Microeconomic Theory I

1. Preferences and Choice

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Administrative matters

- Grades break down and curvature (see syllabus for more details).
- Problem sets (see syllabus for more details).
- Book: there in no book (see syllabus for more details).
- Slides: will be made available before class. You should print them and bring them to class.
- Communication between you and me: use our facebook group (see syllabus for more details).
  - any question regarding the course material should be posted on facebook
  - you should email me for appointments or other personal matters
- No set office hours. You are most welcome to stop by my office whenever you want (I'm usually there), but you are better off by writing me an email in advance.

What previous students say about this course

- “The course is **very demanding**” True!
  - this is your first course in Microeconomics. The purpose of this course is preparing you for more interesting stuff later on.
  - not all the material we cover will be equally exciting. But believe me, it is all necessary.
- ”There is **too much math**” False!
  - The math required is: open/closed sets, sequences, series, calculus ...
  - We will use these simple tools very rigorously (especially in the first part of the course)
  - We will prove theorems and lemmas. You must be familiar with what is a proof and all different types of proofs
  - Read www.math.csusb.edu/notes/proofs/pfnot/node4.html

How to Succeed in Micro I

- The **key to success** in this course are the **problem sets**:
  - You really understand something if you can solve the problem sets **by yourself**.
  - Problem set are **not** application of what we do in class. They are **extensions** of what we do in class.
  - In the midterm and in the final exam, **half of the exercises are cut/paste from the problem sets.**
  - In previous year, students who solved all exercises from problem sets correctly got B/B+
  - They are useful only if you first **try to solve them by yourself.** Only after trying hard alone you should meet with your group.

How to Succeed in Micro I

$\Downarrow$

- The **key to success** in this course are the **TA sessions** and **collaboration** with your classmates:
  - Problem sets have a minimal direct impact on your grade, a large indirect impact on your grade
  - Collaborate with your classmates.
  - The correct solution to an exercise is what the TA explains (no matter the grade you received for your problem set)

How to Succeed in Micro I

$\Downarrow$

- The **key to success** in this course is **class participation**:
  - at the beginning, 80% of the students think they understand but they don't.
  - statistically speaking, you should ask me to repeat very often.
  - if you did not understand something, there is for sure someone else in the class who did not understand either. Asking questions has a *positive externality* on the rest of the class.
  - every class we build on what we did the previous class. Any gap in your preparation will expand at an exponential rate.
  - I will often ask you questions. None of them is rhetorical.
  - coming to class is mandatory. Class participation will be used to break ties between letter grades.

How to Succeed in Micro I

$\Downarrow$

- The **key to success** in this course is **prioritization!**
  - you have to: come to class (and stay focused), re-read your notes after each class to make sure you understand everything, work on the problem sets by yourself, work on the problem sets in group, deal with other courses, ...
  - Do **the most important stuff first**, and stop when you start getting tired or stressed out.
  - For the purposes of this class, you should give priority to ...
  - Read http://www.personal.ceu.hu/staff/Andrea_Canidio/get_an_A.html

What is this course about?

What is Microeconomics?

Economics: how to allocate scarce resources.

Macroeconomics: aggregate problems (inflation, unemployment, economic growth, monetary policy, ...)

Microeconomics: how single firms, single consumers make their choices.

In economics we make two types of statements:

Positive describe a fact, a behavior - can be empirically tested.

Normative an opinion about what should be done.

- During most of the course, we are going to make *positive* statements
- *Normative* statements are going to come up when we talk about welfare

Outline of the Course

1. Starting point: preferences (for consumers) and technology (for firms)
    - taken as **given**
2. Derive the optimal behavior
3. How does optimal behavior change when something in the environment changes? $\rightarrow$ comparative statics
4. Aggregation $\rightarrow$ partial and general equilibrium
5. Aggregation $\rightarrow$ social choice and externalities

The Method

We use mathematical models

- A **model** is a simplified description of reality, leaving out many important aspects.
- All disciplines create models (sociology, political science, medicine, physics, ...).
- **Economics** is the only **social science** using **mathematical models**
- (although mathematical modeling is quickly spreading to other social sciences as well).

The Method

How to use mathematical models:

1. Describe something in mathematical language.
    - Ex. how do you formalize mathematically the concept of choice?
2. Once you have some mathematical objects, you can apply mathematical transformations
    - not all steps in the various transformations we will use have an economic interpretation.
3. We reach another mathematical expression that is informative regarding our original problem.
    - Everything you find is already present in the original description of the problem, just very hard to see.
    - You are not *discovering* anything, you are *uncovering* features of the original description of the problem.

Part 1: Preferences and Choice

Primitives of the Problem: $< X, \succeq >$

- X **consumption set:** the set of goods that is physically possible to consume.
    - depending on the situation: X =all the item on a menu, X =all the goods in a supermarket, $X = \mathbb{R}^L_+$ (L goods, all ≥ 0), ...
    - in general X will contain elements that are affordable and some that are not affordable.
- **Preferences** over the elements of X
    - binary preference ordering:$\succeq$ "**at least as good as**" (or weak preference)
- Derived relationships:
    - $x \succeq y$ & $y \succeq x \Longleftrightarrow x \sim y$ "**indifference**"
    - $x \succeq y$ & $y \not\succeq x \Longleftrightarrow x \succ y$ "**preferred to**" or "**strict preference**"

Definition: Rationality

- We assume that $\succeq$ on X is

complete: $\forall x, y \in X \quad x \succeq y$ or $y \succeq x$

transitive: $\forall x, y, z \in X \quad (x \succeq y \ \& \ y \succeq z) \Rightarrow x \succeq z$

$\succeq$ on X are **rational** if they are **complete** and **transitive**

Exercise 1

Is $\sim$ transitive? If yes, prove your statement. If no, provide a counterexample.

Proof.

- Take $x, y, z \in X$
- Assume $x \sim y$ and $y \sim z$
- We need to show that $x \sim z$
- By definition of $\sim$,

$$x \succeq y, \quad y \succeq x, \quad z \succeq y, \quad y \succeq z$$

- Since $\succeq$ is transitive, these imply $x \succeq z$ and $z \succeq x \implies x \sim z$

□

Exercise 2

Is $\succ$ transitive? If yes, prove your statement. If no, provide a counterexample.

Proof.

- Take $x, y, z \in X$, assume $x \succ y$ and $y \succ z$
- We need to show that $x \succ z$

$$
\begin{array}{lcccc}
x \succ y & \Rightarrow & x \succeq y & \& & y \not\succeq x \\
y \succ z & \Rightarrow & y \succeq z & \& & z \not\succeq y
\end{array}
$$

$$
\begin{array}{lcccc}
x \succeq y & \& & y \succeq z & \Rightarrow & x \succeq z \\
y \not\succeq x & \& & z \not\succeq y & \Rightarrow & ?\ \text{(nothing)}
\end{array}
$$

Exercise 2

- We have to find a contradiction if we assume $z \succeq x$
- $z \succeq x \Rightarrow y \succeq x$ but $y \not\succeq x$ contradiction
- So $z \not\succeq x$
- $x \succeq z \quad \& \quad z \not\succeq x \quad \Rightarrow \quad x \succ z$

□

From Preferences to Choices

Definition

- **Choice function**: a choice function is a function $C(\cdot)$: $2^x \to 2^x$
- Such that $\forall S \neq \emptyset$, $S \in 2^x$ (S: **choice set**)
  - $C(S) \neq \emptyset$
  - $C(S) \subseteq S$

Definition

A choice function **corresponds to preferences** $\succeq$ if and only if:

$$x \in C(S) \Longleftrightarrow \forall y \in S,\ x \succeq y$$

.

From Choices to Preferences

- So far, we first defined the concept of preference, and then we derived the concept of choice.
- However we observe choices, not preferences.
- We can learn something about the underlying preferences by observing choices.

Definitions

1. If $x, y \in S$ & $x \in C(S)$, then x is **revealed at least as good as** y.
2. If $x, y \in S$, $x \in C(S)$, $y \notin C(S)$, then x is **revealed preferred to** y.

From choices to preferences

Definition

The choice function $C(\cdot)$ satisfies the **weak axiom of revealed preferences (WARP) if whenever x is revealed preferred to y, y cannot be revealed preferred to x.**

Example

- $X = \{x, y, z\}$,
- $C(\{x, y\}) = x$
- If $C(.)$ satisfies WARP, what subsets of X can be the solution to $C(\{x, y, z\})$?
- If $C(.)$ satisfies WARP, what subsets of X cannot be the solution to $C(\{x, y, z\})$?

From choices to preferences

Proposition

If $C(\cdot)$ corresponds to preferences that are rational, then $C(\cdot)$ satisfies WARP.

Proof.

- Take x, y such that x is revealed preferred to y
- For some S, $x, y \in S$, $x \in C(S)$, $y \notin C(S)$
- $x \succeq y$ since $C(\cdot)$ corresponds to preferences (1)
- Take S'; if $\begin{cases} x, y \notin S' \\ x \in S', y \notin S' \\ x \notin S', y \in S' \end{cases}$, WARP is satisfied (trivial)

From choices to preferences

- If $x, y \in S'$
  - If $x \in C(S')$, $y \notin C(S')$, again, WARP is satisfied
  - We need to show that if $y \in C(S')$, $x \in C(S')$, too
  - $y \in C(S') \Rightarrow \forall z \in S' \ y \succeq z \ \ (2)$
  - $(1), (2) \Rightarrow \forall z \in S' \ x \succeq z \Rightarrow x \in C(S')$

□

- **Remember:**
  - $\succeq$ rational $\underset{\not\Leftarrow}{\Rightarrow}$ WARP
  - **WARP violated $\Rightarrow$ $\succeq$ not rational**

Demand: Choice Function When the Choice Set is a Budget Set

- $X = \mathbb{R}^L_+$, where L is the # of goods available
- $x \in X$ is a vector ("a bundle")

Definitions

$\forall x \in X$ an **upper contour set** is $G(x) \equiv \{y \in X \mid y \succeq x\}$.
$\forall x \in X$ a **lower contour set** is $B(x) \equiv \{y \in X \mid x \succeq y\}$.

- Note: $B(x) \equiv \{y \in X \mid x \in G(y)\}$

Definition

$I(x) = G(x) \cap B(x)$ is an **indifference curve**.

The Budget Set

- w: wealth
- p: price vector $\left(p \in \mathbb{R}_+^L\right)$

Definition

$S(p, w) = \{x \in X \mid px \leq w\}$ is a **budget set**.

- In $\mathbb{R}_+^2$, $S(p, w) = \{x \in X \mid p_1 x_1 + p_2 x_2 \leq w\}$
- Note the hidden assumption behind this definition that prices are linear

Definition

$x(p, w) \equiv C(S_{p,w})$ is a **demand correspondence** (sometimes called *Mashallian demand*)

Comparative Statics

Comparative Statics: How Demand Changes When Prices or Wealth Change.

We are interested in 4 cases:

1. Changes in both p and w by the same amount
2. Changes in p
3. Changes in w
4. Changes in p associated with a 'slutzky compensated' changes in w (I will define 'slutzki compensated' later).

Comparative Statics

Case 1: Equal change in p and w

Proposition

The demand function is O^0

- define $p' = \alpha p,\ w' = \alpha w \quad (\alpha > 0)$
- $S_{p',w'} = S_{p,w}$ as $\alpha p x \leq \alpha w \Leftrightarrow px \leq w$ (i.e. every x element of $S_{p,w}$ is also an element of $S_{p',w'}$ and vice versa)
- $C(S_{p',w'}) = C(S_{p,w})$
- $x(p,w) = x(\alpha p, \alpha w)$, i.e. $x(p,w)$ is O^0

Comparative Statics

Case 2: Change in one of the prices

Definition

If $x_1(p_1', p_2, p_3, ..., w) < x_1(p_1, p_2, p_3, ..., w)$ for $p_1' > p_1$, i.e. the **own price effect is negative**, x_1 is an **ordinary good**.

Definition

If $x_1(p_1', p_2, p_3, ..., w) > x_1(p_1, p_2, p_3, ..., w)$ for $p_1' > p_1$, i.e. the **own price effect is positive**, x_1 is a **Giffen good**.

Comparative statics

If the demand is differentiable:

We can use derivatives:

- Ordinary: $\frac{\partial x_i(p,w)}{\partial p_i} < 0$
- Giffen: $\frac{\partial x_i(p,w)}{\partial p_i} > 0$

We can use elasticity (**price elasticity**):

$$\varepsilon_{ij}(p,w) = \frac{\partial x_i}{\partial p_j}\frac{p_j}{x_i} \cong \frac{\frac{\Delta x_i}{x_i}}{\frac{\Delta p_j}{p_j}}$$

- We call it **own price elasticity** if $i = j$, **cross price elasticity** if $i \neq j$.
- Independent of the unit of measurement

Comparative statics

Case 3: Change in w

Definitions

x_i is a **normal good** if $\frac{\partial x_i(p,w)}{\partial w} > 0$.
x_i is an **inferior good** if $\frac{\partial x_i(p,w)}{\partial w} < 0$.

- **Income elasticity**:

$$\varepsilon_{iw}(p,w) = \frac{\partial x_i(p,w)}{\partial w}\frac{w}{x_i} \cong \frac{\frac{\Delta x_i}{x_i}}{\frac{\Delta w}{w}}$$

Definitions

x_1 is a **luxury good** if $\varepsilon_{iw} > 1$.
x_1 is an **Inferior good** if $\varepsilon_{iw} < 0$

Comparative Statics

Case 4: Slutzky compensated price change

Definition

A price change Δp is **Slutsky compensated** if wealth is adjusted by the amount

$$\Delta w^s = x(p,w)\Delta p = x(p,w)p' - x(p,w)p$$

$$w' = w + \Delta w^s$$

i.e. wealth is adjusted so that the original consumption bundle $x(p,w)$ is still available at the new prices p'.

Definitions

Substitution effect: $\Delta x^s = x(p', w') - x(p,w)$
Income effect: $\Delta x^w = x(p', w) - x(p', w')$
Total change:$\Delta x = \Delta x^s + \triangle x^w$

Walras' Law

Definition

A demand function $x(p,w)$ **satisfies Walras' Law** whenever

$$px(p,w) = w \quad \forall p, w$$

i.e. the consumer is on his/her budget line.

Proposition

If Walras' Law holds, then WARP $\Longleftrightarrow \triangle p \triangle x^s \leq 0$, with strict inequality whenever $x(p,w) \neq x(p',w')$.

Walras' Law and WARP

Proof.

Preliminaries:

- There is a price change from p to p'; because of Slutsky compensation wealth goes from w to w'; we assume that $x(p,w) \neq x(p',w')$
- $x(p',w')$ is revealed preferred to $x(p,w)$ because $p'x(p,w) = w'$, i.e. $x(p,w)$ is affordable at (p',w') but it is not chosen

Step 1: WARP $\implies \Delta p \Delta x_s < 0$

- At (p,w), $x(p',w')$ was not chosen $\implies$WARP implies that $x(p',w')$ was not available at p,w
  i.e. $px(p',w') > w = px(p,w)$

Walras' Law and WARP

$$px(p', w') > w = px(p, w)$$

$$p'x(p,w) \underbrace{=}_{Slutsky\ Compensation} w' \underbrace{=}_{Walras'\ Law} p'x(p', w')$$

$$px(p', w') + p'x(p, w) > px(p, w) + p'x(p', w')$$
$$px(p', w') - p'x(p', w') > px(p, w) - p'x(p, w)$$

$$\left(p' - p\right)\left(x(p', w') - x(p, w)\right) = \Delta p \Delta x^s < 0$$

Walras' Law and WARP

Step 2: $\Delta p \Delta x^s < 0 \implies$ WARP

- by assumption:

$$
\begin{aligned}
(p' - p)\left(x(p', w') - x(p, w)\right) &< 0 \\
px(p', w') - p'x(p', w') &> px(p, w) - p'x(p, w) \\
px(p', w') + p'x(p, w) &> px(p, w) + p'x(p', w')
\end{aligned}
$$

- Using $p'x(p, w) = p'x(p', w')$,

$$px(p', w') > px(p, w)$$

- $\implies x(p', w')$ is not affordable at (p, w), WARP is not violated

□

Walras' Law and WARP

Corollary

If x is a Giffen good, then x is also inferior.

Proof:

Exercise 3

Attila is indifferent between two goods, x and y: he always consumes the cheapest. Initially, $p_x = 4 \quad p_y = 5 \quad w = 60$. There is a price change to $p'_x = 6$.

1. Find the demand at the old and new prices.
2. Compute total demand change.
3. Find the Slutsky compensation.
4. Find the substitution effect.
5. Find the income effect.