① a) $\Delta\vec{P}_x = \vec{P}_{f_x} - \vec{P}_{i_x}$

$= m\vec{v}_{fx} - m\vec{v}_{ix}$

$= 0 - 2(4\hat{i})$

$= -8\,\hat{i}$ kg·m/s

$\Delta\vec{P}_y = \vec{P}_{f_y} - \vec{P}_{i_y}$

$= m\vec{v}_{fy} - m\vec{v}_{iy}$

$= 2(7\hat{j}) - 2(5\hat{j})$

$= 4\hat{j}$ kg·m/s

$\Delta\vec{P}_{total} = \Delta\vec{P}_x + \Delta\vec{P}_y$

$= (-8\hat{i} + 4\hat{j})$ kg·m/s

$\Delta\vec{p} = \Sigma\vec{F}_{av}\,\Delta t$

$\Sigma\vec{F}_{av} = \dfrac{\Delta\vec{p}}{\Delta t}$

$= \dfrac{-8\hat{i} + 4\hat{j}}{3}$

$= (-2.67\,\hat{i} + 1.33\,\hat{j})$ N

b) $\vec{J} = \Delta\vec{p}$

$= (-8\hat{i} + 4\hat{j})\,\underbrace{\text{N·s}}_{\text{same as kg·m/s}}$

② a) $J = \text{AREA} = \triangle + \square + \triangle$

$$= \tfrac{1}{2}(2)(4) + (1)(4) + \tfrac{1}{2}(2)(4)$$

$$= 12\ \text{N}\cdot\text{s}$$

$$\vec{J} = 12.0\,\hat{i}\ \text{N}\cdot\text{s}$$

b) $\vec{J} = \Delta\vec{P}$

$\vec{P}_i = 0 \quad \therefore \quad \vec{P}_F = 12.0\,\hat{i}\ \text{kg m/s}$

$m = 2\ \text{kg} \rightarrow \quad \vec{v} = \dfrac{\vec{P}}{m} = 6.0\,\hat{i}\ \text{m/s}$

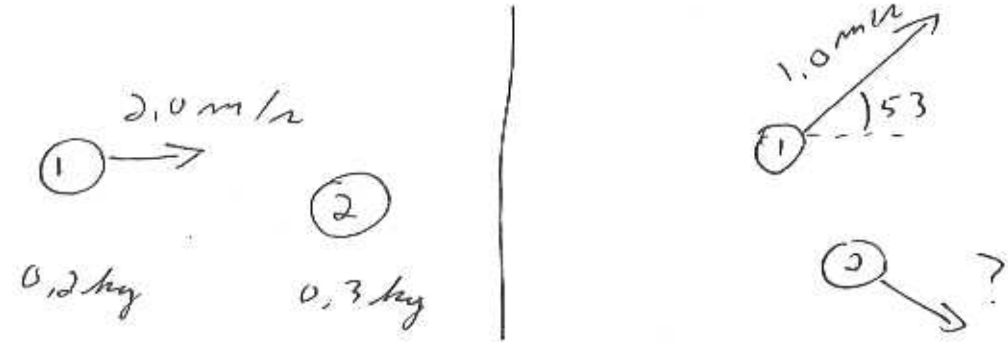
c) $\vec{P}_i = (2)(-2\hat{i}) = -4\hat{i}\ \text{kg m/s}$

$\vec{P}_F = \vec{P}_i + \vec{J} = -4\hat{i} + 12\hat{i}$

$$= +8.0\,\hat{i}\ \text{kg m/s}$$

$$\vec{v}_F = \frac{\vec{P}_F}{m} = +4.0\,\hat{i}\ \text{m/s}$$

③ a)



x DIR

$$m_1 v_1 + \cancelto{0}{m_2 v_2} = m_1 v_1' + m_2 v_2'$$

$$v_2' = \frac{m_1 v_1 - m_1 v_1'}{m_2} = \frac{0,2(2 - 1\cos 53)}{0,3}$$

$$= 0,932 \text{ m/s}$$

y DIR

$$\cancelto{0}{m_1 v_1} + \cancelto{0}{m_2 v_2} = m_1 v_1' + m_2 v_2'$$

$$v_2' = \frac{-m_1 v_1'}{m_2} = \frac{-0,2(1\sin 53)}{0.3}$$

$$= -0,532 \text{ m/s}$$

$$\vec{v}_2' = (0,932\,\hat{\imath} - 0,532\,\hat{\jmath}) \text{ m/s}$$

b) $\boxed{0{,}3 \text{ kg PUCK } (m_2)}$

$$\vec{J} = \Delta\vec{p} = m\,\Delta\vec{v} \qquad \vec{v}_0 = 0$$

$$= 0{,}3\left[\left(0{,}932\hat{\imath} - 0{,}532\hat{\jmath}\right) - 0\right]$$

$$= \left(0{,}280\hat{\imath} - 0{,}160\hat{\jmath}\right)\ \text{N}\cdot\text{s}$$

IMPULSE ON 0,2 kg PUCK : OPPOSITE OF THIS

$$\vec{J} = \left(-0{,}280\hat{\imath} + 0{,}160\hat{\jmath}\right)\ \text{N}\cdot\text{s}.$$

c) $\vec{J} = \vec{F}_{AV}\,\Delta t$

$$\vec{F}_{AV} = \frac{-0{,}280\hat{\imath} + 0{,}160\hat{\jmath}}{.002}$$

$$= \left(-140\hat{\imath} + 80\hat{\jmath}\right)\ \text{N}$$

d) BEFORE

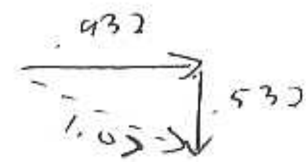
$m_1 \rightarrow 2\ \text{m/s}$ $\qquad m_2 \rightarrow 0$

$$KE_{TOT} = \tfrac{1}{2}(0{,}2)(2)^2 + 0$$

$$= 0{,}40\ \text{J}$$

AFTER

$m_1 \rightarrow 1\ \text{m/s}$ $\qquad m_2 \rightarrow 1{,}07\ \text{m/s}$



$$KE'_{TOT} = \tfrac{1}{2}(0{,}2)(1)^2 + \tfrac{1}{2}(0{,}3)(1{,}07)^2$$

$$= 0{,}272\ \text{J}$$

$KE'_{TOT} < KE_{TOT}$ $\therefore$ INELASTIC

(4)

x DIR

$$m_1 v_1 + \cancelto{0}{m_2 v_2} = m_1 v_1' + m_2 v_2'$$

$$v_2' = \frac{m_1 v_1 - m_1 v_1'}{m_2} = \frac{3(10) - 3(8\cos 35)}{6}$$

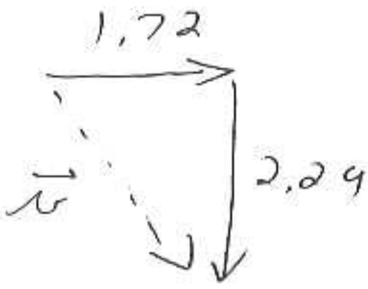
$$= 1.72 \text{ m/s}$$

y DIR

$$\cancelto{0}{m_1 v_1} + \cancelto{0}{m_2 v_2} = m_1 v_1' + m_2 v_2'$$

$$v_2' = \frac{-m_1 v_1'}{m_2} = \frac{-3(8\sin 35)}{6}$$

$$= -2.29 \text{ m/s}$$



$$|\vec{v}| = 2.86 \text{ m/s}$$

(5)

$\boxed{x \text{ DIR}}$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$v_2' = \frac{m_1 v_1 + m_2 v_2 - m_1 v_1'}{m_2}$$

$$= \frac{(0{,}5)(2\cos 30) + (0{,}3)(8\cos 60) - 0{,}5(12\cos 10)}{0{,}3}$$

$$= -12{,}8 \text{ m/s}$$

$\boxed{y \text{ DIR}}$

$$v_2' = \frac{0{,}5(2\sin 30) - 0{,}3(8\sin 60) - 0{,}5(-12\sin 10)}{0{,}3}$$

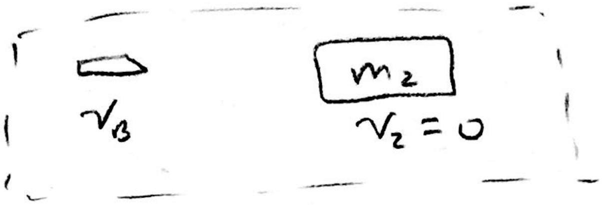
$$= -1{,}79 \text{ m/s}$$

$$\vec{v}_2' = (-12{,}8\,\hat{i} - 1{,}79\,\hat{j}) \text{ m/s}$$

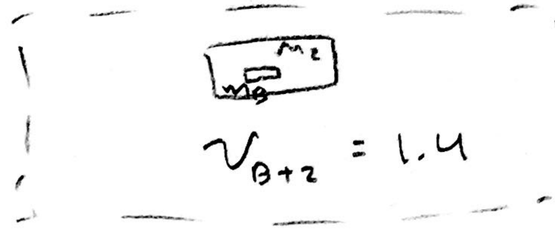
NB KE' > KE! THERE MUST HAVE BEEN SOME SORT OF EXPLOSION WHEN THEY COLLIDED!!

#6)

Collision with mass 2:



INITIAL



FINAL

$$\Sigma P_i = \Sigma P_f$$

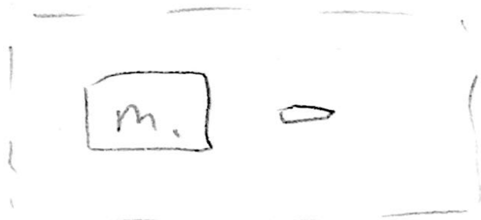
$$m_B v_B + m_2 v_2 = m_{B+2} v_{B+2}$$

$$(3.5E\text{-}3)(v_B) + (1.8)(0) = (3.5E\text{-}3 + 1.8)(1.4)$$

$$\underbrace{v_{B_i} = 721 \text{ m/s}}_{\text{as it leaves } m_1}$$

Collision with mass 1:

INITIAL



FINAL

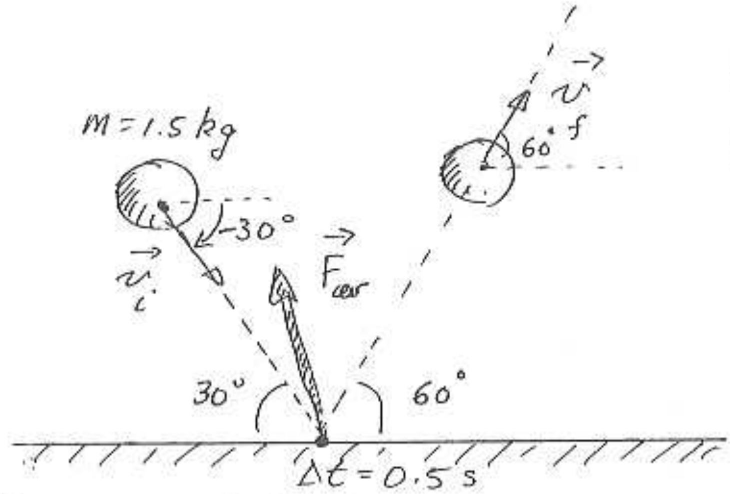
$$\Sigma P_i = \Sigma P_f$$

$$m_{B_i} v_{B_i} + m_{1_i} v_{1_i} = m_{B_f} v_{B_f} + m_{1_f} v_{1f}$$

$$(3.5E\text{-}3) v_{B_i} + 0 = (3.5E\text{-}3)(721) + (1.2)(0.63)$$

$$\underbrace{v_{B_i} = 937 \text{ m/s}}_{\text{as it enters } m_1}$$

#7.



$|\vec{v}_i| = 3.0\,m/s$

$|\vec{v}_f| = 2.0\,m/s$

↑ y → x

$\vec{P}_i = (3.9, -2.25)\ kg\,m/s$

$\vec{P}_f = (1.5,\ 2.60)\ kg\,m/s$

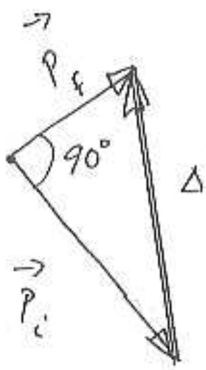
$\Delta\vec{p} = \vec{P}_f - \vec{P}_i = (-2.4, 4.85)\ kg\,m/s$

$$\vec{F}_{av} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(-2.4,\ 4.85)}{0.5} = (-4.8,\ 9.70)\ N$$

$\therefore\ |\vec{F}_{av}| = 10.8N$

direction = 116°. (from x-axis)

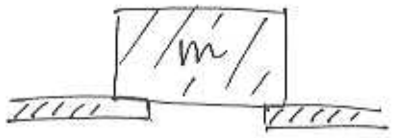
Using a Vector diagram:

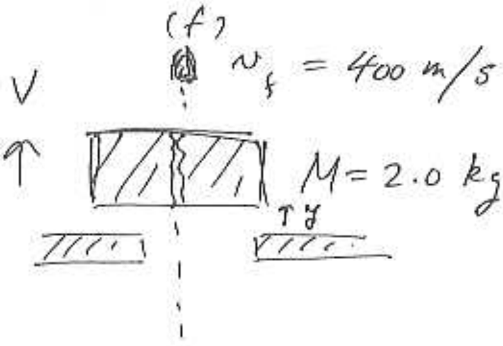


$\Rightarrow |\Delta\vec{P}| = \sqrt{4.5^2 + 3^2} = 5.41$

$$\therefore\ \vec{F}_{av} = \frac{5.41}{\Delta t} = \frac{5.41}{0.5} = 10.8N$$

8.





$\uparrow \vec{v}_i$

$m = .01 kg$

$v_i = 1000 m/s$

(a) $\vec{P}_{Ti} = \vec{P}_{Tf}$

$\therefore\ m v_i = M V + m v_f$

$\therefore\ V = \dfrac{m v_i - m v_f}{M}$ (the speed of the block)

$= \dfrac{10 - 4}{2} = \underline{\underline{3\ m/s}}$

(b) The height the block rises.

$V = 3 - 9.8t$

$\therefore$ at max height $V = 0$.

$\therefore\ t = \dfrac{3}{9.8} = 0.306\ s$.

$\therefore\ y_{max} = 3t - \frac{1}{2}(9.8)t^2$

$= 3(.306) - 4.9(.306)^2 = \underline{\underline{0.459 m}}$.

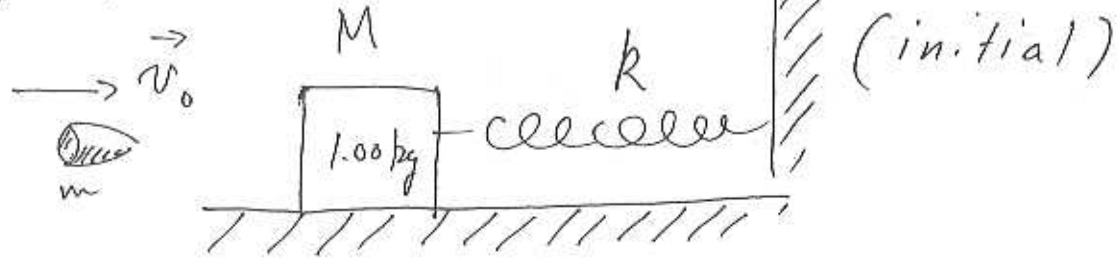
Or we could use

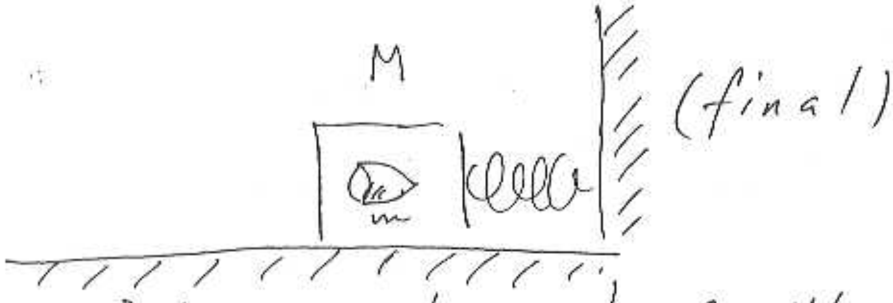
$V^2 = V_0^2 - 2gy$

$\therefore\ y_{max} = \dfrac{-V_0^2}{-2g} = \dfrac{9}{2(9.8)} = \underline{0.459 m}$

b) Or use conservation of energy of block
Ki + Ui = Kf + Uf
(1/2)(m)(Vi^2) = (m)(g)(h)
(1/2)(2)(3^2) = (2)(9.81)(h)
h = 0.459 m (same answer)

#9. (i)





$m = 20\times10^{-3}$ kg $= .02$ kg ; $k = 200$ N/m

$X = 13.3$ cm $= .133$ m

(a) momentum conservation

$$m v_0 = (m+M)V$$

energy is conserved

$$\tfrac{1}{2}(m+M)V^2 = \tfrac{1}{2}kx^2$$

$$\therefore\ V = \left(\frac{kx^2}{m+M}\right)^{1/2} = \sqrt{\frac{200\times(.133)^2}{1.02}}$$

$$= 1.86 \text{ m/s}$$

$$\therefore\ v_0 = \frac{(m+M)V}{m} = \frac{(1.02)(1.86)}{.02} = 94.9\text{ m/s}$$

(b) fraction of initial KE lost.

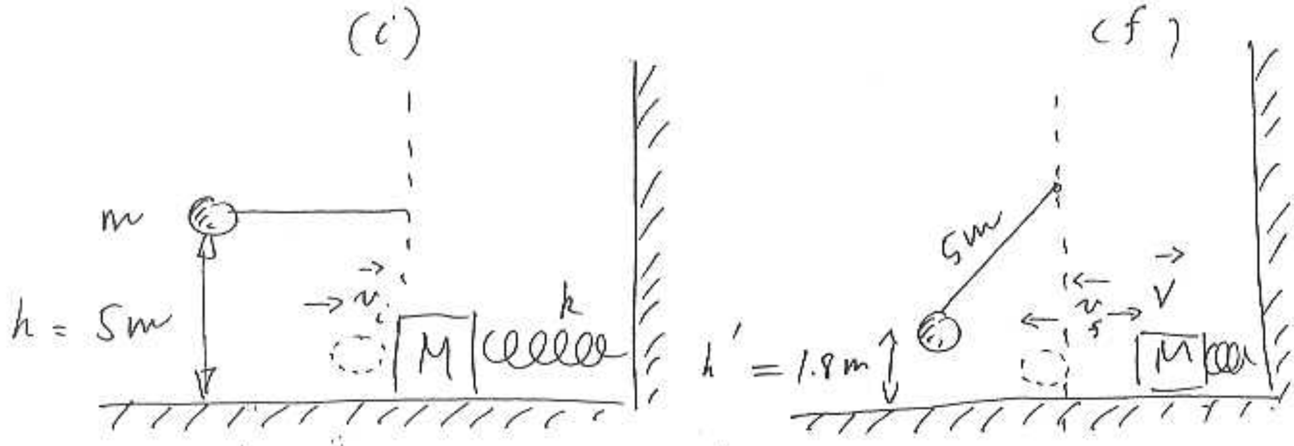
$$K_i = \tfrac{1}{2} m v_0^2 = 0.5(.02)(94.9)^2 = 90.1\text{ J}$$

$$K_f = \tfrac{1}{2}(m+M)V^2 = 0.5(1.02)(1.86)^2 = 1.76\text{ J.}$$

$$f = 1 - \frac{K_f}{K_i} = 1 - \frac{1.76}{90.1} = \underline{\underline{0.98}}$$

or there is a 98% loss of KE.

#10.



$m = 1.0$ kg ; $M = 4.0$ kg ; $k = 1000$ N/m

(a) Energy is conserved:

$$\therefore \; \cancel{m} g h = \frac{1}{2} \cancel{m} v_i^2$$

$$\therefore \; v_i = \sqrt{2gh} = (2 \times 9.8 \times 5)^{1/2} = \underline{\underline{9.9 \text{ m/s}}}$$

(b) After collision:

$$\frac{1}{2} \cancel{m} v_f^2 = \cancel{m} g h'$$

$$\therefore \; v_f = \sqrt{2gh'} = -(2 \times 9.8 \times 1.8)^{1/2} = \underline{\underline{-5.94 \text{ m/s}}}$$

Momentum Conservation:

$$m \vec{v}_i = m \vec{v}_f + M \vec{V} \quad \text{(drop vector symbol)}$$

$$\therefore \; V = \frac{m v_i - m v_f}{M} = \frac{9.9 + 5.94}{4} = 3.96 \text{ (right}$$

(c) $K_{Ti} \stackrel{?}{=} K_{Tf}$ (test this)

$$K_{Ti} = \frac{1}{2}(1)\, 9.9^2 = 49 \text{J}.$$

$$K_{Tf} = \frac{1}{2}(1)(5.94)^2 + \frac{1}{2}(4)(3.96)^2 = 49 \text{J}.$$

$\therefore \; K_{Ti} = K_{Tf} \; \therefore$ collision is elastic

(d) $\cancel{\frac{1}{2}} M V^2 = \cancel{\frac{1}{2}} k x^2$ (Energy conservation)

$$\therefore \; x = \left(\frac{M}{k}\right)^{1/2} V = \left(\frac{4}{1000}\right)^{1/2} 3.96 = \underline{\underline{0.250 \text{m}}}$$