

# Benevolent mediation in the shadow of conflict\*

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## **Abstract**

Before the start of a negotiation, the negotiating parties may try to affect the disagreement outcome by making socially-wasteful investments, such as purchasing weapons or asking for legal opinions. The incentives to make such investments depend on how the negotiation is conducted. We study the problem of a benevolent mediator who wishes to minimize pre-negotiation wasteful investments. Our main result is that the mediator should favor the strongest player, who has the lowest incentive to make wasteful investments. This result is robust to different specifications of the information available to the mediator. We therefore highlight a conflict between fairness and efficiency arising in negotiations.

**JEL classification:** D74; F51; J51.

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## 1 Introduction

Negotiations are often conducted under the shadow of conflict: in case an agreement is not reached, the negotiating parties will fight in a non-cooperative game. For example, two firms may negotiate a settlement under the shadow of a lawsuit; two countries may negotiate a treaty under the shadow of war; a government and a rebel group may negotiate a peace agreement under the shadow of a civil conflict; a wage negotiation may be conducted under the shadow of an industrial conflict.

Because the equilibrium payoffs of the potential conflict define the bargaining power of the negotiating parties, these parties may try to manipulate them. For example, prior to sitting at the bargaining table, a trade union may create a fund to support striking workers. Consequently, all the parties now anticipate a longer period of strikes and industrial actions in case the negotiation breaks down, thus shifting the disagreement point of the negotiation. Similarly, before the negotiation begins, the firm may layoff workers or start investing elsewhere. The same logic extends to other contexts. For example, prior to starting a negotiation, a country may invest in military equipment or a firm may ask for an additional legal opinion.<sup>1</sup> These investments are a form of rent seeking, because they do not increase the total payoff to be shared during the negotiation, but only how this surplus is split. They are, therefore, socially wasteful.

In this paper we consider the problem of a benevolent mediator who wishes to reduce these wasteful pre-negotiation investments. This benevolent mediator could be a person, an institution, or a country called in to mediate, for example, a civil conflict. The mediator anticipates that from the moment the mediation is announced to the moment in which the mediation starts, the parties may waste resources in order to improve the outcome of the negotiation, for example by acquiring military equipment. The mediator also understands that the way the negotiation will be conducted affects the parties' incentives to make wasteful pre-negotiation investments. Hence, he/she may want to announce as soon as possible that the negotiation will be conducted in a way that minimizes these incentives.

We study the extent to which the mediator can affect wasteful, pre-negotiation

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<sup>1</sup> There is ample evidence that conflicts are reactivated prior to the beginning of peace negotiations. For example, the mass killing of civilians (thus permanently weakening the opponents) is significantly more probable during the process of democratization of a country. See Esteban, Morelli, and Rohner (2015).

investments by setting the share of surplus allocated to each player within the negotiation, possibly conditional on the players' actions. We characterize the waste minimizing sharing rule. When the mediator cannot observe the investments made by the bargaining parties, the waste minimizing sharing rule is asymmetric, giving a larger share to the strongest player and inducing the weakest player not to invest at all, where "weak" and "strong" are defined by the outcome of the potential conflict in absence of investments. The intuition is that the weakest player has the strongest incentive to invest. Hence, the mediator reduces the marginal benefit of investing for this player by allocating a larger share of surplus to the strongest player. Furthermore, the more uneven is the distribution of initial strength, the lower the level of pre-negotiation, wasteful investment the mediator can achieve.

Therefore, the model highlights a trade off between equity and efficiency: to minimize wasteful, pre-negotiation investment the mediator should be biased toward the strongest player. This trade off is affected by the information structure. We show that the less the mediator can observe, the more the mediator should be biased in favor of the strongest player. This also implies that, when it comes to the choice of the mediator, the player who is expected to be the strongest will prefer the least informed mediator possible, while the weakest player will have the opposite interest. However, the mediator's actions may have unintended consequences when the player's power is endogenous. If the players expect the mediator to favor the strongest player, they may have an incentive to spend resources to increase their initial power. Hence, rather than decreasing wasteful investment, the actions of the mediator may simply shift wasteful investment to an earlier stage of the game.

We show that the mediator may prevent this from happening by organizing a pre-negotiation contest over the sharing rule. At the start of the negotiation, the mediator can require each player to make *concessions* to the other player, that is, take visible costly actions that benefit the other player. The level of concessions are then used to determine the sharing rule. If appropriately built, this contest can reduce the level of wasteful investment made by the player. Furthermore, under some conditions, the rules of the contest can be made independent from the player's initial power and therefore avoid the shifting of the investment to an earlier period.

Our paper contributes to an important debate in political science and international relations regarding the merits of biased mediation (see Svensson, 2014 for a review). This literature argues that a mediator who is biased may be more likely to achieve an agreement, where "bias" typically refers to a distortion in the mediator's

preferences. For example, in Kydd (2003) a bargaining party is more likely to believe the information transmitted by a mediator if the mediator is biased in favor of this party. In our paper, the mediator is, in principle, unbiased because his goal is to maximize welfare. However, he may choose to be *strategically* biased in order to decrease total waste. Hence, here bias is an equilibrium outcome.

In our model the role of the mediator is to establish the share of the surplus accruing to each player, where the surplus (also “peace dividend”) is the aggregate benefit of finding an agreement rather than triggering a conflict. This is equivalent to imposing a specific solution to the negotiation only if the mediator knows the players’ payoffs in case of conflict and therefore the size of the resulting surplus to be shared. We will be mostly concerned with the case in which the mediator does not have this piece of information. Therefore, using the taxonomy of methods of mediation in Fisher (2012), we model *power mediation*, that is, a mediation in which the mediator has some power over the outcome of the mediation but cannot force on the parties a specific solution (as opposed to an arbitrator).<sup>2</sup>

This modeling choice allows us to make our point in the sharpest possible way. The reason is that once the mediation stage is reached, the outcome of the mediation is always efficient. The only possible inefficiencies are those arising before the negotiation starts. This is in sharp contrast with the existing studies of mediation in economics, in which the mediator’s role is to maximize surplus *within the negotiation*.<sup>3</sup> Here we embed the mediation in a broader game, that also includes pre-negotiation actions by the contenders. Within this broader game, the mediator’s

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<sup>2</sup> Interestingly, we show in the appendix that a mediator who relays offers between players can achieve any sharing of the surplus (without necessarily knowing the size of the surplus) if he can asymmetrically affect the cost of waiting for an offer, for example by delaying the flow of communication between players. We therefore establish an interesting connection between what Fisher (2012) calls *conciliation*, that is, the role of the mediator in establishing a communication link between the players, and power mediation. See also a similar taxonomy by Bercovitch (1997), who distinguishes between a mediator’s communication strategies (i.e., transmitting messages) and manipulative strategies (i.e., influencing the outcome of the negotiation).

<sup>3</sup> See the review by Jackson and Morelli (2011) for different reasons why bargaining failures and an inefficient war may occur. Recent papers that explore the role of third party intervention in reducing the probability of an inefficient breakdown of the negotiations are Goltsman, Hörner, Pavlov, and Squintani (2009), Hörner, Morelli, and Squintani (2015), Balzer and Schneider (2015). Another recent example of third party intervention reducing inefficiencies within the negotiation is Basak (2015), who studies the effect of the degree of asymmetry of information at onset of a negotiation (assumed to be under control of a mediator) on the time required to reach an agreement.

role changes, because maximizing surplus requires considering how the mediator's actions affect the player's pre-negotiation actions.

A number of authors already noted that the way the negotiation is conducted can affect pre-negotiation actions by the players. Both Esteban, Morelli, and Rohner (2015) and Garfinkel, McBride, and Skaperdas (2012) show that the surplus share accruing to each player can have an effect on decisions made prior to the beginning of the negotiation. In Esteban et al. (2015) the surplus share obtained by each party in a negotiation may affect the intensity of the pre-negotiation conflict. They show that an equal surplus-split rule may be welfare decreasing relative to an asymmetric surplus-split rule. Here, we build on this observation and derive the welfare-maximizing surplus split. Garfinkel et al. (2012) notice that by investing in arms players influence the probability of winning in a conflict—and hence the disagreement point—and the share of surplus in the case of a peaceful agreement. Their main result is that when fighting is not sufficiently destructive, arming will be unavoidable within the class of distribution rules considered. Also related is the model in Anbarci, Skaperdas, and Syropoulos (2002), where each party starts by making wasteful investments in armaments. The paper compares the waste produced by three cooperative bargaining solutions: equal sacrifice, equal benefit, and Kalai-Smorodinski. The main result is that if players are symmetric equal sacrifice is the solution generating the lowest waste.

Closer to our paper, Meirowitz, Morelli, Ramsay, and Squintani (2015) consider the role of the mediator in reducing pre-negotiation wasteful investments. They compare mediated and unmediated negotiation and argue that mediated negotiation generates lower pre-negotiation wasteful investment in arms. In their framework, the mediator is only concerned with maximizing the probability that a settlement is reached. To achieve this objective, he will regulate the flow of information among parties, therefore affecting the precision of each player's belief relative to the other player's strength and the incentive to affect this strength via pre-bargaining investments. Hence, the main difference between our model and the one in Meirowitz et al. (2015) is the mediator's objective. Furthermore, Meirowitz et al. (2015) assume that the bargaining players are ex-ante identical, while our paper is mostly concerned with how ex-ante differences affect the optimal negotiation procedure.

Several other authors have developed models in which the bargaining parties may spend resources before the start of the negotiation (see the review by Jackson and Morelli, 2011 and the literature review in Meirowitz et al., 2015). For our purposes,

the most interesting works are those drawing a connection between pre-bargaining wasteful investments and inefficiencies arising within the negotiation.<sup>4</sup> For example, an arms build up prior to the negotiation may increase the chance that an agreement is found and therefore increase the efficiency of the negotiation, either because it makes war more costly or because it reduces the asymmetry of information between players. On the other hand, a military mobilization may decrease the probability of reaching an agreement and the efficiency of the negotiation because it generates a *hands-tying effect*: a decrease in the cost of starting a war that operates as a public commitment device. These channels are muted in our model. How the mediator can maximize welfare when the inefficiencies arising in the pre-negotiation stage affect the inefficiencies arising during the negotiation is a research question that we leave for future work.

The remainder of the paper is organized as follows. The next section describes the model. The following three sections analyze the mediator's problem under different assumptions regarding what the mediator can observe: in Section 3 the mediator has full information; in Section 4 the mediator does not observe the wasteful investments made by players prior to the start of the negotiation; in Section 5 the mediator observes neither players' investments nor their initial power levels. Section 6 discusses what happens when the players' power levels are endogenous. Section 7 allows the mediator to organize a pre-negotiation contest over the sharing rule. Unless otherwise noted, all proofs are in Appendix B.

## 2 The model

A total payoff  $S$  needs to be shared between two players, 1 and 2. First the players make offensive and defensive investments, then, in a second stage, the players negotiate over the total payoff  $S$  with the help of a mediator. In case the negotiation breaks down, a conflict between the two players will break out.

**Period 1.** In the first period of the game, both players can make an offensive investment  $o_i$  and a defensive investment  $d_i$ . An offensive investment by player  $i$  decreases the payoff of player  $-i$  in case of a conflict. A defensive investment by player  $i$  increases the payoff of this same player in case of a conflict. As examples of offensive

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<sup>4</sup> See Powell (1993), Kydd (2000), Slantchev (2005), Meirowitz and Sartori (2008), Jackson and Morelli (2009).

investment, a player may purchase ballistic missiles or collect evidence against the opponent to be used in a court case. As examples of defensive investments, a player may purchase antimissile system and bunkers, or move assets to jurisdictions where they are harder to seize in case the outcome of a lawsuit is negative. In the case of industrial conflict, the use of a resistance fund in a labor strike reduces the harm of conflict to workers and hence should be considered “defensive” accordingly with our conceptualization. In the same context, a firm may invest resources to make the relocation of the factory a credible threat, which should be considered “offensive.” We assume that these two types of investments are always socially wasteful, in the sense that they have no effect on the total payoff to be allocated  $S$ .

Note that many investments are simultaneously offensive and defensive, in the sense that they simultaneously increase a player’s utility and decrease the opponents utility in case of conflict (for example, hiring a very expensive but competent lawyer or purchasing tanks). Some other investments decrease both players utilities in case of conflicts, and are therefore mutually destructive (for example, nuclear weapons). Whether an investment is offensive, defensive (or both) depends also on the details of the conflict game. For example, in a tournament, each player’s investment is simultaneously offensive and defensive, because when a player increases his/her probability of winning the opponent’s probability of winning automatically decreases. In a war of attrition, some investments may increase the length of a conflict and therefore be mutually destructive. In general, for a given model of conflict, any type of investment can be expressed as a combination of offensive and defensive investments based on its effect on the players’ payoffs. For this reason, as a first approximation here we only consider purely offensive or purely defensive investments.

Formally, call  $\phi_i$  the ex-ante “power” of player  $i$ —the payoff achieved by this player in the conflict game in case no investment is made. The ex-ante “power” of a player may depend on natural elements (e.g. the presence of mountains may make one country harder to attack) or by the merit of the legal dispute. It may also depend on prior investments in offensive/defensive technology (see Section 6 for more on this interpretation). Let us denote by  $\underline{u}_i$  the payoff of player  $i$  in the conflict game, taking into account her ex-ante power  $\phi_i$ , own defensive investment  $d_i$  and the opponent’s offensive investment  $o_{-i}$ , that is,  $\underline{u}_i = u(\phi_i, d_i, o_{-i})$ . We specify

this payoff function to

$$\underline{u}_i = \phi_i e^{-o_j} (2 - e^{-d_i}), \quad i, j = 1, 2.$$

The key feature of this expression is weak separability: the marginal rate of substitution between  $d_i$  and  $\phi_i$  is independent of  $o_{-i}$  and the one between  $o_{-i}$  and  $\phi_i$  is independent of  $d_i$ . This implies that the rate at which player  $i$  can compensate for being weak ex-ante (i.e., having low  $\phi_i$ ) by making an investment in defense is independent on  $o_{-i}$ . Similarly, the rate at which player  $i$  can compensate for player  $-i$  being strong ex-ante (i.e., having high  $\phi_{-i}$ ) by making an investment in offense is independent on  $d_{-i}$ . This key feature is justified by the fact that  $\phi_i$  can be the result of investments in offensive and defensive technology made in prior periods, the marginal effect of which should not depend on the current level of investments.<sup>5</sup> Clearly, weak separability is satisfied if and only if  $u_i = \psi[f(\phi_i)g(d_i)h(o_{-i})]$ . Our assumed payoff function is a member of this class, that we use for the sake of convenience.

Finally, we impose the restriction  $S \geq 2(\phi_1 + \phi_2)$ , so that sharing  $S$  in a negotiation dominates the conflict payoff *for every possible level of defensive and offensive investment*, even when offensive investments are both zero and defensive investments are at their maximum. The marginal cost of investing in defensive and offensive technology are  $c_d$  and  $c_o$ , assumed constant.

**Period 2.** In period 2 the players negotiate with the help of a mediator. No agreement is possible without mediation, because the two parties do not trust each other and every direct communication between them is cheap talk. However, once the mediator is involved, then any proposal becomes credible and the players may reach an agreement. Therefore, the mediator here is a third party—a large country, an arbitrator with some legal power—who can coerce players into implementing the agreement they signed.<sup>6</sup>

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<sup>5</sup> Under the alternative assumption, the marginal rate of substitution between  $d_i$  and  $\phi_i$  is a function of  $o_{-i}$ . But then  $o_{-i}$  should affect the relative benefit not only of current defensive investments, but also past defensive investments. It follows that  $\phi_i$  should be a function of the investments  $o_{-i}$  and  $d_i$ , a complication that we wish to avoid. Note also that our specification abstracts away issues related to risk, that are not central to our analyses.

<sup>6</sup> The negotiations between the Colombian government and the FARC provide a good example. They have taken place in Cuba because the assumed capacity of this government to “force” the

The mediator can choose to favor either one of the players. More formally, call the ex-post surplus (also called "peace dividend") to be shared in the negotiation

$$S - \underline{u}_1 - \underline{u}_2.$$

We assume that the mediator chooses the share of ex-post surplus accruing to player 1, which we call  $\gamma \in [0, 1]$ , with the remaining share  $1 - \gamma$  going to player 2. This is depicted in Figure 1. Given the mediator's choice, the players payoffs are

$$U_1 = \underline{u}_1 + \gamma(S - \underline{u}_1 - \underline{u}_2), \quad U_2 = \underline{u}_2 + (1 - \gamma)(S - \underline{u}_1 - \underline{u}_2).$$

Notice that by choosing the sharing rule the mediator can determine the payoff achieved by each player only if he observes  $\underline{u}_1$  and  $\underline{u}_2$ . When this is the case, the mediator's role is similar to that of an arbitrator (or rather, a "final offer arbitrator"). If instead  $\underline{u}_1$  and  $\underline{u}_2$  are not observed by the mediator, the players' payoffs will depend both on the sharing rule announced by the mediator and on the players' actions.<sup>7</sup> As we discussed in the introduction, this situation corresponds to power mediation (Fisher, 2012), because the mediator can influence—but not fully determine—the outcome of the mediation.

Finally, note that choosing the sharing rule is equivalent to choosing the weight attached to a player's utility in a Generalized Nash Bargaining (GNB) problem. Hence, equivalently, we can think of the mediator as choosing a specific bargaining solution among all those that satisfy Pareto optimality, independence of irrelevant alternatives, invariance to rescaling of utility (which are the axioms that characterize the GNB solution).<sup>8</sup> Furthermore, in Appendix A we provide a possible

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FARC to implement the commitments. In Germany, for many years the negotiations between unions and employers have been chaired by the government because its capacity to force the commitment of the parties.

<sup>7</sup> In a more general model, offensive and defensive investments may affect not only the players' outside options, but also the outcome of the negotiation—that is, the share of ex-post surplus going to each player—possibly jointly with some actions taken by the mediator (as in, for example, Meirowitz et al., 2015). The model can easily be extended in that direction, but we prefer here to focus on the effect of the investments on the players' outside options.

<sup>8</sup> It is interesting to note that Luce and Raiffa (1957) interpreted the axioms of a bargaining solution as normative properties of a negotiation/arbitration scheme (see Luce and Raiffa, 1957, pp. 122–123). According to this interpretation, starting from the axioms of the Generalized Nash Bargaining solution we introduce an additional normative statement about how the mediator should behave (i.e., minimize pre-negotiation waste).



**The mediator's objective.** We assume that the mediator is benevolent: his objective is to minimize socially-wasteful expenditure in offensive and defensive technology,  $c_o(o_1 + o_2) + c_d(d_1 + d_2)$ . He will set a specific ex-post surplus split  $\gamma$  so to achieve this goal. Depending on what the mediator can observe, the ex-post surplus split can be contingent on the initial power  $\phi_1$  and  $\phi_2$ , and/or on the investment made by each player. Figure 2 summarizes the timing of the game.

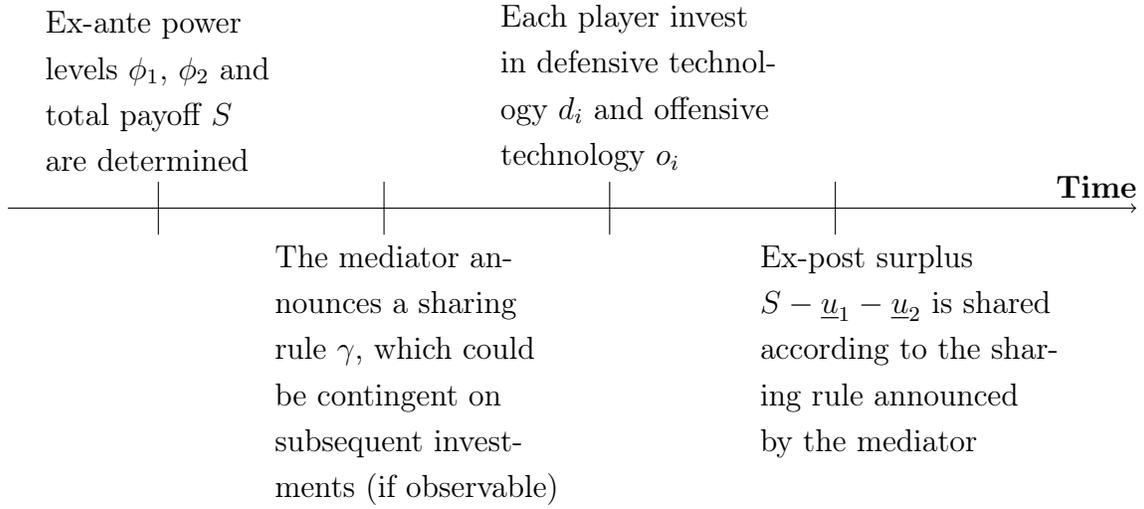


Fig. 2: Timeline

### 3 Full information

Without loss of generality, assume that  $\phi_1 \geq \phi_2$ . When the mediator observes everything, he can announce that he will implement a sharing rule  $\gamma$  in case of zero investment, and simultaneously announce that he will allocate zero ex-post surplus to the player making positive investments in either defensive or offensive technology. Given this announcement, there is an equilibrium in which the ex-post surplus share is  $\gamma$  and there is no investment whenever no level of defensive investment delivers higher payoff to either player, that is whenever:

$$\begin{aligned} \phi_1 + \gamma(S - \phi_1 - \phi_2) &\geq \max_{d_1} \{ (2 - e^{-d_1})\phi_1 - c_d d_1 \} \\ &= 2\phi_1 - \min \left\{ \phi_1, c_d \left( 1 + \log \left( \frac{\phi_1}{c_d} \right) \right) \right\} \end{aligned} \quad (1)$$

$$\begin{aligned} \phi_2 + (1 - \gamma)(S - \phi_1 - \phi_2) &\geq 2 \max_{d_2} \{(2 - e^{-d_2})\phi_2 - c_d d_2\} \\ &= 2\phi_2 - \min \left\{ \phi_2, c_d \left( 1 + \log \left( \frac{\phi_2}{c_d} \right) \right) \right\}. \end{aligned} \quad (2)$$

The mediator may also announce that in case *both* players make a positive investment all ex-post surplus will go to one of them, for example player 2. Given this announcement, if (1) holds player 1 will never want to invest, and hence in equilibrium we never have both players investing.

It follows that when the above two inequalities hold at a specific  $\gamma$  there exists an announcement for which no investment is the unique equilibrium. It is also easy to see that, when either (1) or (2) are violated at a specific  $\gamma$ , then there is no announcement for which no investment is an equilibrium, the reason being that allocating zero ex-post surplus to a player in case of positive investment is the harshest punishment that the mediator can impose. Simple algebra show that, under the assumption  $S > 2(\phi_1 + \phi_2)$ , there always exists a  $\gamma$  that satisfies both conditions and therefore generates zero investment.<sup>9</sup>

**Proposition 1.** *In equilibrium, the mediator can implement any sharing rule:*

$$\gamma \in \left[ \frac{\phi_1 - \min \left\{ \phi_1, c_d \left( 1 + \log \left( \frac{\phi_1}{c_d} \right) \right) \right\}}{S - \phi_1 - \phi_2}, 1 - \frac{\phi_2 - \min \left\{ \phi_2, c_d \left( 1 + \log \left( \frac{\phi_2}{c_d} \right) \right) \right\}}{S - \phi_1 - \phi_2} \right] \neq \emptyset$$

*and generate zero investment. No other sharing rule can be implemented and generate zero investment.*

*Proof.* In the text. □

We can now address the question of whether the mediator can eliminate all investment *and* be fair at the same time, that is whether

$$\frac{1}{2} \in \left[ \frac{\phi_1 - \min \left\{ \phi_1, c_d \left( 1 + \log \left( \frac{\phi_1}{c_d} \right) \right) \right\}}{S - \phi_1 - \phi_2}, 1 - \frac{\phi_2 - \min \left\{ \phi_2, c_d \left( 1 + \log \left( \frac{\phi_2}{c_d} \right) \right) \right\}}{S - \phi_1 - \phi_2} \right]$$

Simple algebra shows that

$$\frac{1}{2} < 1 - \frac{\phi_2 - \min \left\{ \phi_2, c_d \left( 1 + \log \left( \frac{\phi_2}{c_d} \right) \right) \right\}}{S - \phi_1 - \phi_2},$$

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<sup>9</sup> This is easy to check by considering the largest possible defensive investment  $d_1 = d_2 = 2$  (or alternatively the smallest possible cost of investing  $c_d = 0$ ).

which implies that, there is a *tradeoff between efficiency and fairness* if and only if

$$\frac{\phi_1 - \min \left\{ \phi_1, c_d \left( 1 + \log \left( \frac{\phi_1}{c_d} \right) \right) \right\}}{S - \phi_1 - \phi_2} > \frac{1}{2}$$

or

$$3\phi_1 + \phi_2 > S + 2 \min \left\{ \phi_1, c_d \left( 1 + \log \left( \frac{\phi_1}{c_d} \right) \right) \right\}.$$

Hence, the tradeoff between efficiency and fairness is more likely to emerge whenever the cost  $c_d$  is small,  $S$  is small, or  $\phi_2$  is large. The reason is that a small  $S$  or a large  $\phi_2$  reduce the ex-post surplus to be shared in the negotiation and the benefit for player 1 of accepting  $\gamma = \frac{1}{2}$  rather than deviating. This tradeoff is also more likely to emerge whenever  $\phi_1$  is large. Again, as  $\phi_1$  increases the ex-post surplus decreases. Furthermore,  $\phi_1$  large also increases the benefit for player 2 to deviate and make a large defensive investment. When this tradeoff emerges, to eliminate the incentives to invest the mediator needs to be biased in favor of player 1, who is the strongest player and therefore has the strongest incentive to invest.

## 4 Unobservable investments

If instead the mediator does not observe the investment made by the players, he can only announce an unconditional surplus split  $\gamma$ , determined after the ex-ante power levels  $\phi_1$  and  $\phi_2$  are realized.<sup>10</sup>

Player 1's problem and Player 2's problem are, respectively:

$$\max_{o_1, d_1} \left\{ \phi_1 e^{-o_2} (2 - e^{-d_1}) + \gamma \left[ S - \phi_1 e^{-o_2} (2 - e^{-d_1}) - \phi_2 e^{-o_1} (2 - e^{-d_2}) \right] \right\} - c_o o_1 - c_d d_1.$$

$$\max_{o_2, d_2} \left\{ \phi_2 e^{-o_1} (2 - e^{-d_2}) + (1 - \gamma) \left[ S - \phi_1 e^{-o_2} (2 - e^{-d_1}) - \phi_2 e^{-o_1} (2 - e^{-d_2}) \right] \right\} - c_o o_2 - c_d d_2.$$

Figures 3 and 4 illustrate the relevant tradeoffs in the choice of investment mix. Figure 3 compares the effect of a given offensive investment by player 1 on his

<sup>10</sup> Because the players observe each other's investment, the mediator may try to elicit this information from them. Note, however, that in case the players' reports on the investment levels do not match (so that at least one of the two players is lying), the mediator is unable to punish both players at the same time. The reason is that the mediator's instrument is  $\gamma$ , and therefore if one player is punished by receiving a low surplus share, the other player must be rewarded. It follows that there is no equilibrium in which the players report truthfully. We do, however, show in Section 7 that the mediator may benefit from organizing a contest for  $\gamma$  in which the sharing rule depends on some costly actions taken by the players.

final payoff, for different values of  $\gamma$ . Figure 4 does the same for a given defensive investment by player 1. They show that if  $\gamma$  is large—i.e., the sharing rule is extremely biased in favor of player 1—player 1’s investment in offensive weapons produces a much higher benefit than defensive weapons. We have the opposite effect if  $\gamma$  is low. Intuitively, as the share of ex-post surplus received increases, a player’s payoff depends more and more on the opponent’s outside option rather than on his own outside option. In the limit case in which all ex-post surplus is allocated to player  $i$ , the final payoff for both players only depends on player  $-i$ ’s outside option. As a consequence, the incentive to degrade the opponent and make an offensive investment increases with the share of ex-post surplus received. Similarly, as the share of ex-post surplus received *decreases*, a player’s payoff depends more and more on his own outside option rather than on his opponent’s. It follows that, as the share of ex-post surplus received decreases, the incentive to make a defensive investment increases.

The first-order conditions of the problem yield:

$$o_1 = \log \left( \frac{\gamma\phi_2(2 - e^{-d_2})}{c_o} \right) \text{ if } \frac{\gamma\phi_2(2 - e^{-d_2})}{c_o} \geq 1 \text{ else } o_1 = 0, \quad (3)$$

$$d_1 = \log \left( \frac{(1 - \gamma)\phi_1 e^{-o_2}}{c_d} \right) \text{ if } \frac{(1 - \gamma)\phi_1 e^{-o_2}}{c_d} \geq 1 \text{ else } d_1 = 0, \quad (4)$$

$$o_2 = \log \left( \frac{(1 - \gamma)\phi_1(2 - e^{-d_1})}{c_o} \right) \text{ if } \frac{(1 - \gamma)\phi_1(2 - e^{-d_1})}{c_o} \geq 1 \text{ else } o_2 = 0, \quad (5)$$

$$d_2 = \log \left( \frac{\gamma\phi_2 e^{-o_1}}{c_d} \right) \text{ if } \frac{\gamma\phi_2 e^{-o_1}}{c_d} \geq 1 \text{ else } d_2 = 0. \quad (6)$$

Note that taking (3) and (6) together we have that  $o_1$  and  $d_2$  are best response of each other and similarly if we take (4) and (5) together we define the reciprocal response of  $d_1$  and  $o_2$ .

By (3) and (5), the incentive to invest in offensive technology increases in the ex-post surplus share received, in the opponent’s ex-ante power, and in the opponent’s defensive investment. As discussed earlier, as the share of ex-post surplus received increases, a player’s payoff depends more and more on the opponent’s outside option, increasing the incentive to make offensive investment. Conditions (3) and (5) show that this effect is stronger the higher the opponent’s outside option—which is determined by the combination of his ex-ante power and his defensive investment.

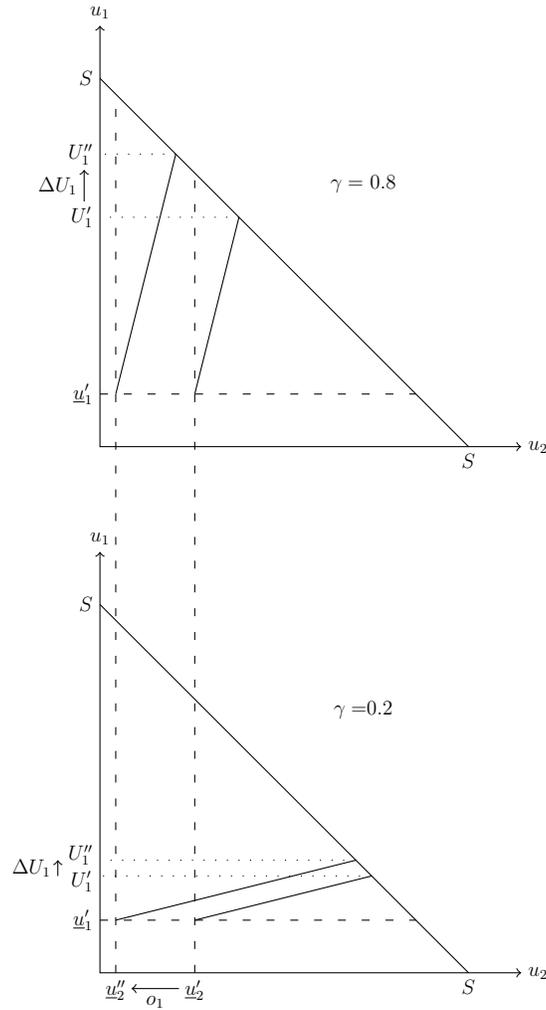


Fig. 3: Benefit of player 1’s offensive investment for different values of  $\gamma$ .

That is, an increase in the share received by player  $i$  has a smaller impact on this player’s utility when player  $-i$  is weak than when player  $-i$  is strong.

Similarly, by 4 and 6, the incentive to invest in defensive technology decreases with the share of ex-post surplus received, increases with own ex-ante power and decreases with the opponent’s offensive investment. Again, as the surplus share received decreases, a player’s payoff depends more and more on his own outside option, increasing the level of defensive investment. This effect is stronger the stronger is the player—where his strength is determined by his ex-ante power and the opponent’s investment in offensive technology.

Putting the best responses together, we can characterize the Nash equilibrium

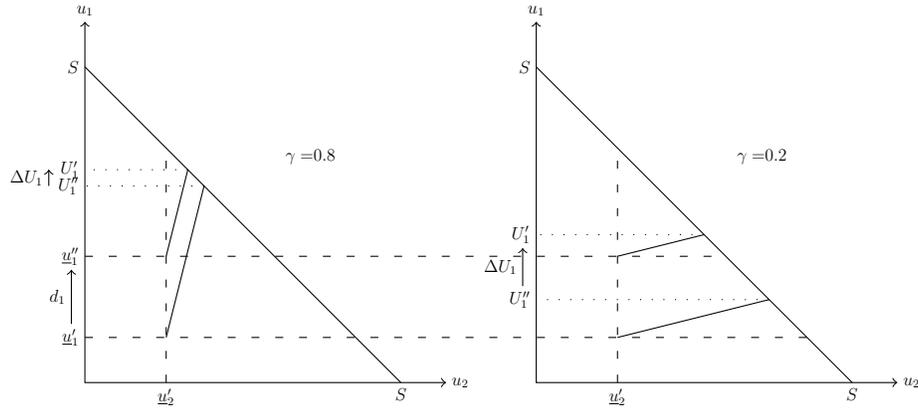


Fig. 4: Benefit of player 1's defensive investment for different values of  $\gamma$ .

of the game:

**Lemma 1.** *The Nash equilibrium of the game is as follows:*

- If  $c_o \leq c_d$ ,

$$o_1 = \max \left\{ \log \left( \frac{\gamma \phi_2}{c_o} \right), 0 \right\}, o_2 = \max \left\{ \log \left( \frac{(1-\gamma) \phi_1}{c_o} \right), 0 \right\}$$

and  $d_1 = d_2 = 0$ .

- If  $c_o > c_d$ ,

– for  $(o_1, d_2)$

\* for  $\gamma \geq \frac{c_o + c_d}{2\phi_2}$  we have  $o_1 = \log \left( \frac{2\gamma\phi_2}{c_o + c_d} \right)$  and  $d_2 = \log \left( \frac{c_o + c_d}{2c_d} \right)$ ;

\* for  $\frac{c_d}{\phi_2} \leq \gamma \leq \frac{c_o + c_d}{2\phi_2}$  we have  $o_1 = 0$  and  $d_2 = \log \left( \frac{\gamma\phi_2}{c_d} \right)$ .

\* for  $\gamma < \frac{c_d}{\phi_2}$ , we have  $o_1 = 0$  and  $d_2 = 0$ .

– for  $(d_1, o_2)$

\* for  $1 - \gamma \geq \frac{c_o + c_d}{2\phi_1}$  we have  $o_2 = \log \left( \frac{2(1-\gamma)\phi_1}{c_o + c_d} \right)$  and  $d_1 = \log \left( \frac{c_o + c_d}{2c_d} \right)$ ;

\* for  $\frac{c_d}{\phi_1} \leq 1 - \gamma \leq \frac{c_o + c_d}{2\phi_1}$  we have  $o_2 = 0$  and  $d_1 = \log \left( \frac{(1-\gamma)\phi_1}{c_d} \right)$ .

\* for  $1 - \gamma < \frac{c_d}{\phi_1}$ , we have  $o_2 = 0$  and  $d_1 = 0$ .

The Nash equilibrium depends on the cost of offensive and defensive investments. When  $c_d \geq c_o$ , in equilibrium there never is any investment in defensive technology.

Remember that defensive investment is decreasing in offensive investment. When the cost of offensive investment is low relative to the cost of defensive investment, each player will make a large investment in offensive technology and, in equilibrium, drive the incentive to invest in defensive technology of the other player to zero.

If instead  $c_d < c_o$ , for extreme sharing rules  $c_d < c_o$ , (i.e.  $\gamma \geq \frac{c_o+c_d}{2\phi_2}$  or  $\gamma \leq 1 - \frac{c_o+c_d}{2\phi_1}$ ), one player invests only in offensive technology while the other invests only in defensive technology. Instead, for intermediate sharing rules, players only make defensive investments and no offensive investments.<sup>11</sup> Intuitively, because of the cost advantage, players are more likely to make a defensive investment, the more so the larger the share of ex-post surplus going to the other player. However, the offensive investment made by player  $i$  increases with the defensive investment made by player  $-i$ , which implies that for extreme sharing rules one player makes a defensive investment while the other player makes an offensive investment.

The Nash equilibrium also depends on  $\gamma$ . Suppose the mediator allocates most of the ex-post surplus to player 2 by setting  $\gamma$  close to zero. Depending on  $c_o$  and  $c_d$ , Player 1 may make positive defensive investment but no offensive investment. Player 2 will make a positive offensive investment and no defensive investment. As  $\gamma$  increases, the players substitute one type of investment with the other. For  $\gamma$  close to one, the situation is reversed, with Player 1 making only an offensive investment, and Player 2 possibly only a defensive investment. The key observation is that the choice of  $\gamma$  determines whether the fight will be over Player 1's outside option (with Player 2 attacking it and Player 1 defending it) or over Player 2's outside option (with Player 1 attacking it and Player 2 defending it). The following proposition characterizes the solution to the mediator's problem.

**Proposition 2.** *Whenever*

$$\min\{c_o, c_d\} \geq \frac{\phi_1\phi_2}{\phi_1 + \phi_2}, \quad (7)$$

*then any*

$$\gamma^* \in \left[ 1 - \frac{\min\{c_o, c_d\}}{\phi_1}, \frac{\min\{c_o, c_d\}}{\phi_2} \right]$$

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<sup>11</sup> The reader may wonder why a player may make a defensive investment when the other player is not making any offensive investment. To understand this, remember that players may have made offensive or defensive investments before the game start. These investments are embedded into the initial power levels  $\phi_1$  and  $\phi_2$ . Hence, the result here is that, for intermediate sharing rule players make additional defensive investment but no additional offensive investment.

drives wasteful investment to zero. Whenever (7) is violated, the mediator minimizes waste by setting

$$\gamma^* = 1 - \frac{\min\{c_o, c_d\}}{\phi_1}.$$

At the waste-minimizing  $\gamma$  player 2's offensive investment and player 1's defensive investments are zero. If  $c_o \leq c_d$  player 1's offensive investment is positive but player 2's defensive investment is zero. If  $c_d < c_o$ , player 2's defensive investment is positive.

Condition (7) implies that, for given  $\phi_1 + \phi_2$ , the distribution of initial power is sufficiently uneven. In this case, the mediator can completely eliminate the players' incentives to invest. Instead, whenever the distribution of initial power levels is sufficiently equal so that (7) is violated, the mediator is unable to eliminate wasteful investment. As we discussed earlier, the choice of the sharing rule determines whether the fight is over player 1 or player 2 outside option. The proposition shows that total waste is minimized when the mediator sets  $\gamma^* = 1 - \frac{\min\{c_o, c_d\}}{\phi_1}$ , which is the sharing rule that eliminates the fight over player 1's outside option. This sharing rule may generate a fight over player 2's outside option. However, because of the difference in initial power levels, player 1's incentive to perform offensive investment is lower than player 2, and the opposite holds for the incentives to perform defensive investments. Hence, total waste is minimized when the fight is over player 2's outside option, with player 1 attacking and player 2 defending, rather than player 1's.

If the distribution of power is uneven so that 7 holds,  $\gamma^* = \frac{1}{2}$  eliminates all investments if and only if  $\phi_1 \leq 2 \min\{c_o, c_d\}$ . If instead (7) is violated simple algebra shows that  $\gamma^*$  is greater than  $1/2$ . Hence, there is no trade off between fairness and efficiency if and only if  $\phi_1 \leq 2 \min\{c_o, c_d\}$ . In this case, both players' initial power levels are low (remember that  $\phi_2 \leq \phi_1$ ), and the incentives to invest of both players are also low. If instead  $\phi_1 > 2 \min\{c_o, c_d\}$  fairness and efficiencies are mutually exclusive, and the mediator minimizes waste by favoring the strongest player.<sup>12</sup>

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<sup>12</sup> Contrary to the case of observable investments (Section 3), here  $S$  and  $\phi_2$  play no role in determining the emergence of the tradeoff between fairness and efficiency. The reason is that, here, the sharing rule does not change with the investments made by the players. Hence, the benefit of shifting a player's outside option is independent on the other player's outside option or the total payoff  $S$ . When the sharing rule reacts to the investment as in Section 3, instead, the

## 5 Unobservable ex-ante power levels

Assume now that the mediator does not observe neither the investment levels, nor the ex-ante power levels, nor  $S$ . For simplicity, let us only consider the case  $c_o \leq c_d$ .

For given  $\gamma$ , the optimal investments by each player are the same as derived in Section 4. It follows that, for a given belief over the distribution of ex-ante power levels, the mediator solves:<sup>13</sup>

$$\min_{\gamma} \left\{ \Pr \left( \phi_2 > \frac{c_o}{\gamma} \right) E \left[ \log \left( \frac{\gamma \phi_2}{c_o} \right) \middle| \phi_2 > \frac{c_o}{\gamma} \right] + \Pr \left( \phi_1 > \frac{c_o}{1-\gamma} \right) E \left[ \log \left( \frac{(1-\gamma)\phi_1}{c_o} \right) \middle| \phi_1 > \frac{c_o}{1-\gamma} \right] \right\}.$$

That is, the mediator minimizes each player's probability of investing times the level of investment in case a player invests. The mediator's objective function can be written explicitly whenever  $\phi_1$  and  $\phi_2$  are drawn from two Pareto distributions.

**Lemma 2.** *Assume that  $\phi_1$  and  $\phi_2$  are drawn from two Pareto distributions with parameters  $\kappa_1 > 0$  and  $\kappa_2 > 0$ , and minimum values  $\underline{\phi}_1 > 0$  and  $\underline{\phi}_2 > 0$  respectively. Then, the mediator minimizes*

$$\begin{cases} \left( \frac{\phi_2 \gamma}{c_o} \right)^{\kappa_2} \frac{1}{\kappa_2} + \left( \log \left( \frac{\phi_1 (1-\gamma)}{c_o} \right) + \frac{1}{\kappa_1} \right) & \text{if } \gamma \leq \min \left\{ 1 - \frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2} \right\} \\ \left( \frac{\phi_2 \gamma}{c_o} \right)^{\kappa_2} \frac{1}{\kappa_2} + \left( \frac{\phi_1 (1-\gamma)}{c_o} \right)^{\kappa_1} \frac{1}{\kappa_1} & \text{if } 1 - \frac{c_o}{\underline{\phi}_1} \leq \gamma \leq \frac{c_o}{\underline{\phi}_2} \\ \left( \log \left( \frac{\phi_2 \gamma}{c_o} \right) + \frac{1}{\kappa_2} \right) + \left( \log \left( \frac{\phi_1 (1-\gamma)}{c_o} \right) + \frac{1}{\kappa_1} \right) & \text{if } \frac{c_o}{\underline{\phi}_2} \leq \gamma \leq 1 - \frac{c_o}{\underline{\phi}_1} \\ \left( \log \left( \frac{\phi_2 \gamma}{c_o} \right) + \frac{1}{\kappa_2} \right) + \left( \frac{\phi_1 (1-\gamma)}{c_o} \right)^{\kappa_1} \frac{1}{\kappa_1} & \text{if } \max \left\{ 1 - \frac{c_o}{\underline{\phi}_1}, \frac{c_o}{\underline{\phi}_2} \right\} < \gamma \end{cases} \quad (8)$$

Without loss of generality, we assume that  $\underline{\phi}_1 > \underline{\phi}_2$ . As the parameters  $\kappa_1$  and  $\kappa_2$  increase, the masses of the two distributions become more and more concentrated near their minimum value. It follows that for  $\kappa_1$  and  $\kappa_2$  arbitrarily large, the mediator's problem converges to the one studied in the previous section. On the other hand, as the parameters  $\kappa_1$  and  $\kappa_2$  decrease, the tails of the two Pareto distributions become thicker, with higher  $\phi_i$  becoming more likely and therefore increasing the expected investment levels by the two players. In particular, when  $\kappa_i < 2$  the tails of the distribution are so thick that  $Var[\phi_i]$  is not well defined; when  $\kappa_i < 1$  the tails of the distribution are even thicker and also  $E[\phi_i]$  is not well defined. As  $\kappa_1 \rightarrow 0$ , the mediator's belief becomes an improper prior.

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size of the ex-post surplus is a relevant element in determining the existence of the tradeoff.

<sup>13</sup>  $S$  is also unobserved by the mediator. However, the way in which it is determined is not relevant as long as for every realization of  $\phi_1$  and  $\phi_2$  the realization of  $S$  is such that  $2(\phi_1 + \phi_2) \leq S$ , so that also here finding an agreement is preferred to war for every level of investment.

We interpret  $\kappa_i$  as a measure of how informed the mediator is about player  $i$ . If the mediator is well informed, then  $\kappa_i$  is large, the tail of the Pareto distribution is thin and the probability that player  $i$  turns out to be extremely powerful is low. On the other hand, the mediator could be completely uninformed: the only thing he may know is that player  $i$ 's power is above a certain threshold. In this case  $\kappa_i$  is small, the tail of the Pareto distribution is thick and there is a non-negligible probability that player  $i$  turns out to be extremely powerful.

The following proposition characterizes the solution to the mediator's problem.

**Proposition 3.** *The waste-minimizing sharing rule is weakly increasing in  $\underline{\phi}_1$ , weakly decreasing in  $\underline{\phi}_2$ , weakly increasing in  $\kappa_2$ , weakly decreasing in  $\kappa_1$ .<sup>14</sup> Furthermore:*

- for  $\kappa_1, \kappa_2 \rightarrow \infty$ , the players power levels are  $\underline{\phi}_1$  and  $\underline{\phi}_2$  with almost certainty, and the waste minimizing sharing rule converges to the one in Lemma ??.
- for  $\kappa_1, \kappa_2 \leq 1$  the waste minimizing sharing rule is

$$\gamma^* = \begin{cases} 1 & \text{if } \frac{1}{\kappa_2} - \frac{1}{\kappa_1} < \log\left(\frac{\underline{\phi}_1}{\underline{\phi}_2}\right) \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

- for  $\underline{\phi}_1, \underline{\phi}_2 \rightarrow \infty$  the waste minimizing sharing rule converges to (9),
- for  $\underline{\phi}_1, \underline{\phi}_2 \leq c_o$  (so that for every  $\gamma$  there is a positive probability that neither player invests), the waste minimizing sharing rule is

$$\gamma^* = \gamma : \left(\frac{\underline{\phi}_2}{c_o}\right)^{\kappa_2} \gamma^{\kappa_2-1} = \left(\frac{\underline{\phi}_1}{c_o}\right)^{\kappa_1} (1-\gamma)^{\kappa_1-1}$$

Also here, keeping  $\kappa_1$  and  $\kappa_2$  constant, as the expected strength of player  $i$  relative to player  $-i$  increases the mediator will increase the share of ex-post surplus received by player  $i$ . Furthermore, keeping  $\underline{\phi}_1$  and  $\underline{\phi}_2$  constant, the surplus share received by player  $i$  decreases with  $\kappa_i$  and increases with  $\kappa_{-i}$ . That is, each player prefers when the mediator has a precise belief about his opponent's power level, but an uninformative belief about his own power level. This is again due to the fact that

<sup>14</sup> If the solution to the mediator's problem is not unique, then both the smallest and the largest waste-minimizing  $\gamma$  are weakly increasing in  $\underline{\phi}_1$  and  $\kappa_2$ , and weakly decreasing in  $\underline{\phi}_2$  and  $\kappa_1$ .

for given  $\underline{\phi}_1$  and  $\underline{\phi}_2$ , the expected strength of player  $i$  relative to player  $-i$  decreases with  $k_i$  and increases with  $k_{-i}$ .

Finally, as  $\underline{\phi}_1, \underline{\phi}_2$  increase or  $\kappa_1, \kappa_2$  decrease, the two players become more likely to invest. Hence, for  $\underline{\phi}_1, \underline{\phi}_2$  sufficiently high or  $\kappa_1, \kappa_2$  sufficiently low, the waste minimizing sharing rule becomes extreme, allocating all ex-post surplus to one of the two players. In less extreme cases, the waste-minimizing sharing rule is intermediate, because each player has a low probability of investing.

A related question is whether the players would prefer to have a more knowledgeable mediator, who has more precise information about both players' power levels. For example, the mediator may be given the ability to gather intelligence and inspect both players, leading to an increase in both  $\kappa_1$  and  $\kappa_2$ .<sup>15</sup> To explore this possibility, let us assume  $\kappa_1 = \kappa_2 \equiv \kappa$ , meaning that the mediator's prior beliefs over  $\phi_1$  and  $\phi_2$  are equally precise.

By the previous Proposition, if  $\underline{\phi}_1 \leq c_o$  and  $\underline{\phi}_2 \leq c_o$ , the solution to the mediator's problem is simply

$$\gamma^* = \begin{cases} \left[ \left( \frac{\underline{\phi}_2}{\underline{\phi}_1} \right)^{\frac{\kappa}{\kappa-1}} + 1 \right]^{-1} & \text{if } \kappa > 1 \\ 1 & \text{Otherwise} \end{cases}$$

Again, the player expected to be stronger receives a larger share of ex-post surplus. Note also that, as the mediator belief becomes more imprecise (lower  $\kappa$ ), player 1 receives a larger share of ex-post surplus. Whenever  $\kappa \leq 1$ , the objective function is strictly concave and the mediator's problem has a corner solution  $\gamma = 1$ .

Hence, the player who is expected to be stronger prefers a less informative belief (in the sense of lower  $\kappa$  and thicker tails), while the opposite is true for the player who is expected to be weaker. This implies that, for example, the player expected to be weaker would want the mediator to have the ability to gather information and inspect both players, so to have a more precise belief about their power levels. The player expected to be stronger instead would oppose this.

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<sup>15</sup> See, for example, the inspections of IRAN's nuclear sites prior to the 2015 framework agreement.

## 6 Endogenous “originary” power levels.

The fact that in order to reduce wasteful investment the mediator should favor the strongest player opens a possible issue. If the initial power levels are endogenous, then each player has the incentive to become the strongest player in order to obtain a more favorable share of the ex-post surplus, potentially leading to very high level of wasteful investment.

Consider, for example, the model discussed in Section 4 in which the initial power levels are observable by players and mediator. Assume now that the investments are done in two steps: the players can invest before the mediator announces the sharing rule as well as after the announcement. The initial investments are observable by the mediator. By Proposition 2, the mediator sets  $\gamma^* = 1 - \frac{\min\{c_d, c_o\}}{\max\{\phi_1, \phi_2\}}$ , where we allow for either player 1 or 2 to be the strongest depending on their initial investment.

Hence, if the players have the opportunity to make an investment before the mediator announces the sharing rule, the player who is the weakest ex-ante will benefit from making an offensive investment. This way, in the moment the mediator announces the sharing rule, the other player will be weaker than at the start of the game and therefore will receive a lower share of ex-post surplus. Possibly, the weakest player may become the strongest player and receive the majority of the ex-post surplus. The other player will anticipate this and may invest as well. The expectation of the mediator’s intervention leads to wasteful investments before the intervention of the mediator.

The same logic applies also when the mediator does not observe the initial power levels. To show this, we add a stage to the model presented in the previous section: before the power levels are realized, each player can spend resources to affect their minimum power levels. The timeline is now as follows:

1. each player makes his initial offensive and defensive investments. Call these investments  $\hat{o}_i$  and  $\hat{d}_i$  respectively, and assume that they are observable by the mediator. The minimum power levels are determined by

$$\underline{\phi}_i = \hat{\phi}_i e^{-\hat{o}_i} (2 - e^{-\hat{d}_i})$$

where  $\hat{\phi}$  is each player’s “original ex-ante minimum power.”

2. the mediator announces the sharing rule,

3. the power levels  $\phi_1, \phi_2$  are drawn from two Pareto distributions with minimum values  $\phi_1, \phi_2$  and parameters  $\kappa_1, \kappa_2$ . The power levels are not observed by the mediator,<sup>16</sup>
4. the parties make additional investments  $d_i$  and  $o_i$ , also not observable by the mediator,
5. the negotiation starts.

By Proposition 3 whenever  $\kappa_1 = \kappa_2 \equiv \kappa > 1$ , investing in  $\hat{d}_i$  and  $\hat{o}_i$  always results in an increase in the share of ex-post surplus received. As a consequence, if the costs  $\hat{c}_o$  and  $\hat{c}_d$  are not too large, the mediator's intervention may induce the players to invest in  $\hat{d}_i$  and  $\hat{o}_i$ . As  $\kappa$  decreases the sharing rule implemented by the mediator becomes more and more sensitive to the players' relative power, and therefore the incentive to invest in  $\hat{d}_i$  and  $\hat{o}_i$  increases. In the limit case  $\kappa \leq 1$  the mediator allocates the entire ex-post surplus to the player who is expected to be stronger. It follows that in the equilibrium of the ex-ante investment game each player invests with positive probability for any value of  $\hat{c}_o, \hat{c}_d$ . The reason is that, if a player expects the other player not to invest, this player has the incentive to invest a tiny amount and capture the entire ex-post surplus.

To conclude, notice that the results derived in this section hold also when the mediator can announce the sharing rule at the beginning of the game (i.e., before  $\hat{d}_i$  and  $\hat{o}_i$  are set) but cannot commit to it. That is because the mediator will always revise the sharing rule after observing  $\hat{d}_i$  and  $\hat{o}_i$ , leading to the same conclusions we have obtained above. This implies the following remark.

**Remark 1.** *A benevolent mediator who lacks the power to commit to a sharing rule may cause a higher level of social waste than a mediator who simply implements an exogenously given sharing rule.*

Note that, by definition, a mediator who can commit to a sharing rule at the beginning of the game ought to be able to achieve a (weakly) lower level of wasteful investment than a mediator who lacks commitment. Hence, the remark holds

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<sup>16</sup> For simplicity, we focus on the case in which players do not know their power levels when making their initial, observable investment. Otherwise the game becomes a signaling game in which the level of investment may reveal something about each player's power level. The fact that in signaling games players may perform socially wasteful investments is well understood, and not a point we wish to reiterate here.

whenever the exogenously given sharing rule is the one that would be chosen by a benevolent mediator with the power to commit.

## 7 Contest for $\gamma$ : pre-negotiation concessions

We saw that when the mediator has full information he can always eliminate all waste, but this outcome may not be achievable if he does not observe the players' investments. In this section we explore whether the mediator can compensate for this lack of information by conducting a contest for  $\gamma$ , that is, announcing that the sharing rule will depend on visible, costly actions taken by the players. We allow these costly actions to benefit the other player, and therefore interpret them as *concessions*.

Before the negotiation begins the mediator asks each player to make concessions to the other player. Call  $b_1$  the level of concessions made by player 1 and  $b_2$  the level of concessions made by player 2. If player  $i$  makes concessions  $b_i$ , player  $i$  bears a cost equal to  $b_i$  while player  $-i$  enjoys a benefit equal to  $\alpha \cdot b_i \geq 0$ , where  $\alpha \in [0, 1]$ .<sup>17</sup> Note that whenever  $\alpha < 1$ , making a concession generates a welfare loss. The concessions are used by the mediator to set the sharing rule  $\gamma$ , which is now

$$\gamma = f(b_1, b_2)$$

with  $f$  continuous and differentiable in both arguments, increasing and concave in  $b_1$ , decreasing and convex in  $b_2$ . We assume that the function  $f(b_1, b_2)$  and  $\alpha$  are announced by the mediator at the beginning of the game. That is, the mediator can require the players to make specific concessions (corresponding to a given  $\alpha$ ; given costs/benefits to the player making/receiving the concession) and then use the level of concessions to determine  $\gamma$  in a way that is fully anticipated by the players at the beginning of the game.

We first present our argument under the assumption that the initial power levels  $\phi_1$  and  $\phi_2$  are observed by the mediator and can be used to design the function  $f(b_1, b_2)$ . We later argue that if the mediator does not observe the initial power

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<sup>17</sup> Our results can be easily extended to more general expressions for the cost and benefit of concessions. However, for ease of notation, here we assume simple linear functions. Also, in our specification, the cost and benefits of concessions are independent from the underlying conflict. In a more general environment, one could allow the cost and benefit of a concession to depend on the players' initial power levels and on their investments

levels, the contest for  $\gamma$  can be constructed in such a way to induce the players to truthfully reveal  $\phi_1$  and  $\phi_2$ .

### Only offensive investments

In the choice of concession levels, player 1 solves

$$\max_{b_1} \{f(b_1, b_2)(S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1}) - b_1 + \alpha b_2\},$$

with FOC:

$$\frac{\partial f(b_1, b_2)}{\partial b_1} (S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1}) = 1. \quad (10)$$

Similarly, player 2 solves:

$$\max_{b_2} \{(1 - f(b_1, b_2))(S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1}) - b_2 + \alpha b_1\},$$

with FOC

$$- \frac{\partial f(b_1, b_2)}{\partial b_2} (S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1}) = 1. \quad (11)$$

Assuming that both players maximization problems have an internal solution, conditions (10) and (11) define the equilibrium level of concessions  $b_1(o_1, o_2)$  and  $b_2(o_1, o_2)$ .<sup>18</sup> Note that both  $b_1(o_1, o_2)$  and  $b_2(o_1, o_2)$  must be increasing in the offensive investments  $o_1$  and  $o_2$ . The reason is that the higher the offensive investments, the larger the ex-post surplus to be shared in the negotiation, the benefit of obtaining a more favorable surplus split, and therefore the intensity of the competition over  $\gamma$ .

Given this, when deciding on the level of offensive investment, player 1 solves

$$\max_{o_1} \{ \phi_1 e^{-o_2} + f(b_1(o_1, o_2), b_2(o_1, o_2))(S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1}) - c_o o_1 - b_1(o_1, o_2) + \alpha b_2(o_1, o_2) \},$$

with FOC<sup>19</sup>

$$\frac{\partial f(\cdot, \cdot)}{\partial b_2} \frac{\partial b_2(\cdot, \cdot)}{\partial o_1} (S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1}) + \gamma \phi_2 e^{-o_1} + \alpha \frac{\partial b_2(\cdot, \cdot)}{\partial o_1} = c_o.$$

<sup>18</sup> Remember that  $f(b_1, b_2)$  is chosen by the mediator. Hence, making sure that the equilibrium exists, is unique, and each player's problem is interior will be part of the mediator's problem.

<sup>19</sup> By the envelope theorem, we can ignore the effect of  $o_1$  on  $b_1$ .

Using equation 11, the above FOC becomes:

$$\gamma\phi_2 e^{-o_1} - \frac{\partial b_2(\cdot, \cdot)}{\partial o_1}(1 - \alpha) = c_o.$$

Therefore, player 1 anticipates that by investing in  $o_1$ , he will increase the concessions made by player 2 during the contest. This has two effects. First, it directly increases player 1 utility. Second, it increases the share of ex-post surplus accruing to player 2, therefore hurting player 1. If  $\alpha < 1$  the negative effect dominates, and player 1 decreases his investment in offensive technology to reduce the intensity of the contest over  $\gamma$ . If instead  $\alpha = 1$  the two effects cancel out and the contest for  $\gamma$  has no impact on  $o_1$ .

Similarly, player 2's FOC is

$$(1 - \gamma)\phi_1 e^{-o_2} - \frac{\partial b_1(\cdot, \cdot)}{\partial o_2}(1 - \alpha) = c_o.$$

Also in this case, if  $\alpha < 1$  the introduction of the contest reduces the players incentive to invest in offensive technology, while if  $\alpha = 1$  the introduction of the contest has no impact on player 2 investment.

The key observation is that the shape of the function  $f(b_1, b_2)$  matters in two ways. The first derivatives of  $f(b_1, b_2)$  determine the players' concessions and, whenever  $\alpha < 1$ , the welfare loss generated by the contest for  $\gamma$ . The second derivatives of  $f(b_1, b_2)$  determine  $\frac{\partial b_1(\dots)}{\partial o_2}$  and  $\frac{\partial b_2(\dots)}{\partial o_1}$ , that, in turn, determine how each player's concessions react to the other player's investment and the incentive to make offensive investments. The next proposition shows that these two channels can be controlled separately by the mediator in order to achieve zero waste in equilibrium.

**Proposition 4.** *If  $\alpha < 1$ , the mediator can achieve full efficiency and implement any sharing rule  $\gamma \in [0, 1]$ .*

The mediator can achieve zero concessions in equilibrium by setting

$$\frac{\partial f(0, 0)}{\partial b_1} = -\frac{\partial f(0, 0)}{\partial b_2} = \frac{1}{S - \underline{u}_1 - \underline{u}_2}$$

At the same time the mediator sets both  $\frac{\partial^2 f(0, 0)}{\partial b_1^2}$  and  $\frac{\partial^2 f(0, 0)}{\partial b_2^2}$  low, so that concessions are very sensitive to the players' offensive investment. That is, both players expect that if they set positive offensive investment, they will generate a large concession from the opponent. If  $\alpha < 1$  this expectation draws offensive investment to zero.

This mechanism works for any  $\gamma$ , including  $\gamma = \frac{1}{2}$ . It follows that, here, there is no conflict between efficiency and fairness.

Few points are worth noting. First, despite the fact that there is no waste in equilibrium, the contest is effective only if  $\alpha < 1$ . That is, concessions need to be an inefficient way to transfer surplus among players. They cannot be monetary transfers, but should rather be “in kind” transfers. The proposition shows that the mediator can achieve full efficiency as long as he can require the players to make concessions that are wasteful.

Second, although there is no welfare loss in equilibrium, the contest should generate inefficiencies off equilibrium (i.e., for positive offensive investment). Interestingly, here the mediator can easily commit to destroying welfare off equilibrium. The reason is that the mediator does not observe the player’s offensive investment. Hence, following a positive offensive investment, the mediator has no incentive to modify the function  $f(b_1, b_2)$  so to avoid costly concessions.

Third, whereas in the absence of the contest the mediator favors the strongest player potentially leading to a wasteful race to become the strongest player (see Section 6), here any sharing rule can be implemented and achieve zero waste. Hence, the mediator can credibly announce that, even if the players spend resources to modify their initial power level, the sharing rule will not change. This, therefore, eliminates any incentive to become the strongest player. Not only, but similarly to what has been discussed in Section 3, the mediator can credibly announce that the sharing rule will penalize the players in case they spend resources to modify their initial power levels. Hence, the mediator can achieve zero waste also when the initial power level is endogenous.

Finally, the fact that the mediator can observe the initial power levels is here without loss of generality, because the mediator can elicit them from the two players. The mediator can announce that  $\frac{\partial f(\cdot, \cdot)}{\partial b_1} = -\frac{\partial f(\cdot, \cdot)}{\partial b_2}$  (so that the player’s concessions levels are always identical) and  $f(b_1, b_2)|_{b_1=b_2} = \gamma$  (so that the sharing rule implemented is constant). It follows that the players cannot manipulate the allocation of the ex-post surplus by misreporting their power levels. The only effect of misreporting their initial power levels is to, potentially, cause positive concessions and positive offensive investment in equilibrium. However, it is easy to see that no player can benefit from inducing positive offensive investments and positive concession. By reporting the initial power levels, each player can manipulate the function  $f(\cdot, \cdot)$ . Because player  $i$ ’s concessions and investment are optimal given  $f(\cdot, \cdot)$ , by an en-

velope argument manipulating  $f(.,.)$  affects player  $i$ 's utility only because it may induce player  $-i$  to change his behavior. It is however evident that player  $i$  cannot do better than reporting truthfully and inducing player  $-i$  to set both concessions and investments to zero.

## Offensive and defensive investment.

In this case player 1 solves

$$\max_{b_1} \{f(b_1, b_2)(S - \phi_1 e^{-o_2}(2 - e^{-d_1}) - \phi_2 e^{-o_1}(2 - e^{-d_2})) - b_1 + \alpha b_2\}$$

with FOC

$$\frac{\partial f(b_1, b_2)}{\partial b_1} (S - \phi_1 e^{-o_2}(2 - e^{-d_1}) - \phi_2 e^{-o_1}(2 - e^{-d_2})) = 1. \quad (12)$$

Similarly, the FOC corresponding to  $b_2$  os:

$$-\frac{\partial f(b_1, b_2)}{\partial b_2} (S - \phi_1 e^{-o_2}(2 - e^{-d_1}) - \phi_2 e^{-o_1}(2 - e^{-d_2})) = 1. \quad (13)$$

Again, if both players maximization problems have an internal solution, conditions (12) and (13) define the equilibrium level of concessions  $b_1(o_1, o_2, d_1, d_2)$  and  $b_2(o_1, o_2, d_1, d_2)$ . Note that both  $b_1(o_1, o_2, d_1, d_2)$  and  $b_2(o_1, o_2, d_1, d_2)$  are increasing in the offensive investments  $o_1$  and  $o_2$ , but are decreasing in the defensive investments  $d_1$  and  $d_2$ . The intuition is the same discussed earlier. The incentive to make concessions increases with the size of the ex-post surplus to be shared in the negotiation. The ex-post surplus increases with the players' offensive investment but decreases with the players' defensive investment.

Hence, when deciding on the investments levels, player 1 solves

$$\max_{o_1, d_1} \left\{ \left\{ \phi_1 e^{-o_2}(2 - e^{-d_1}) + f(b_1(o_1, o_2), b_2(o_1, o_2))(S - \phi_1 e^{-o_2}(2 - e^{-d_1}) - \phi_2 e^{-o_1}(2 - e^{-d_2})) \right\} - c_o o_1 - c_d d_1 - b_1(o_1, o_2) + \alpha b_2(o_1, o_2) \right\},$$

with FOC for  $o_1$ :

$$\frac{\partial f(.,.)}{\partial b_2} \frac{\partial b_2(.,.)}{\partial o_1} (S - \phi_1 e^{-o_2}(2 - e^{-d_1}) - \phi_2 e^{-o_1}(2 - e^{-d_2})) + \gamma \phi_2 e^{-o_1}(2 - e^{-d_2}) + \alpha_2 \frac{\partial b_2(.,.)}{\partial o_1} = c_o$$

which, again, using 13 becomes:

$$\gamma \phi_2 e^{-o_1}(2 - e^{-d_2}) - \frac{\partial b_2(.,.)}{\partial o_1} (1 - \alpha) = c_o.$$

The FOC for  $d_1$  is:

$$\frac{\partial f(\cdot, \cdot)}{\partial b_2} \frac{\partial b_2(\cdot, \cdot)}{\partial d_1} (S - \phi_1 e^{-o_2} (2 - e^{-d_1}) - \phi_2 e^{-o_1} (2 - e^{-d_2})) + (1 + \gamma) \phi_1 e^{-o_2} e^{-d_1} + \alpha \frac{\partial b_2(\cdot, \cdot)}{\partial d_1} = c_d$$

$$(1 - \gamma) \phi_1 e^{-o_2} e^{-d_1} - \frac{\partial b_2(\cdot, \cdot)}{\partial d_1} (1 - \alpha) = c_d$$

Similarly for player 2, the FOCs for  $o_2$  is:

$$(1 - \gamma) \phi_1 e^{-o_2} (2 - e^{-d_1}) - \frac{\partial b_1(\cdot, \cdot)}{\partial o_2} (1 - \alpha) = c_o,$$

and for  $d_2$  is:

$$\gamma \phi_2 e^{-o_1} e^{-d_2} - \frac{\partial b_1(\cdot, \cdot)}{\partial d_2} (1 - \alpha) = c_d.$$

Also here, when  $\alpha < 1$  the contest over  $\gamma$  decreases the benefit of making an offensive investment. However, the contest over  $\gamma$  simultaneously increases the benefit of making a defensive investment. The intuition is the reverse of what discussed in the previous section. A defensive investment decreases the ex-post surplus to be shared in the contest and therefore the incentive of both players to perform monetary payments. Hence, by making a defensive investment, player  $i$  can decrease  $b_{-i}$  and obtain a higher surplus share during the negotiation. If instead  $\alpha_i = 1$ , the contest for  $\gamma$  has no impact on the investment made by player 1. The next proposition shows that, because of these tradeoffs, contrary to the previous case here the mediator may not be able to eliminate all waste.

**Proposition 5.** *The mediator is able achieve full efficiency and implement the surplus split  $\gamma \in [0, 1]$  if and only if there exists  $A_1 \geq 0$  and  $A_2 \geq 0$  such that*

$$\begin{aligned} 1 - \min \left\{ \frac{c_d}{\phi_1} - (1 - \alpha)A_1, \frac{c_o}{\phi_1} + (1 - \alpha)A_2 \right\} &\leq \\ \gamma &\leq \min \left\{ \frac{c_o}{\phi_2} + (1 - \alpha)A_1, \frac{c_d}{\phi_2} - (1 - \alpha)A_2 \right\} \end{aligned} \quad (14)$$

We showed earlier that without the contest for  $\gamma$  there exists a sharing rule that eliminates all waste if and only if  $\frac{\phi_1 \phi_2}{\phi_1 + \phi_2} \leq \min\{c_o, c_d\}$  (see Proposition 2). Unsurprisingly therefore, under this same condition there exist  $\gamma, A_1, A_2$  for which (14) holds, so that the mediator can eliminate all waste via a contest for  $\gamma$  in all situations in which he can eliminate all waste also without the contest for  $\gamma$ . More interestingly, as long as as concessions are socially wasteful (that is,  $\alpha < 1$ ) the

possibility of running the contest for  $\gamma$  increases the mediator's ability to eliminate waste.<sup>20</sup> However, contrary to the case of only offensive investment, here there are parameter values for which the mediator may not be able to achieve efficiency.<sup>21</sup>

Whenever there is a range of  $\phi_1$  and  $\phi_2$  for which the mediator can credibly maintain the same sharing rule and achieve zero waste, the mediator may be able to eliminate the incentives to manipulate the initial power levels. If the players manipulate their initial power levels and remain within this range, the surplus share allocated to each player will not change, and hence there is no "race" to become the most powerful player. Not only, but also here the mediator may credibly announce that the sharing rule will penalize a player if he spends resources to modify his initial power levels, eliminating the incentives to spend resources in this type of manipulation. However, here the ability of the mediator to achieve this outcome depends on the parameters of the model. It is easy to see that when  $c_o$  and  $c_d$  are large, the range of  $\gamma$  that can be implemented is large, and the mediator is better able to eliminate the players incentives to manipulate their initial power levels. The opposite is true if  $c_o$  and  $c_d$  are small.

Finally, when the initial power levels are not observed by the mediator, the possibility of achieving zero waste depends on the parameters. If (14) holds at the same  $\gamma$  for every possible  $\phi_1$  and  $\phi_2$ , then the logic discussed for the case of only offensive investment continues to hold. The mediator can ask the players to report their power levels, which are then used to determine the shape of  $f(.,.)$  so to generate zero waste. Because the sharing rule can be made independent from the reports, the players have no incentive to misreport the power levels. If instead there is no  $\gamma$  that satisfies (14) for all possible values of  $\phi_1$  and  $\phi_2$ , then this argument will fail, because the mediator cannot commit to maintain the same sharing rule for all possible reports, making truthful reporting impossible.

## 8 Conclusions

We analyze the problem of a benevolent mediator who can set the sharing rule of the mediation so to minimize total pre-negotiation waste. The main result is that the mediator should penalize the weakest player, who is the one with the

<sup>20</sup> For example, if  $\frac{\phi_1\phi_2}{\phi_1+\phi_2} \leq \frac{c_o+c_d}{2}$  there are  $A_1$ ,  $A_2$  and  $\gamma$  that satisfy (14).

<sup>21</sup> For example, if  $\phi_1 = \phi_2$  and  $c_o = c_d$  sufficiently small, then (14) is violated for every  $\gamma \in [0, 1]$  and it is not possible to achieve efficiency.

strongest incentive to undertake wasteful investments. This results remains true under different assumptions on what the mediator can observe, and highlights a conflict between fairness and efficiency. However, the fact that the mediator will favor the strongest player, by itself may provide incentives for wasteful investment prior to the intervention of the mediator. We discuss how the mediator may avoid this problem by organizing a contest for the sharing rule.

Relative to the existing literature on mediation in political science, our paper shows that the mediator can be biased not because of his preferences, but strategically to minimize social waste. Relative to the existing literature on mediation in economics, our paper highlights that the mediator's actions have effects on the players' behavior not only within the negotiation, but also prior to it.

Our analysis suggests several lines for future research. For example, our framework can be used to explore the choice between mediated and unmediated negotiation. Despite the fact that the mediator will favor the strongest player, the weakest player may nevertheless prefer a mediated negotiation over an unmediated one, because of the reduction in wasteful investment. Also, we showed that the precision of the mediator's information affects the sharing rule implemented. Our results suggest that the weakest player benefits from a more informed mediator while the opposite is true for the strongest player, but the full analyses of the strategic choice of transparency remains to be completed. Finally, the ability of the mediator to commit may be key to the reduction of wasteful investment. Analyzing different ways in which the mediator can acquire this commitment remains an open problem.

## A Appendix A: the bargaining game

In period-2, the players exchange offers that can be either accepted or rejected. The mediator does not necessarily observe the content of each offer—which could be placed inside a sealed envelop. However, he can manipulate the outcome of the negotiation by deciding on the order of offers and on the time required for an offer to flow between players.<sup>22</sup>

Consider a continuous-time bargaining game a la Rubinstein (1982). Each player's instantaneous discount factor is  $\beta$ , so that any payoff achieved in  $t$  periods is discounted today by the factor  $e^{-\beta t}$ . The mediator can announce  $t_1$  and  $t_2$ , which are the time required for an offer to go from player 1 to player 2, and from player 2 to player 1 respectively. Define the discount factors

$$\begin{aligned}\delta_1 &= e^{\beta t_1} \\ \delta_2 &= e^{\beta t_2}\end{aligned}$$

It follows from Rubinstein (1982) that, once  $t_1$  and  $t_2$  are chosen, if player 1 makes the first offer the only subgame perfect equilibrium of this game is  $\gamma = \frac{1-\delta_2}{1-\delta_1\delta_2}$ . Hence, any surplus split can be achieved with arbitrarily small waiting time  $\delta_1 \delta_2$ , provided that the *relative* waiting time is chosen appropriately. Furthermore, if the mediator can manipulate also the order of play, then any surplus split can be achieved via finite  $t_1, t_2$ . The agreement is reached as soon as the first offer is delivered to the other player.

## B Appendix B: mathematical derivations

*Proof of Proposition 2.* For  $c_d \geq c_o$ , we argued in the text that there is no defensive investments. Also,  $o_1^*$  is zero if  $\gamma \leq \frac{c_o}{\phi_2}$ , and  $o_2^*$  is zero if  $\gamma \geq 1 - \frac{c_o}{\phi_1}$ . Hence, wasteful investment can be completely eliminated with any  $\gamma \in [1 - \frac{c_o}{\phi_1}, \frac{c_o}{\phi_2}]$  whenever

$$c_o \geq \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}. \quad (15)$$

Suppose now that

$$c_o < \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}.$$

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<sup>22</sup> The point is that the mediator can make waiting for an offer more or less costly. Here we assume that this is done by extending the wait of a player. The mediator could, equivalently, use other tools to make the wait more painful.

For  $\gamma < \frac{c_o}{\phi_2}$  we have that  $o_1 = 0$  and  $o_2$  is strictly decreasing in  $\gamma$ . For  $\gamma > 1 - \frac{c_o}{\phi_1}$  we have that  $o_2 = 0$  and  $o_1$  is strictly increasing in  $\gamma$ . Therefore, it has to be that the waste minimizing  $\gamma \in [\frac{c_o}{\phi_2}, 1 - \frac{c_o}{\phi_1}]$ .

For this range of values, the mediator solves:

$$\begin{aligned} \min_{\gamma \in [\frac{c_o}{\phi_2}, 1 - \frac{c_o}{\phi_1}]} \left\{ \log \left( \frac{\gamma \phi_2}{c_o} \right) + \log \left( \frac{(1 - \gamma) \phi_1}{c_o} \right) \right\} = \\ \min_{\gamma \in [\frac{c_o}{\phi_2}, 1 - \frac{c_o}{\phi_1}]} \left\{ \log(\gamma(1 - \gamma)) + \log \left( \frac{\phi_1 \phi_2}{c_o^2} \right) \right\}. \end{aligned}$$

Hence, the mediator minimizes  $\gamma(1 - \gamma)$  over the relevant interval. It can be verified that when  $\phi_1 \geq \phi_2$ —as we assume throughout—and  $c_o < \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}$  this minimum is always reached at  $\gamma^* = 1 - \frac{c_o}{\phi_1}$ .

For  $c_d < c_o$ , consider the total expenditure fighting over player 2's outside option, with player 1 attacking and player 2 defending:

$$c_o \cdot o_1 + c_d \cdot d_2 = \begin{cases} 0 & \text{if } \gamma \leq \frac{c_d}{\phi_2} \\ c_d \left( \log(\gamma) + \log \left( \frac{\phi_2}{c_d} \right) \right) & \text{if } \frac{c_d}{\phi_2} \leq \gamma \leq \frac{c_o + c_d}{2\phi_2} \\ c_d \log \left( \frac{c_o + c_d}{2c_d} \right) + c_o \left( \log(\gamma) + \log \left( \frac{2\phi_2}{c_o + c_d} \right) \right) & \text{otherwise,} \end{cases}$$

Similarly, consider the total expenditure fighting over player 1's outside option:

$$c_o \cdot o_2 + c_d \cdot d_1 = \begin{cases} 0 & \text{if } 1 - \gamma \leq \frac{c_d}{\phi_1} \\ c_d \left( \log(1 - \gamma) + \log \left( \frac{\phi_1}{c_d} \right) \right) & \text{if } \frac{c_d}{\phi_1} \leq 1 - \gamma \leq \frac{c_o + c_d}{2\phi_1} \\ c_d \log \left( \frac{c_o + c_d}{2c_d} \right) + c_o \left( \log(1 - \gamma) + \log \left( \frac{2\phi_1}{c_o + c_d} \right) \right) & \text{otherwise,} \end{cases}$$

It is easy to verify that whenever  $c_d \geq \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}$ , then any  $\gamma \in [1 - \frac{c_d}{\phi_1}, \frac{c_d}{\phi_2}]$  achieves zero waste. If instead  $c_d < \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}$ , then  $\frac{c_d}{\phi_2} < 1 - \frac{c_d}{\phi_1}$  and we have

$$c_o \cdot (o_1 + o_2) + c_d \cdot (d_1 + d_2) = \begin{cases} \text{strictly decreasing} & \text{if } \gamma \leq \frac{c_d}{\phi_2} \\ \text{strictly concave} & \text{if } \frac{c_d}{\phi_2} \leq \gamma \leq 1 - \frac{c_d}{\phi_1} \\ \text{strictly increasing} & \text{otherwise.} \end{cases} \quad (16)$$

Hence, total expenditure  $c_o \cdot (o_1 + o_2) + c_d \cdot (d_1 + d_2)$  is minimized either at  $\gamma = \frac{c_d}{\phi_2}$ , where the expenditures fighting over 2's outside options is zero, or at  $\gamma = 1 - \frac{c_d}{\phi_1}$  where the expenditures fighting over 1's outside options is zero. At these two values total expenditures are

$$[c_o \cdot (o_1 + o_2) + c_d \cdot (d_1 + d_2)]|_{\gamma = \frac{c_d}{\phi_2}} = c_d \log \left( \min \left\{ \frac{c_o + c_d}{2c_d}, \phi_1 \left( \frac{1}{c_d} - \frac{1}{\phi_2} \right) \right\} \right) + c_o \log \left( \max \left\{ 0, \phi_1 \left( \frac{1}{c_d} - \frac{1}{\phi_2} \right) \frac{2c_d}{c_o + c_d} \right\} \right)$$

$$[c_o \cdot (o_1 + o_2) + c_d \cdot (d_1 + d_2)]|_{\gamma = 1 - \frac{c_d}{\phi_1}} = c_d \log \left( \min \left\{ \frac{c_o + c_d}{2c_d}, \phi_2 \left( \frac{1}{c_d} - \frac{1}{\phi_1} \right) \right\} \right) + c_o \log \left( \max \left\{ 0, \phi_2 \left( \frac{1}{c_d} - \frac{1}{\phi_1} \right) \frac{2c_d}{c_o + c_d} \right\} \right)$$

Because  $c_d < \frac{\phi_1 \phi_2}{\phi_1 + \phi_2}$ , then  $\phi_2 \left( \frac{1}{c_d} - \frac{1}{\phi_1} \right) < \phi_1 \left( \frac{1}{c_d} - \frac{1}{\phi_2} \right)$  and total waste is minimized whenever  $\gamma = 1 - \frac{c_d}{\phi_1}$ . □

*Proof of Lemma 2.* To start, note that if  $\phi_i$  is Pareto-distributed with minimum  $x$  and parameter  $\kappa$ , then  $\log \left( \frac{\phi_i}{x} \right)$  is exponentially distributed with parameter  $\kappa$ . To see this, consider

$$\Pr \left\{ \log \left( \frac{\phi_i}{x} \right) \leq y \right\} = \Pr \{ \phi_i \leq e^y x \}$$

Because  $\phi_i$  is distributed according to a Pareto distribution, the above expression becomes

$$1 - \left( \frac{x}{xe^y} \right)^\kappa = 1 - e^{-y\kappa}$$

which is the CDF of an exponential distribution with parameter  $\kappa$ .

Knowing this, we can compute

$$E \left[ \log \left( \frac{\gamma \phi_2}{c_o} \right) \middle| \phi_2 > \frac{c_o}{\gamma} \right] = \begin{cases} \frac{1}{\kappa_2} & \text{if } \underline{\phi_2} \leq \frac{c_o}{\gamma} \\ E \left[ \log \left( \frac{\gamma \phi_2}{c_o} \right) \right] = \log \left( \frac{\phi_2 \gamma}{c_o} \right) + E \left[ \frac{\phi_2}{\phi_2} \right] = \log \left( \frac{\phi_2 \gamma}{c_o} \right) + \frac{1}{\kappa_2} & \text{otherwise} \end{cases}$$

and similarly for  $E \left[ \log \left( \frac{(1-\gamma)\phi_1}{c_o} \right) \middle| \phi_1 > \frac{c_o}{(1-\gamma)} \right]$ . Finally, using the definition of Pareto distribution we compute

$$\Pr \left( \phi_2 > \frac{c_o}{\gamma} \right) = \begin{cases} \left( \frac{\phi_1 \gamma}{c_o} \right)^{\kappa_2} & \text{if } \underline{\phi_2} \leq \frac{c_o}{\gamma} \\ 1 & \text{otherwise} \end{cases}$$

and similarly for  $\Pr \left( \phi_1 > \frac{c_o}{1-\gamma} \right)$ . □

*Proof of Proposition 3.* The mediator minimizes

$$\begin{cases} A(\gamma) \equiv \left( \frac{\phi_2 \gamma}{c_o} \right)^{\kappa_2} \frac{1}{\kappa_2} + \left( \log \left( \frac{\phi_1 (1-\gamma)}{c_o} \right) + \frac{1}{\kappa_1} \right) & \text{if } \gamma \leq \min \left\{ 1 - \frac{c_o}{\phi_1}, \frac{c_o}{\phi_2} \right\} \\ B(\gamma) \equiv \left( \frac{\phi_2 \gamma}{c_o} \right)^{\kappa_2} \frac{1}{\kappa_2} + \left( \frac{\phi_1 (1-\gamma)}{c_o} \right)^{\kappa_1} \frac{1}{\kappa_1} & \text{if } 1 - \frac{c_o}{\phi_1} \leq \gamma \leq \frac{c_o}{\phi_2} \\ C(\gamma) \equiv \left( \log \left( \frac{\phi_2 \gamma}{c_o} \right) + \frac{1}{\kappa_2} \right) + \left( \log \left( \frac{\phi_1 (1-\gamma)}{c_o} \right) + \frac{1}{\kappa_1} \right) & \text{if } \frac{c_o}{\phi_2} \leq \gamma \leq 1 - \frac{c_o}{\phi_1} \\ D(\gamma) \equiv \left( \log \left( \frac{\phi_2 \gamma}{c_o} \right) + \frac{1}{\kappa_2} \right) + \left( \frac{\phi_1 (1-\gamma)}{c_o} \right)^{\kappa_1} \frac{1}{\kappa_1} & \text{if } \max \left\{ 1 - \frac{c_o}{\phi_1}, \frac{c_o}{\phi_2} \right\} < \gamma \end{cases}$$

Whenever  $\kappa_1, \kappa_2 \rightarrow \infty$ , the uncertainty about the players power level disappears. The solution to the mediator's problem is the one derived in Section 4.

Taking the derivative of the mediator's objective function with respect to  $\gamma$  we get:

$$\begin{cases} A'(\gamma) \equiv \left(\frac{\phi_2 \gamma}{c_o}\right)^{\kappa_2} \frac{1}{\gamma} - \frac{1}{1-\gamma} & \text{if } \gamma \leq \min\{1 - \frac{c_o}{\phi_1}, \frac{c_o}{\phi_2}\} \\ B'(\gamma) \equiv \left(\frac{\phi_2 \gamma}{c_o}\right)^{\kappa_2} \frac{1}{\gamma} - \left(\frac{\phi_1(1-\gamma)}{c_o}\right)^{\kappa_1} \frac{1}{1-\gamma} & \text{if } 1 - \frac{c_o}{\phi_1} \leq \gamma \leq \frac{c_o}{\phi_2} \\ C'(\gamma) \equiv \frac{1}{\gamma} - \frac{1}{1-\gamma} & \text{if } \frac{c_o}{\phi_2} \leq \gamma \leq 1 - \frac{c_o}{\phi_1} \\ D'(\gamma) \equiv \frac{1}{\gamma} - \left(\frac{\phi_1(1-\gamma)}{c_o}\right)^{\kappa_1} \frac{1}{1-\gamma} & \text{if } \max\{1 - \frac{c_o}{\phi_1}, \frac{c_o}{\phi_2}\} < \gamma. \end{cases} \quad (17)$$

which is continuous in  $\gamma$ . We solve the mediator's problem by considering few separate cases:

- $\kappa_1, \kappa_2 \leq 1$ . In this case  $A(\gamma)$ ,  $B(\gamma)$ ,  $C(\gamma)$  and  $D(\gamma)$  are all concave. By continuity of 17, the solution can only be at the extremes, and hence the waste-minimizing sharing rule is

$$\gamma^* = \begin{cases} 1 & \text{if } \frac{1}{\kappa_2} - \frac{1}{\kappa_1} < \log\left(\frac{\phi_1}{\phi_2}\right) \\ 0 & \text{otherwise} \end{cases}$$

- $\kappa_1, \kappa_2 > 1$ . In this case  $A'(0) < 0$  and  $D'(1) > 0$  and therefore the solution is never an extreme value. If, furthermore  $c_o > \phi_1 > \phi_2$ , then the mediator problem is to minimize  $B(\gamma)$ , which is convex. Hence the solution to the mediator's problem is

$$\gamma^* : B'(\gamma^*) = 0$$

If instead  $\phi_1, \phi_2 \rightarrow \infty$ , the mediator's objective function converges to  $C(\gamma)$ , which is concave. By continuity, the solution to the mediator's problem converges to

$$\gamma^* = \begin{cases} 1 & \text{if } \frac{1}{\kappa_2} - \frac{1}{\kappa_1} < \log\left(\frac{\phi_1}{\phi_2}\right) \\ 0 & \text{otherwise} \end{cases}$$

which is the  $\gamma$  minimizing  $C(\gamma)$ .

To characterize the solution to the mediator's problem in all other cases, we take the derivative of 17 with respect to  $\phi_1$ ,  $\phi_2$ ,  $\kappa_1$ ,  $\kappa_2$  and then invoke Topkis's theorem.

The derivative of 17 with respect to  $\underline{\phi}_1$  is:

$$\begin{cases} 0 & \text{if } \gamma < 1 - \frac{c_o}{\underline{\phi}_1} \\ -\kappa_1 \left( \frac{\underline{\phi}_1}{c_o} (1 - \gamma) \right)^{\kappa_1 - 1} \left( \frac{1}{c_o} \right)^{\kappa_1} & \text{otherwise} \end{cases}$$

which is weakly negative. The derivative of 17 with respect to  $\underline{\phi}_2$  is:

$$\begin{cases} \kappa_2 \left( \frac{\underline{\phi}_2}{c_o} \gamma \right)^{\kappa_2 - 1} \left( \frac{1}{c_o} \right)^{\kappa_2} & \text{if } \gamma < \frac{c_o}{\underline{\phi}_2} \\ 0 & \text{otherwise} \end{cases}$$

which is weakly positive. The derivative of 17 with respect to  $\kappa_2$  is

$$\begin{cases} \left( \frac{\underline{\phi}_2}{c_o} \right)^{\kappa_2} \gamma^{\kappa_2 - 1} \log \left( \frac{\underline{\phi}_2 \gamma}{c_o} \right) & \text{if } \gamma \leq \frac{c_o}{\underline{\phi}_2} \\ 0 & \text{otherwise.} \end{cases}$$

which is weakly negative. The derivative of 17 with respect to  $\kappa_1$  is

$$\begin{cases} - \left( \frac{\underline{\phi}_1}{c_o} \right)^{\kappa_1} (1 - \gamma)^{\kappa_1 - 1} \log \left( \frac{\underline{\phi}_1 (1 - \gamma)}{c_o} \right) & \text{if } 1 - \frac{c_o}{\underline{\phi}_1} \leq \gamma \\ 0 & \text{otherwise.} \end{cases}$$

which is weakly positive. By Topkis's theorem, therefore, the waste-minimizing sharing rule is weakly increasing in  $\underline{\phi}_1$ ,  $\kappa_2$ ; weakly decreasing in  $\underline{\phi}_2$ ,  $\kappa_1$ .

□

*Proof of Proposition 4.* If the mediator announces a  $f(b_1, b_2)$  such that

$$\frac{\partial f(0, 0)}{\partial b_1} = - \frac{\partial f(0, 0)}{\partial b_2} = \frac{1}{S - \phi_1 - \phi_2}$$

and offensive investments are zero, then both equilibrium concessions are zero.

Furthermore, by the implicit function theorem

$$\begin{aligned} b'_1(o_2) \frac{\partial^2 f(b_1, b_2)}{\partial b_1^2} &= \frac{-\phi_1 e^{-o_2}}{(S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1})} \\ -b'_2(o_1) \frac{\partial^2 f(b_1, b_2)}{\partial b_2^2} &= \frac{-\phi_2 e^{-o_1}}{(S - \phi_1 e^{-o_2} - \phi_2 e^{-o_1})} \end{aligned}$$

which, evaluated at zero offensive investment and zero concessions become

$$b'_1(o_2) = A_1 \phi_1$$

$$b'_2(o_1) = A_2\phi_2$$

where

$$A_1 \equiv -\frac{1}{\frac{\partial^2 f(0,0)}{\partial b_1^2}(S - \phi_1 - \phi_2)} > 0$$

$$A_2 \equiv \frac{1}{\frac{\partial^2 f(0,0)}{\partial b_2^2}(S - \phi_1 - \phi_2)} > 0.$$

Note that the mediator can set  $A_1$  and  $A_2$  to any strictly positive value by manipulating  $\frac{\partial^2 f(0,0)}{\partial b_1^2}$  and  $\frac{\partial^2 f(0,0)}{\partial b_2^2}$

It follows that offensive investment by player 2 is zero whenever

$$(1 - \gamma)\phi_1 - A_1\phi_1(1 - \alpha) \leq c_o$$

which is always satisfied for any  $\gamma = f(0,0)$  by setting  $A_1$  sufficiently large, that is, if  $-\frac{\partial^2 f(0,0)}{\partial b_1^2}$  is sufficiently small (remember that  $\frac{\partial^2 f(0,0)}{\partial b_1^2} < 0$ ). Similar steps show that player 1 offensive investment is also zero if

$$\gamma\phi_2 - A_2\phi_2(1 - \alpha) \leq c_o$$

which is also satisfied if  $A_2$  is sufficiently large, that is, if  $\frac{\partial^2 f(0,0)}{\partial b_2^2}$  is sufficiently small.  $\square$

*Proof of Proposition 5.* Following the same steps as in Proposition 4, we can show that, also here, if offensive investment is zero concessions are zero whenever

$$\frac{\partial f(0,0)}{\partial b_1} = -\frac{\partial f(0,0)}{\partial b_2} = \frac{1}{S - \phi_1 - \phi_2}$$

Furthermore, by the implicit function theorem we have that, when all investments are zero and concessions are zero:

$$\frac{\partial b_1(\cdot, \cdot, \cdot, \cdot)}{\partial o_2} = \phi_1 A_1$$

$$\frac{\partial b_1(\cdot, \cdot, \cdot, \cdot)}{\partial d_2} = -\phi_2 A_1$$

$$\frac{\partial b_2(\cdot, \cdot, \cdot, \cdot)}{\partial o_1} = \phi_2 A_2$$

$$\frac{\partial b_2(\cdot, \cdot, \cdot, \cdot)}{\partial d_1} = -\phi_1 A_2$$

where

$$A_1 = -\frac{1}{\frac{\partial^2 f(0,0)}{\partial b_1^2} (S - \phi_1 - \phi_2)^2} \geq 0.$$

$$A_2 = \frac{1}{\frac{\partial^2 f(0,0)}{\partial b_2^2} (S - \phi_1 - \phi_2)^2} \geq 0.$$

Note that the mediator can achieve any value  $A_1 \geq 0$  and  $A_2 \geq 0$  by manipulating the function  $f(., .)$ .

Hence, using the players' FOCs, investments will be zero whenever

$$\gamma\phi_2 - \phi_2 A_1(1 - \alpha) \leq c_o.$$

$$(1 - \gamma)\phi_1 + \phi_1 A_1(1 - \alpha) \leq c_d$$

$$(1 - \gamma)\phi_1 - \phi_1 A_2(1 - \alpha) \leq c_o,$$

$$\gamma\phi_2 + \phi_2 A_2(1 - \alpha) \leq c_d.$$

where  $\gamma = f(0, 0)$  is the surplus share implemented in equilibrium.

These four conditions together are equivalent to

$$\begin{aligned} 1 - \min \left\{ \frac{c_d}{\phi_1} - (1 - \alpha)A_1, \frac{c_o}{\phi_1} + (1 - \alpha)A_2 \right\} &\leq \\ \gamma &\leq \min \left\{ \frac{c_o}{\phi_2} + (1 - \alpha)A_1, \frac{c_d}{\phi_2} - (1 - \alpha)A_2 \right\} \end{aligned} \quad (18)$$

Hence, there is a contest with zero investments and zero transfers if and only if  $\alpha < 1$  and there is  $A_1 \geq 0$  and  $A_2 \geq 0$  that satisfies the above condition.

□

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