = 2 \times 10^{-7} s$	$\Delta t = 2 \times 10^{-7} s$
$\Delta x = ?$	$a = -3.51 \times 10^{13} m/s^2$
	$\Delta y = ?$

Kinematics equations:

$$\Delta x = vt$$

$$\Delta x = 0.4m$$

$$\Delta y = v_o t + \frac{1}{2} at^2$$

$$\Delta y = -0.103m$$

the initial position: $\vec{r}_o = (0,0)m$

the final position: $\vec{r} = (0.400, -0.103)m$

c) The velocity is a vector, so we have to find the x and y components:

In the x-direction:

$$v_x = 2 \times 10^6 m/s \text{ (constant)}$$

In the y-direction

$$v_y = v_o + at$$

$$v_y = -4.02 \times 10^6 m/s$$

The velocity: $\vec{v} = (2.00 \times 10^6 \hat{i} - 4.02 \times 10^6 \hat{j}) m/s$

QUESTION 8

We first have to find the x and y components of the velocity vector:

$$\vec{v} = (6.00 \times 10^6 \cos 45^\circ \hat{i} + 6.00 \times 10^6 \sin 45^\circ \hat{j}) = (4.24 \hat{i} + 4.24 \hat{j}) \times 10^6 \text{ m/s}$$

a) In the y-direction:

$$F_y = ma_y = qE_y \Rightarrow a_y = \frac{qE_y}{m}$$

$$a_y = \frac{(-1.6 \times 10^{-19})(2 \times 10^3)}{9.11 \times 10^{-31}} = -3.51 \times 10^{14} \hat{j} \text{ m/s}^2$$

$$v_{oy} = 4.24 \times 10^6 \text{ m/s}$$

$$\Delta y = 0.02 \text{ m}$$

$$\Delta t = ?$$

$$\Delta y = v_{oy}t + \frac{1}{2}a_y t^2 \quad \text{solve for the time } t \dots$$

$$2 \times 10^{-2} = 4.24 \times 10^6 t + \frac{1}{2}(-3.51 \times 10^{14})t^2$$

$$1.76 \times 10^{14} t^2 - 4.24 \times 10^6 t + 2 \times 10^{-2} = 0$$

Solve using the quadratic equation...

$$t = \frac{4.24 \times 10^6 \pm \sqrt{(4.24 \times 10^6)^2 - 4(1.76 \times 10^{14})(2 \times 10^{-2})}}{2(1.76 \times 10^{14})}$$

$$t = 6.46 \times 10^{-9} \text{ s} \quad \text{or} \quad t = 1.77 \times 10^{-8} \text{ s}$$

The time $t = 6.46 \times 10^{-9} \text{ s}$ corresponds to the time taken for the charge to travel $y = 2 \text{ cm}$ on its way up. The time $t = 1.77 \times 10^{-8} \text{ s}$ corresponds to the time taken for the charge to travel $y = 2 \text{ cm}$ on its way down. So it hits the top plate after $6.46 \times 10^{-9} \text{ s}$.

How far has it moved horizontally in that time?

In the x-direction

$$v_{ox} = 4.24 \times 10^6 \text{ m/s}$$

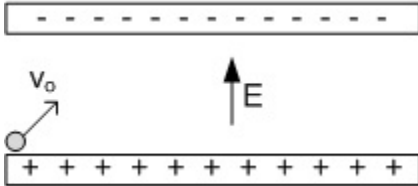
$$a_x = 0 \quad (\vec{E}_x = 0)$$

$$\Delta t = 6.46 \times 10^{-9} \text{ s}$$

$$\Delta x = v_{ox} \Delta t$$

$$4.24 \times 10^6 (6.46 \times 10^{-9}) = 2.74 \times 10^{-2} \text{ m}$$

b)



The electron just misses the top plate. It leaves the field at the point [4; 2] cm

x-direction	y-direction
$v_{ox} = 4.24 \times 10^6 \text{ m/s}$	$v_{oy} = 4.24 \times 10^6 \text{ m/s}$
$\Delta x = 0.04 \text{ m}$	$\Delta y = 0.02 \text{ m}$
	$a = ? \quad \& \quad \Delta t = ?$

$$\Delta x = v \Delta t \Rightarrow \Delta t = \frac{\Delta x}{v} = 9.43 \times 10^{-9} \text{ s}$$

$$\Delta y = v_o t + \frac{1}{2} a t^2$$

$$a_y = \frac{2(\Delta y - v_o t)}{t^2} = -4.50 \times 10^{14} \hat{j} \text{ m/s}^2$$

The force acting on the charge is related to the acceleration of the charge and the electric field in which the charge travels.

$$\vec{F} = m\vec{a} = q\vec{E}$$

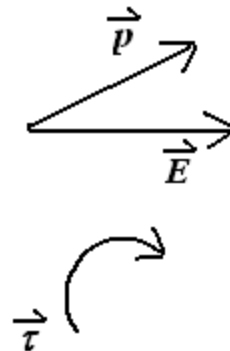
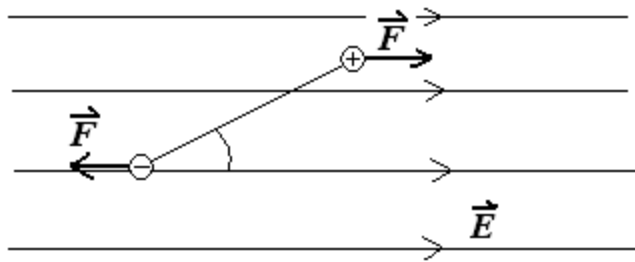
$$\vec{E}_y = \frac{m a_y}{q} = \frac{(9.11 \times 10^{-31})(-4.50 \times 10^{14})}{-1.6 \times 10^{-19}}$$

$$= 2.56 \times 10^3 \hat{j} \text{ N/C}$$

$$\boxed{\vec{E}_{\min} = (0\hat{i} + 2.56 \times 10^3 \hat{j}) \text{ N/C}}$$

QUESTION 9

Visualize the situation:



a)

$$\begin{aligned} p &= qd \\ &= (1e)(4 \times 10^{-10} \text{ m}) \\ &= 4 \times 10^{-10} \text{ e} \cdot \text{m} \end{aligned}$$

$$\vec{p} = (4.00 \times 10^{-10} \text{ e} \cdot \text{m}, 35^\circ)$$

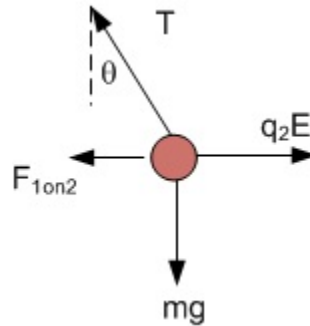
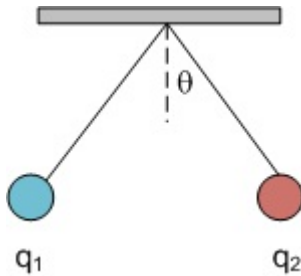
b)

$$\begin{aligned} \tau &= pE \sin \theta \\ &= (4 \times 10^{-10} \text{ e} \cdot \text{m})(4.8 \times 10^{-16} \text{ N/e}) \sin 35 \\ &= 1.10 \times 10^{-24} \text{ N} \cdot \text{m} \end{aligned}$$

$$\vec{\tau} = -1.10 \times 10^{-25} \hat{k} \text{ N} \cdot \text{m} \quad (\text{clockwise})$$

QUESTION 10

You must draw a free body diagram. Both charges will have the same f.b.d. so let's draw it for q_2 . Four forces are present: the tension [T], the weight [mg], the electric force between the two charges [F_{1on2}] and, the electric force due to the charge placed in the external electric field [q_2E].



$$1) \quad \Sigma F_x = q_2 E - T \sin \theta - F_{1on2} = 0$$

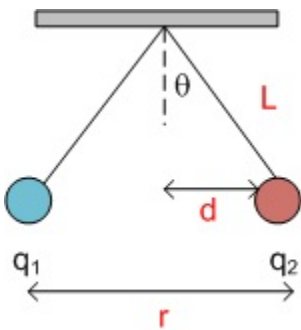
$$\text{with } F_{1on2} = \left| \frac{kq_1 q_2}{r^2} \right|$$

$$2) \quad \Sigma F_y = T \cos \theta - m_2 g = 0$$

$$\Rightarrow T = \frac{m_2 g}{\cos \theta}$$

$$\begin{aligned} \Sigma F_x &= q_2 E - \left[\frac{m_2 g}{\cos \theta} \right] \sin \theta - F_{1on2} = 0 \\ &= q_2 E - m_2 g \tan \theta - F_{1on2} = 0 \end{aligned}$$

The distance r between the charges can be obtained by geometry.



$$d = L \sin \theta$$

$$r = 2d = 3.42 \text{ cm}$$

Solve for the electric field from the above equation:

$$\Sigma F_x = q_2 E - m_2 g \tan \theta - F_{1on2} = 0$$

$$\begin{aligned} \Rightarrow E &= \frac{F_{1on2} + m_2 g \tan \theta}{q_2} \\ &= \frac{\left| \frac{k(5 \times 10^{-8})^2}{0.0342^2} \right| + 2.00 \times 10^{-3} \cdot 9.81 \tan 10}{5 \times 10^{-8}} \\ &= 4.54 \times 10^5 \text{ N/C} \end{aligned}$$

