Unemployment insurance, human capital and financial markets

PRELIMINARY AND INCOMPLETE

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Abstract

I characterize optimal unemployment insurance in the presence of human capital life-cycle trends and incomplete financial markets. Each worker is subject to unemployment risk, and exerts unobservable effort either to keep her job (if employed) or to find one (if unemployed). Human capital accumulates when she is employed, while depreciates when unemployed. She has access to financial (incomplete) markets, where she can buy or sell risk-free bonds at a constant interest rate to self-insure against unemployment risk. Trading in the financial market is not observable. I show by numerical examples that the optimal allocation can be decentralized with a system of unemployment insurance savings accounts (UISA). The deposit rate of the saving accounts is almost flat during employment spells, while the withdrawal rate slightly decreases during unemployment.

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1 Introduction

This paper provides a first attempt at analyzing optimal unemployment insurance with lifetime human capital trends and hidden access to financial markets. There has been a lot of discussion about the possibility of substituting the current unemployment insurance systems with an unemployment insurance savings accounts (UISAs) scheme (see e.g. Altman and Feldstein (2007)). Chile has recently implemented UISAs (Reyes Hartley, van Ours and Vodopivec (2010)), and other Latino-American countries are following their example.

In this paper, I show that a UISA system similar to the one implemented in Chile arises naturally as a decentralized equilibrium that implements the optimal allocation in an economy with human capital trends and hidden access to financial markets. Each worker in the economy has an initial endowment of human capital which increases through learning-by-doing when she is working, while depreciates when she is unemployed. The worker has access to financial markets, and can save or borrow at a constant return rate. When the worker is employed, she has to exert unobservable effort that affects her job retaining probability. When unemployed, the hidden effort affects the probability of finding a new job.

The optimal allocation is obtained by solving a constrained optimization problem where the planner takes into account incentive compatibility. This optimal allocation has a continuum of decentralized implementations, from the extreme in which access to financial markets is prevented completely and the planner expropriates the agent of her income and transfers part of it, to the other extreme in which the worker is left without any transfers from the planner, but can have access to financial markets.

The latter implementation is equivalent to a UISA scheme. Therefore I focus my attention to it. Numerical examples show that this scheme features an almost flat deposit rate when the worker is employed, and a slightly decreasing withdrawal rate when unemployed.

This finding is in opposition to the result of Shimer and Werning (2008) that simple policies with constant benefits financed by constant taxes are optimal (or very close to optimality). In their work, they do have hidden savings, however they do not keep track of the human capital. The result of this paper therefore implies that any policy recommendation that does not take into account the human capital trends is biased and far from optimal. Moreover, this paper constitutes a rationale for setting up a system based on unemployment insurance saving accounts (UISAs) of the type implemented in Chile, where withdrawals from the UISA when unemployed is decreasing through unemployment spell.

A secondary contribution of this paper is methodological: given that the problem is difficult to analyze with standard techniques, the solution strategy followed here implements the recursive Lagrangean method suggested by Mele (2010), which is particularly useful for dynamic agency problems with several endogenous state variables.

The seminal work of Hopenhayn and Nicolini (1997) suggests that an incentive compatible system of unemployment insurance must have decreasing unemployment benefit during unemployment spell. Two assumptions are crucial: first, there is no human capital accumulation or depreciation in their setup, and second, the worker has no access to financial markets.

However, it is a well documented fact that human capital depreciates during unemployment spells while increases during employment tenure[1]. Moreover, there is also evidence that transition probabilities depend on the length of unemployment or employment spell[2].

On the other hand, the assumption of no access to financial markets seems extreme too[3]. However, there are few works that look at the effects of human capital and savings on the optimal unemployment insurance system. Pavoni (2009) shows that, in a model of UI with human capital depreciation and only two possible levels of effort, the result

[1] For example, Keane and Wolpin (1997) find an annual human capital depreciation rate for white US males during unemployment of 9.6% for blue collars and 36.5% for white collars. Pavan (2009) estimates the contribution of human capital trends on wage dynamics in a model that takes into account a distinction between general, career-specific and firm-specific human capital. Liu (2009) shows that, in a model with human capital and job search, 40% of wage growth over the life cycle can be explained by human capital accumulation

[2] van den Berg and van Ours (1994,1996) and Bover, Arellano and Bentolila (2002) find negative duration dependence in the unemployment hazard rate. It is also well documented that workers with different levels of human capital have different unemployment hazard rates

[3] Gruber (2001) shows that median US workers can finance around two thirds of income loss associated with unemployment. We also know that the diffusion of credit cards and informal credit markets can help workers to self-insure themselves against unemployment risk. All these forms of insurance are partial, but they can provide liquid funds during unemployment. Chetty (2008) makes the distinctions between two channels through which the unemployment benefits affect workers' behavior: there is a liquidity effect (i.e., unemployment benefits help workers to finance their unemployment spell) and standard moral hazard effect (i.e., unemployment benefits discourage intense job search). He estimate that, given current benefits in US, the liquidity effect accounts for 60 percent of the marginal effect on unemployment duration.

of Hopenhayn and Nicolini (1997) of a decreasing profile for unemployment benefits survives, but there is a point at which benefits stop decreasing and are kept constant by the principal. This is due to depreciation of human capital during unemployment spells: for very low levels of human capital, the benevolent government has no interest in inducing the worker to find a new job, since both the probability of finding a job and the wage he will be receiving are too low. In my paper, there is continuum effort and the average trend for the human capital is increasing, therefore this result disappears.

Rendahl (2009) analyzes a model of optimal unemployment insurance in which workers can save and borrow, but they face a debt limit. He shows how the benefits must depend on the wealth of the unemployed, giving therefore support for means-tested insurance schemes. Abraham and Pavoni (2008) characterize optimal allocation in a model with repeated moral hazard and hidden access to financial markets. In their setup, there is no human capital. The agent can save or borrow at a constant exogenous interest rate, but the principal cannot observe asset trades in the market. Under these assumptions, optimal unemployment benefits do not need to be decreasing, and they might even be increasing during unemployment[4]. A similar result is obtained by Shimer and Werning (2008) in a McCall search model.

Other papers have focused on the feasibility of an UISA system. Altman and Feldstein (2005) show that an UISA system for US is implementable and brings savings for the government with respect to the current system. Setty (2011) compares a standard unemployment insurance system with a UISA system in a calibrated model for the US economy and finds that UISAs are welfare improving with respect to a UI system for plausible parametrizations. However, in these papers the UISA system is assumed exogenous and not optimally chosen. In particular, the UISA system is given by a constant deposit rate and a constant withdrawal rate, while I characterize the optimal nonlinear deposit and withdrawal rate.

In this paper, I develop a model of optimal unemployment insurance with human capital trends and hidden asset markets, building on the work of Pavoni (2009) and Abraham and Pavoni (2008). In my model, the worker can be employed or unemployed. I do not assume employment is an absorbing state as in Hopenhayn and Nicolini (1997): there can be alternate spells of unemployment and employment. The transition proba-

[4]The intuition is that it is consumption that must be decreasing during unemployment spell, but since the government cannot perfectly control the consumption profile due to the possibility to save or borrow, this can be obtained even with a quite flat unemployment benefit schedule for some reasonable parameterization.

bility depends on unobservable effort[5] and on the level of human capital of the worker (as in Pavoni (2009)). Moreover the worker has access to a hidden asset market as in Abraham and Pavoni (2008): she can buy or sell a risk-free bond with constant exogenous interest rate.

This model is extremely difficult to analyze: the standard promised utilities approach is not easily applicable to it, given the large number of state variables and the presence of an hidden endogenous state (bond holdings). Therefore, an important methodological contribution is the use of recursive Lagrangean techniques in the spirit of Mele (2010). I show how the model can be solved by writing the Lagrangean of the government's dynamic optimization problem. I then prove that the Lagrangean has a recursive structure, where the state variables are human capital, employment states and two costate variables derived from the Lagrange multipliers. Given this recursive structure, it is very easy to solve the model using standard numerical techniques. In particular, I use a collocation algorithm to solve the Lagrangean first-order conditions. However, this methodology crucially depends on the validity of the first-order approach, but there are no sufficient conditions that guarantee it. Therefore, I use a verification procedure similar to Abraham and Pavoni (2009) to numerically check if the first-order approach is justified.

I then provide numerical examples of decentralization of the optimal allocation through an optimal nonlinear UISA scheme. The scheme has two main features: the deposit rate is almost flat, while the withdrawal rate is decreasing during unemployment. On average, the system has a positive balance and therefore it is possible to lend money to those few unemployed workers that have negative balances.

Moreover, the system is "progressive": high skilled worker have higher deposit rates and lower withdrawal rates. This is not due to some redistributive concerns, but it is entirely driven by incentives considerations: the high skilled workers face lower withdrawal in order to put higher effort and find a job faster.

Finally, the system gives higher withdrawal rates to young workers, since old workers are both richer and with higher human capital and therefore have incentives to put low search effort.

The paper is organized as follows. Section 2 presents the problem in presence of human capital trends and hidden access to financial markets, and shows how to solve

[5] Effort during unemployment is the classical search effort. During employment spells, it can be interpreted as job retention effort, as in Wang and Williamson (1996).

it with the Lagrangean approach developped in Mele (2010). Section 3 proves the recursivity of the Lagrangean. Section 4 characterizes analytically the optimal contract. Section 5 explains the implementation of the optimal allocation with a UISA system. Section 6 presents numerical examples and Section 7 concludes.

2 Unemployment insurance with human capital and hidden savings

In this Section, I present the basic framework. The worker is endowed with human capital, and his labor wage depends on it. Initial human capital is given and equal to h_{-1}. There are two possible states of the world: $S \equiv \{U, E\}$ where U means unemployed and E means employed. Human capital increases during employment spells, while it depreciates during unemployment periods, according to the following process:

$$h_t\left(s^t\right) = \begin{cases} (1+\rho)\, h_{t-1}\left(s^{t-1}\right) & s_t = E \\ \\ (1-\delta)\, h_{t-1}\left(s^{t-1}\right) & s_t = U \end{cases}$$

Notice that human capital evolves according to an exogenous process. We can interpret the increase in human capital while working as learning-by-doing effects, while the depreciation during unemployment comes from obsolescence of knowledge, for example.

The wage is function of the human capital of the worker

$$y_t\left(s_t, h_{t-1}\left(s^{t-1}\right)\right) = \begin{cases} F\left(h_{t-1}\left(s^{t-1}\right)\right) & s_t = E \\ \\ 0 & s_t = U \end{cases}$$

As in Pavoni (2009), I assume that the unobservable effort of the worker affects the transition probability between the two possible state of nature, but those probabilities also depend on the human capital through the function $\pi\left(s_{t+1} \mid s_t, h_{t-1}\left(s^{t-1}\right), a_t\left(s^t\right)\right)$. I assume independence across periods and therefore I denote by $\Pi\left(s^t \mid s_0, h^{t-1}\left(s^{t-1}\right), a^t\left(s^t\right)\right)$ the probability of a history s^t. Another natural assumption is that $\pi(\cdot)$ is increasing in human capital, as the empirical evidence would suggest. In this setup, the worker exerts effort when employed to increase the probability to keep the job, while when unemployed effort increases the probability of finding a job. This work therefore departs from Pavoni (2009) by assuming continuous effort choice. I also depart from Hopenhayn and Nicolini (1997): they assume that employment is an absorbing state, while I allow for multiple spells of employment and unemployment, as in Zhao (2001) and in Wang and Williamson (1996, 2002).

In terms of consumption-saving decisions, there are two main departures from Hopenhayn and Nicolini (1997). Firstly, the worker can buy or sell a risk-free bond at a constant interest rate R. Her holdings of this bond at the beginning of period t will be indicated as $b_{t-1}(s^{t-1})$. In this way, the worker can self-insure herself only partially, given that financial markets are not complete[6]. Secondly, bond holdings are not observable by the planner. In this sense, the model is very similar to Abraham and Pavoni (2009). This last assumption makes the characterization of the optimal scheme very challenging: not only there is a moral hazard problem since search effort is unobservable, but we cannot say which worker is rich or poor given that bond holdings are unobservable. In particular, there is a continuum of wealth types and therefore the standard promised utility approach is unfeasible. Abraham and Pavoni (2009) suggest a way to overcome this issue by using a first-order approach, i.e. using the agent's first-order conditions with respect to unobservable variables as a substitute for the incentive-compatibility constraint in the planner problem.

I follow a different route. In order to characterize the optimal scheme, I use the recursive Lagrangean techniques developed in Mele (2010). This methodology is particularly useful in dynamic agency problems with observable endogenous state variables, and it turns out that it also very convenient in the presence of unobservable endogenous states (which is the case in my framework). The method consists of adopting a first-order approach, as in Abraham and Pavoni (2008). However, the planner problem is solved by means of Lagrangean techniques, which happen to have a recursive structure and therefore are easily solvable with standard numerical techniques. Mele (2010) shows that this approach is suitable for a dynamic agency model with hidden savings, provided that we verify numerically the validity of the first-order approach.

Given my assumptions, I can derive agent's first-order conditions with respect to

[6] In order to be able to pick only the effect of self-insurance, the assumption is that there is a borrowing constraint that never binds (i.e., the worker can borrow almost freely). If the borrowing constraint were occasionally binding, there will be a second effect on the consumption-saving decision that depends on liquidity constraints. I plan to explore what happens if borrowing constraints are occasionally binding, as for example in Rendahl (2009).

unobservable variables. The agent's problem is:

$$
\begin{aligned}
& V\left(s_0, h_{-1}, b_{-1} ; \tau^{\infty}\right)= \\
& =\max _{\left\{c_t\left(s^t\right), a_t\left(s^t\right), b_t\left(s^t\right)\right\}_{t=0}^{\infty}}\left\{\sum_{t=0}^{\infty} \sum_{s^t} \beta^t\left[u\left(c_t\left(s^t\right)\right)-v\left(a_t\left(s^t\right)\right)\right] \times\right. \\
& \left.\times \Pi\left(s^t \mid s_0, h^{t-2}\left(s^{t-2}\right), a^{t-1}\left(s^{t-1}\right)\right)\right\} \\
& \text {s.t.} \quad c_t\left(s^t\right)+b_t\left(s^t\right)=y_t\left(s_t, h_{t-1}\left(s^{t-1}\right)\right)+\tau_t\left(s^t\right)+R b_{t-1}\left(s^{t-1}\right) \\
& h_t\left(s^t\right)=\begin{cases}(1+\rho) h_{t-1}\left(s^{t-1}\right) & s_t=E \\
(1-\delta) h_{t-1}\left(s^{t-1}\right) & s_t=U\end{cases} \\
& h_t\left(s^t\right) \geq 0 \qquad h_{-1}, b_{-1} \quad \text{given}
\end{aligned}
$$

We interpret $\tau_t\left(s^t\right)$ in the following way: when the worker is unemployed, $\tau_t\left(s^t\right)$ is the unemployment benefit; when the worker is employed, $\tau_t\left(s^t\right)$ is a tax or a transfer from the principal. We can derive agent's first order conditions with respect to the effort:

$$
\begin{aligned}
v'\left(a_t\left(s^t\right)\right)= & \sum_{j=1}^{\infty} \beta^j \sum_{s^{t+j} \mid s^t} \frac{\pi_a\left(s_{t+1} \mid s_t, a_t\left(s^t\right)\right)}{\pi\left(s_{t+1} \mid s_t, a_t\left(s^t\right)\right)} \times \\
& \times\left[u\left(c_{t+j}\left(s^{t+j}\right)\right)-v\left(a_{t+j}\left(s^{t+j}\right)\right)\right] \Pi\left(s^{t+j} \mid s^t, h^{t+j-1}\left(s^{t+j-1} \mid s^{t-1}\right), a^{t+j}\left(s^{t+j} \mid s^t\right)\right)
\end{aligned} \tag{1}
$$

and the first order condition with respect to bonds:

$$
u'\left(c_t\left(s^t\right)\right)=\beta R \sum_{s^{t+1} \mid s^t} u'\left(c_{t+1}\left(s^{t+1}\right)\right) \pi\left(s_{t+1} \mid s_t, h_{t-1}\left(s^{t-1}\right), a_t\left(s^t\right)\right) \tag{2}
$$

Mele (2010) shows that we can see the planner problem as a social welfare maximization, in which the weight assigned to the worker depends on the minimal lifetime utility that the planner want to deliver through the unemployment insurance system. Call this weight γ. Let $\beta^t \lambda_t\left(s^t\right)$ the Lagrange multiplier for (1), and $\beta^t \eta_t\left(s^t\right)$ the Lagrange multiplier for (2). Moreover, define:

$$
\begin{aligned}
& \phi_{t+1}\left(s^t, \widehat{s}\right)=\phi_t\left(s^t\right)+\lambda_t\left(s^t\right) \frac{\pi_a\left(s_{t+1}=\widehat{s} \mid s_t, h_{t-1}\left(s^{t-1}\right), a_t\left(s^t\right)\right)}{\pi\left(s_{t+1}=\widehat{s} \mid s_t, h_{t-1}\left(s^{t-1}\right), a_t\left(s^t\right)\right)} \quad \forall \widehat{s} \in S \\
& \phi_0\left(s^0\right)=\gamma
\end{aligned}
$$

and

$$
\zeta_{t+1}\left(s^t, \widehat{s}\right)=\eta_t\left(s^t\right) \quad \forall \widehat{s} \in S \quad \text{and} \quad \zeta_0\left(s^0\right)=0
$$

The value of γ in this context is the analogous of the initial lifetime utility that the optimal contract delivers to the worker (see for example Hopenhayn and Nicolini (1997)). It is easy to show that there is a one-to-one correspondence between the choice of initial lifetime utility and the parameter γ. With few lines of tedious algebra, I can write the Lagrangean of this problem in the following form:

$$\begin{aligned}
&\mathcal{L}\left(s_0, \gamma, c^\infty, a^\infty, h^\infty, \lambda^\infty, \eta^\infty, \phi^\infty, \zeta^\infty\right) = \\
&= \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ y\left(s_t\right) - c_t\left(s^t\right) + \phi_t\left(s^t\right)\left[u\left(c_t\left(s^t\right)\right) - \upsilon\left(a_t\left(s^t\right)\right)\right] + \right. \\
&\quad \left. - \lambda_t\left(s^t\right)\upsilon'\left(a_t\left(s^t\right)\right) + \left[\eta_t\left(s^t\right) - R\zeta_t\left(s^t\right)\right] u_c\left(c_t\left(s^t\right)\right)\right\} \times \\
&\quad \times \Pi\left(s^t \mid s_0, h^{t-2}\left(s^{t-2}\right), a^{t-1}\left(s^{t-1}\right)\right)
\end{aligned} \tag{3}$$

where

$$h_t\left(s^t\right) = \begin{cases} (1+\rho)\, h_{t-1}\left(s^{t-1}\right) & s_t = E \\ (1-\delta)\, h_{t-1}\left(s^{t-1}\right) & s_t = U \end{cases}, \qquad h_{-1} \quad \text{given}$$

3 Recursivity

The Lagrangean (3) has a recursive representation that uses ϕ_t, ζ_t, h_{t-1} and the occupational status s_t as state variables. In order to show that, let me work with a slightly more general version of the problem:

$$\begin{aligned}
&W_\theta^{SWF}\left(s_0, h_{-1}\right) = \\
&\max_{\{a_t(s^t), c_t(s^t)\}_{t=0}^{\infty} \in \Gamma^{HA}} \overline{\phi}^0 \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left[y\left(h_{t-1}, s_t\right) - c_t\left(s^t\right)\right] \Pi\left(s^t \mid s_0, h^{t-2}, a^{t-1}\left(s^{t-1}\right)\right) + \\
&\qquad + \gamma \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left(u\left(c_t\left(s^t\right)\right) - \upsilon\left(a_t\left(s^t\right)\right)\right) \Pi\left(s^t \mid s_0, h^{t-2}, a^{t-1}\left(s^{t-1}\right)\right) \\
&s.t. \quad \upsilon'\left(a_t\left(s^t\right)\right) = \sum_{j=1}^{\infty} \beta^j \sum_{s^{t+j} \mid s^t} \frac{\pi_a\left(s_{t+1} \mid s_t, h_{t-1}, a_t\left(s^t\right)\right)}{\pi\left(s_{t+1} \mid s_t, h_{t-1}, a_t\left(s^t\right)\right)} \times \\
&\qquad \times \left[u\left(c_{t+j}\left(s^{t+j}\right)\right) - \upsilon\left(a_{t+j}\left(s^{t+j}\right)\right)\right] \Pi\left(s^{t+j} \mid s^t, h^{t+j-2}, a^{t+j-1}\left(s^{t+j-1} \mid s^t\right)\right) \\
&\qquad \forall s^t, t \geq 0 \\
&\qquad u'\left(c_t\left(s^t\right)\right) = \beta R \sum_{s_{t+1}} u'\left(c_{t+1}\left(s^t, s_{t+1}\right)\right) \pi\left(s_{t+1} \mid s_t, h_{t-1}, a_t\left(s^t\right)\right)
\end{aligned}$$

I can associate a Lagrangean to this generalized problem:

$$\begin{aligned} & L\left(s_0, \gamma, c^\infty, a^\infty, h^\infty, \lambda^\infty, \eta^\infty, \phi^\infty, \zeta^\infty\right) \\ & \quad = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ \phi^0 \left[y\left(s_t, h_{t-1}\right) - c_t\left(s^t\right)\right] + \phi_t\left(s^t\right)\left[u\left(c_t\left(s^t\right)\right) - \upsilon\left(a_t\left(s^t\right)\right)\right] + \right. \\ & \qquad \left. - \lambda_t\left(s^t\right)\upsilon'\left(a_t\left(s^t\right)\right) + \left[\eta_t\left(s^t\right) - R\zeta_t\left(s^t\right)\right] u_c\left(c_t\left(s^t\right)\right)\right\} \times \\ & \qquad \times \Pi\left(s^t \mid s_0, h^{t-2}\left(s^{t-2}\right), a^{t-1}\left(s^{t-1}\right)\right) \end{aligned} \tag{4}$$

where

$$h_t\left(s^t\right) = \begin{cases} (1+\rho)\, h_{t-1}\left(s^{t-1}\right) & s_t = E \\ (1-\delta)\, h_{t-1}\left(s^{t-1}\right) & s_t = U \end{cases}, \qquad h_{-1} \quad \text{given}$$

$$\phi_{t+1}\left(s^t, \widehat{s}\right) = \phi_t\left(s^t\right) + \lambda_t\left(s^t\right) \frac{\pi_a\left(s_{t+1} = \widehat{s} \mid s_t, h_{t-1}\left(s^{t-1}\right), a_t\left(s^t\right)\right)}{\pi\left(s_{t+1} = \widehat{s} \mid s_t, h_{t-1}\left(s^{t-1}\right), a_t\left(s^t\right)\right)} \quad \forall \widehat{s} \in S$$

$$\phi_0\left(s^0\right) = \gamma$$

and

$$\zeta_{t+1}\left(s^t, \widehat{s}\right) = \eta_t\left(s^t\right) \quad \forall \widehat{s} \in S \quad \text{and} \qquad \zeta_0\left(s^0\right) = 0$$

Define $\widetilde{r}(a, c, h, s) \equiv y(h, s) - c$ and notice that it must be uniformly bounded by debt limits, therefore there exists a lower bound $\underline{\kappa}$ such that $\widetilde{r}(a, c, h, s) \geq \underline{\kappa}$. We can then define $\kappa < \frac{\underline{\kappa}}{1-\beta}$, $\varphi^1(\phi, \lambda, h, s') \equiv \phi + \lambda \frac{\pi_a(s'|s,h,a)}{\pi(s'|s,h,a)}$, $\varphi^2(\zeta, \eta, s') \equiv \eta$, $\Psi(\phi, \zeta, \lambda, \eta, h, s') \equiv \begin{bmatrix} \varphi^1(\phi, \lambda, h, s') \\ \varphi^2(\zeta, \eta, s') \end{bmatrix}$, $r_0^P(a, c, h, s) \equiv \widetilde{r}(a, c, h, s)$, $r_1^P(a, c, h, s) \equiv \widetilde{r}(a, c, h, s) - \kappa$, $r_0^{ICC}(a, c, h, s) \equiv u(c) - \upsilon(a)$, $r_1^{ICC}(a, c, h, s) \equiv -\upsilon'(a)$, $r_0^{EE}(a, c, h, s) \equiv -Ru'(c)$, $r_1^{EE}(a, c, h, s) \equiv u'(c)$, $\theta \equiv \begin{bmatrix} \phi^0 & \phi & \zeta \end{bmatrix} \in \mathbb{R}^3$, $\chi \equiv \begin{bmatrix} \lambda^0 & \lambda & \eta \end{bmatrix}$, and

$$\ell(h, s') \equiv \begin{cases} (1+\rho)\, h & s' = E \\ (1-\delta)\, h & s' = U \end{cases}$$

I can therefore write the objective function of the Lagrangean as:

$$\begin{aligned} r(a, c, \theta, \chi, h, s) & \equiv \theta r_0(a, c, h, s) + \chi r_1(a, c, h, s) \\ & \equiv \begin{bmatrix} \phi^0 & \phi & \zeta \end{bmatrix} \begin{bmatrix} r_0^P(a, c, h, s) \\ r_0^{ICC}(a, c, h, s) \\ r_0^{EE}(a, c, h, s) \end{bmatrix} + \begin{bmatrix} \lambda^0 & \lambda & \eta \end{bmatrix} \begin{bmatrix} r_1^P(a, c, h, s) \\ r_1^{ICC}(a, c, h, s) \\ r_1^{EE}(a, c, h, s) \end{bmatrix} \end{aligned}$$

which is homogenous of degree 1 in (θ, χ). The Lagrangean can be written as:

$$L_\theta\left(s_0, h_{-1}, \gamma, c^\infty, a^\infty, \chi^\infty, \theta^\infty\right) =$$
$$= \sum_{t=0}^{\infty} \sum_{s^t} \beta^t r\left(a_t\left(s^t\right), c_t\left(s^t\right), \theta_t\left(s^t\right), \chi_t\left(s^t\right), h_{t-1}, s_t\right) \Pi\left(s^t \mid s_0, h^{t-2}\left(s^{t-2}\right), a^{t-1}\left(s^{t-1}\right)\right)$$

where

$$\theta_{t+1}\left(s^t, \widehat{s}\right) = \Psi\left(\theta_t\left(s^t\right), \chi_t\left(s^t\right), h_{t-1}\left(s^{t-1}\right), \widehat{s}\right) \quad \forall \widehat{s} \in S$$

$$\theta_0\left(s^0\right) = \begin{bmatrix} \overline{\phi}^0 & \gamma & 0 \end{bmatrix}$$

We can associate a functional equation to this Lagrangean

$$J(s, \theta, h) = \min_{\chi} \max_{a,c} \left\{ r(a, c, \theta, \chi, h, s) + \beta \sum_{s'} \pi\left(s' \mid s, h, a\right) J\left(s', \theta'\left(s'\right), h'\left(s'\right)\right) \right\} \tag{5}$$

$$\begin{aligned} s.t. \quad \theta'\left(s'\right) &= \Psi\left(\theta, \chi, s'\right) \quad \forall s' \\ h'\left(s'\right) &= \ell\left(h, s'\right) \quad \forall s' \end{aligned}$$

Notice that (5) looks like a Bellman equation, except for the *min max* operator in the right hand side. In the following Proposition, I show that this operator is a contraction.

Proposition 1 *Fix an arbitrary constant* $K > 0$ *and let* $K_\theta = \max\{K, K\|\theta\|\}$. *The operator*

$$(T_K f)(s, \theta, h) \equiv$$
$$\min_{\{\chi > 0 : \|\chi\| \leq K_\theta\}} \max_{a,c} \left\{ r(a, c, \theta, \chi, h, s) + \beta \sum_{s'} \pi\left(s' \mid s, h, a\right) f\left(s', \theta'\left(s'\right), h'\left(s'\right)\right) \right\}$$
$$\begin{aligned} s.t. \quad \theta'\left(s'\right) &= \Psi\left(\theta, \chi, s'\right) \quad \forall s' \\ h'\left(s'\right) &= \ell\left(h, s'\right) \quad \forall s' \end{aligned}$$

is a contraction.

Proof. *See Mele (2010), Proposition 2.* ■

Proposition 1 implies that the value of the optimal unemployment insurance mechanism can be found by solving the functional equation (5) with standard dynamic programming algorithms (value function iteration, for example). Mele (2010) shows that we can characterize analytically and numerically the optimal allocations by using the sequential version of the Lagrangean. Since we proved there is a recursive solution, we can restrict ourselves to policy and value functions that are Markovian.

4 Characterization of the optimal allocations

The recursive Lagrangean approach allows a simple characterization of the optimal contract by means of the Lagrangean first-order conditions. It is possible to use the latter both for theoretical analysis and for numerical simulations. This section provides few analytical results about the optimal insurance scheme.

Set $\overline{\phi}^0 = 1$. In this way, the recursive formulation is equivalent to the original problem. Deriving first-order conditions is straightforward. Let $y_t = h_{t-1}$ if the worker is employed, and $y_t = 0$ if unemployed. Assuming that the value function of the recursive formulation is differentiable[7], the Benveniste-Scheinkman envelope conditions are

$$J_\phi(s, \phi, \zeta, h) = u(c) - \upsilon(a) \tag{6}$$

$$J_\zeta(s, \phi, \zeta, h) = -Ru'(c) \tag{7}$$

Equation (7) shows that the derivative of the value function with respect to ζ is always negative. To simplify the analysis, it is helpful to have i.i.d. employment status. Define $\pi(s_{t+1} = E \mid s_t, a_t) \equiv \pi(a_t)$, i.e. there is no persistence for employment status. Let

$$LR_t\left(s_{t+1} \mid s_t, a_t\left(s^t\right)\right) \equiv \frac{\pi_a\left(s_{t+1} \mid s_t, a_t\left(s^t\right)\right)}{\pi\left(s_{t+1} \mid s_t, a_t\left(s^t\right)\right)}.$$

Moreover, notice that

$$\begin{aligned} J(s_{t+1}, \phi_{t+1}, \zeta_{t+1}, h_t) = \sum_{j=1}^{\infty} \beta^{j-1} \sum_{s^{t+j} | s^{t+1}} \Big\{ y(s_{t+j}) - c_{t+j}\left(s^{t+j}\right) - \\ - \lambda_{t+j}(s_{t+j})\, \upsilon'\left(a_{t+j}\left(s^{t+j}\right)\right) + \phi_{t+j}\left(s^{t+j}\right)\left[u\left(c_{t+j}\left(s^{t+j}\right)\right) - \upsilon\left(a_{t+j}\left(s^{t+j}\right)\right)\right] + \\ + \left[\eta_{t+j}\left(s^{t+j}\right) - R\zeta_{t+j}\left(s^{t+j}\right)\right] u_c\left(c_{t+j}\left(s^{t+j}\right)\right)\Big\} \Pi\left(s^{t+j} \mid s^{t+1}, h^t, a^{t+1}\right) \end{aligned}$$

[7]Given the nature of the problem, it is likely that the value function is not differentiable. However, the analysis of Benveniste-Scheinkman conditions may be insightful on some aspects of the optimal allocation.

is the value of the government at $t+1$, and

$$V\left(s_{t+1},\phi_{t+1},\zeta_{t+1},h_t\right)=\sum_{j=1}^{\infty}\beta^{j-1}\sum_{s^{t+j}|s^{t+1}}\left[u\left(c_{t+j}\right)-\upsilon\left(a_{t+j}\right)\right]\Pi\left(s^{t+j}\mid s^{t+1},h^t,a^{t+1}\right)$$

is the value of the worker at $t+1$. Given these definitions, first-order conditions with respect to choice variables can be written as

$$c_t:\quad 0=-1+\phi_t u_c\left(c_t\right)+\left[\eta_t-R\zeta_t\right]u_{cc}\left(c_t\right)$$

$$\begin{aligned}a_t:\quad & 0=-\lambda_t\upsilon''\left(a_t\right)-\phi_t\upsilon'\left(a_t\right)+\\ & +\beta\pi_a\left(a_t\right)\left[J\left(\{E\},\phi_{t+1},\zeta_{t+1},h_t\right)-J\left(\{U\},\phi_{t+1},\zeta_{t+1},h_t\right)\right]+\\ & +\beta\lambda_t\sum_{s_{t+1}=\{E,U\}}\frac{\partial\left(LR_t\left(s_{t+1}\mid a_t\right)\right)}{\partial a_t}\times\\ & \qquad\times\left[u\left(c_{t+1}\left(s_{t+1}\right)\right)-\upsilon\left(a_{t+1}\left(s_{t+1}\right)\right)\right]\pi\left(s_{t+1}\mid s_t,a_t\right),\end{aligned}$$

while first-order conditions with respect to Lagrange multipliers are simply the constraints of the government's dynamic problem:

$$\lambda_t:\quad 0=-\upsilon'\left(a_t\right)+\beta\pi_a\left(a_t\right)\left[V\left(\{E\},\phi_{t+1},\zeta_{t+1},h_t\right)-V\left(\{U\},\phi_{t+1},\zeta_{t+1},h_t\right)\right]$$

and

$$\eta_t:\quad 0=u'\left(c_t\right)-\beta R\sum_{s_{t+1}=E,U}u'\left(c_{t+1}\left(s_{t+1}\right)\right)\pi\left(s_{t+1}\mid s_t,a_t\right).$$

Combining Benveniste-Scheinkman envelope conditions (6)-(7) with the above first-order conditions, and using the notational shortcut $W'(h)$ for next period's value of W calculated in the state $h\in E,U$, and $W'_k(h)$ for next period's value of the partial derivative of W with respect to state k calculated in the employment status $h\in\{E,U\}$, it is possible to derive the following equations:

$$c:\quad 0=-1+\phi u'(c)+[\eta-R\zeta]u''(c)\tag{8}$$

$$\begin{aligned}a:\quad & 0=-\lambda\upsilon''(a)-\phi\upsilon'(a)+\beta\pi_a(a)\left[J'(E)-J'(U)\right]+\\ & +\beta\lambda\left[\pi(a)J'_\phi(E)\frac{\partial LR(E\mid a)}{\partial a}+(1-\pi(a))J'_\phi(U)\frac{\partial LR(U\mid a)}{\partial a}\right],\end{aligned}\tag{9}$$

$$\lambda:\quad 0=-\upsilon'(a)+\beta\pi_a(a)\left[V'(E)-V'(U)\right],\tag{10}$$

and

$$\eta:\quad 0=u'(c)+\beta\left[\pi(a)J'_\zeta(E)+(1-\pi(a))J'_\zeta(U)\right].\tag{11}$$

The analysis of equations (8)-(11) characterizes the optimal insurance scheme.

4.1 The effect of hidden assets accumulation

In order to understand the effect of hidden assets accumulation, it is useful to see what changes in equations (8)-(11) when comparing the case of hidden assets with the case of no access to financial markets.

Notice that, when there is no financial market, (8) reduces to

$$c: \quad 0 = -1 + \phi u'(c). \tag{12}$$

Using (12) and the law of motion for the Pareto weight ϕ, the following condition holds:

$$\frac{1}{u'(c_t)} = \beta R \left[\frac{1}{u'\left(c_{t+1}^E\right)} \pi^E (h_{t-1}, a_t) + \frac{1}{u'\left(c_{t+1}^U\right)} \left(1 - \pi^E (h_{t-1}, a_t)\right) \right]$$

which implies (by Jensen's inequality):

$$u'(c_t) < \beta R \left[u'\left(c_{t+1}^E\right) \pi^E (h_{t-1}, a_t) + u'\left(c_{t+1}^U\right) \left(1 - \pi^E (h_{t-1}, a_t)\right) \right] \tag{13}$$

This is the well known inverted Euler equation by Rogerson (1985a). This condition states that, when the worker cannot save (or equivalently, when the savings are observable), the optimal allocation is *saving-constrained*: while the worker would like to save in order to finance her consumption tomorrow, the government wants to avoid it since savings reduce the incentives to look for a job. Therefore, the optimal unemployment benefit puts a wedge between consumption today and consumption tomorrow.

The characterization is very different in the case of hidden access to credit markets[8]. The standard Euler equation is at work here, therefore (13) does not hold. Moreover, notice from (8) that the Euler equation has an ambiguous effect on consumption. Rewrite the equation as

$$1 = \phi u'(c) + [\eta - R\zeta] u''(c) \tag{14}$$

Without hidden savings, the equation (14) does not have the last part in squared brackets. Therefore, only the Pareto weight ϕ of the worker's lifetime utility determines optimal consumption, and clearly consumption is increasing in ϕ. When there are hidden assets, *ceteris paribus* consumption is still increasing in ϕ. However, η and ζ have also an effect. Remember that ζ is equal to the past value for η, hence the difference $\eta - R\zeta$ is always zero if and only if $\eta_t = R\eta_{t-1}$ for any t. It is easy to see that

[8] The analysis here is similar to Abraham and Pavoni (2009).

this is impossible, since it is in contradiction with the Euler equation[9]. Therefore, in general for given ϕ and h, it must be that $\eta - R\zeta \neq 0$, and when $\eta - R\zeta < 0$ (respectively, > 0), consumption is higher (respectively, lower) than without hidden assets.

4.2 The effect of human capital

Human capital has a strong effect on self-insurance incentives. In particular, if on average human capital has an increasing trend, wages increase during lifetime, therefore the worker has an incentive to borrow in order to finance today a higher consumption level. If at the contrary the human capital trend is decreasing, the worker has an incentive to save. This mechanism has clear implications on the incentives to look for a job. Savings relax the incentive compatibility constraints, therefore reducing effort. Borrowing, at the contrary, makes the incentive compatibility constraint more stringent, and hence increases effort.

Moreover, there is another effect which has been pointed out in Pavoni (2009): during unemployment spells the human capital depreciates, therefore the wage that the unemployed would get if he eventually finds a job is decreasing during the unemployment period. This mechanism, *ceteris paribus*, reduces her effort to find a job. At the contrary, an opposite effect is at work when the worker is employed: her wage is increasing during her tenure, therefore *ceteris paribus* her effort to keep her job increases. However, notice that transition probabilities are increasing in human capital. Hence a second effect is at work: when unemployed, the job finding probability decreases because human capital decreases, and this actually increases the incentives to put more effort. The same thing with opposite sign happens when the worker is employed. Therefore, it is a quantitative matter if effort is increasing or decreasing during unemployment and employment spells.

5 Decentralization

It is easy to see that this model has a continuum of possible decentralizations. In fact, given a sequence $\{c_t(s^t), a_t(s^t), b_{t-1}(s^{t-1}), \tau_t(s^t)\}_{t=0}^{\infty}$ that solves the problem, there is another sequence which is identical except for $(b_{t+i}(s^{t+i})+\epsilon, \tau_{t+i}(s^{t+i})+\epsilon, \tau_{t+i+1}(s^{t+i+1})-\epsilon R)$ for some i that implements the same allocation.

[9]If $\eta_t = R\eta_{t/1}$ for any t, then (8) is the same as in the case without hidden assets, and (13) must hold.

However, there are two natural decentralizations. The first is the one in which the government sets the sequence of transfers $\{\tau_t(s^t)\}_{t=0}^{\infty}$ such that $\{b_t(s^t)\}_{t=-1}^{\infty}$ is always zero. This amounts to set $\tau_t(s^t) = c_t^*(s^t) - y_t(s^t)$. This is equivalent to a standard unemployment insurance system, where the government sets the unemployment benefit (usually as a function of the previous wage and the duration of the unemployment spell) and the payroll tax that finances it.

The other natural decentralization sees the government setting the transfers to zero in each period, and imposing restrictions on savings such that $b_t(s^t) - Rb_{t-1}(s^{t-1}) = y_t(s^t) - c_t^*(s^t)$. This is a mandatory unemployment saving account system with regulated deposits and withdrawals, in which the returns are reinvested in the account when working.

We focus on the latter implementation. Notice that, depending on the worker being employed or unemployed, the difference $y_t(s^t) - c_t^*(s^t)$ can be positive or negative. Therefore, we refer to it as the deposit when the worker is employed, and the withdrawal when the worker is unemployed.

6 Numerical examples

In this section, I present some numerical examples that help understanding the qualitative features of the UISA system. I assume that one period is equivalent to one month. The utility function for consumption is assumed to be CRRA with intertemporal elasticity of substitution $\sigma = 2$, while effort disutility is quadratic: $\upsilon(a) \equiv \alpha a^{\varepsilon}$, with $\alpha = .5$ and $\varepsilon = 2$. The human capital parameters ρ and δ are set to $\rho = \delta = .00025$, implying an average wage growth rate in line with the one for the median worker in US (around 12% during lifetime, see Huggett, Ventura and Yaron (2006)). The transition probabilities are:

$$\Pr\{s_{t+1} = \{E\} | s = \{i\}, a, h\} \equiv 1 - e^{-\nu_i a h}$$

and $\nu_U = 1.5$, $\nu_E = 10$. These parameters are chosen such that the transition probabilities are close to data. The discount factor is set to $\beta = .995$. The interest rate $R = \frac{1}{\beta}$ for simplicity. I scale the human capital such that the wage is equal to human capital, and a unit of human capital is equivalent to 1000 dollars.

Figure 1 shows the optimal UISA system for an unemployed worker with human capital equivalent to a monthly wage of 1000 dollars that becomes unemployed immediately after he enters in the labor force. The optimal allocation for consumption is decreasing during the unemployment spell, in line with all recent research on UI. In the NE panel I plot the withdrawal rate as a fraction of the wage before falling in the unemployment pool (i.e., 1000 dollars), which clearly decreases through time for incentive reasons. Effort increases during the unemployment spell, and this causes lifetime expected utility to decrease.

Figure 2 shows the evolution of asset holdings in the saving account. Given positive withdrawals, the account balance is reduced. If the account balance reaches zero, then the unemployed must borrow money [10].

Figure 3 plots the same variables of Figure 1 for the case in which the worker is employed when he enters in the labor force and remains employed for 3 years. All variables move specularly to the unemployment case, however the interesting feature is that the deposit rate is very flat. Figure 4 shows the UISA balance during employment, which is increasing given the positive deposit rate.

Figure 5 and Figure 6 show how the withdrawal and deposit rates change for different initial human capital levels. In particular, it seems that the system has some progressive flavor: the deposit rate is higher for workers with higher human capital, while the withdrawal rate is lower for more skilled workers.

Finally, I look at what happen to the withdrawal rate for different employment tenures. I then calculate what is the withdrawal rate in the first period of unemployment after the worker has been employed from time zero for n months. Figure 7 shows the initial withdrawal rate as a function of the previous employment tenure n expressed in months. It is decreasing for incentive reasons, because longer employment tenures are associated with higher savings which ceteris paribus would reduce effort.

7 Conclusions

The debate about unemployment insurance systems is divided between the traditional model of unemployment benefits and the recent proposal of UISAs. It is therefore important to understand the economic rationale behind each of them. I showed that

[10]This might seem an unfortunate feature of the UISA system. However, the system as a whole has always a positive balance, since most of the workers are employed and they deposit. I verified that the system has positive balance by simulating an economy with 100000 workers for 40 years. It turns out that there is always enough money to lend to the few unemployed that have negative balance.

UISAs arise as decentralized equilibria in a model of alternate spells of unemployment with human capital trends and hidden access to financial markets.

The optimal nonlinear UISA is actually quite simple: the deposit rate is almost constant, while withdrawal rate decreases during unemployment spells. Moreover, it is progressive for incentive reasons, and it gives higher withdrawal rates to young workers.

Work in progress is trying to calibrate the model in a more precise way in order to make welfare comparisons with the current UI system.

This model however assumed partial equilibrium. It is important to understand the welfare gains in general equilibrium where the interest rate can adjust to supply and demand for assets. This is left for future research.

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A Figures

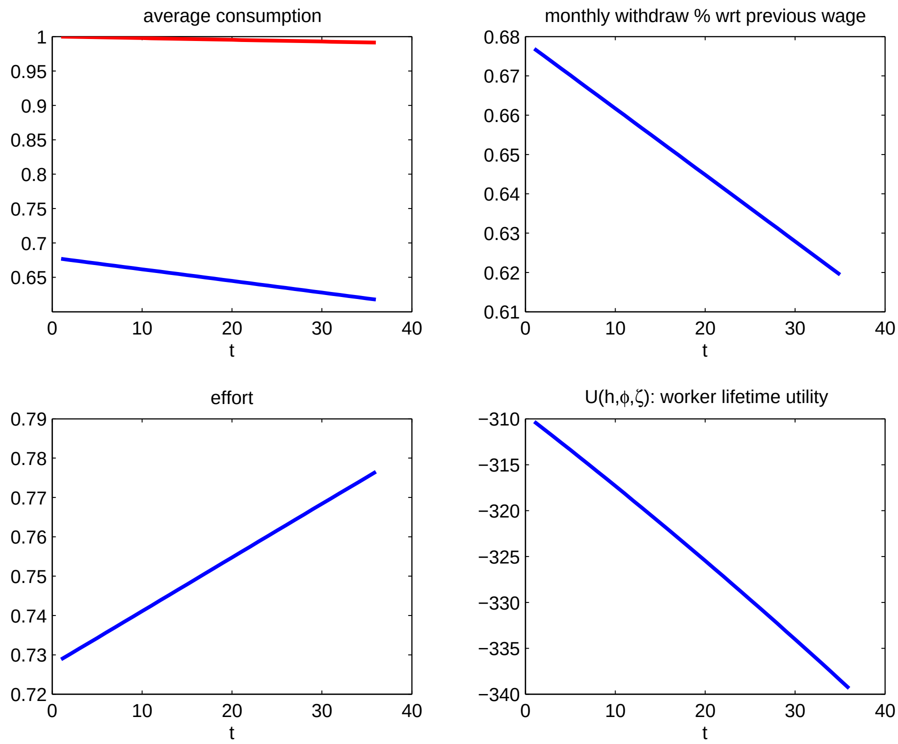

Figure 1: Consumption, monthly withdrawal, effort and lifetime utility for unemployed. Note: red solid line in the NW panel is human capital

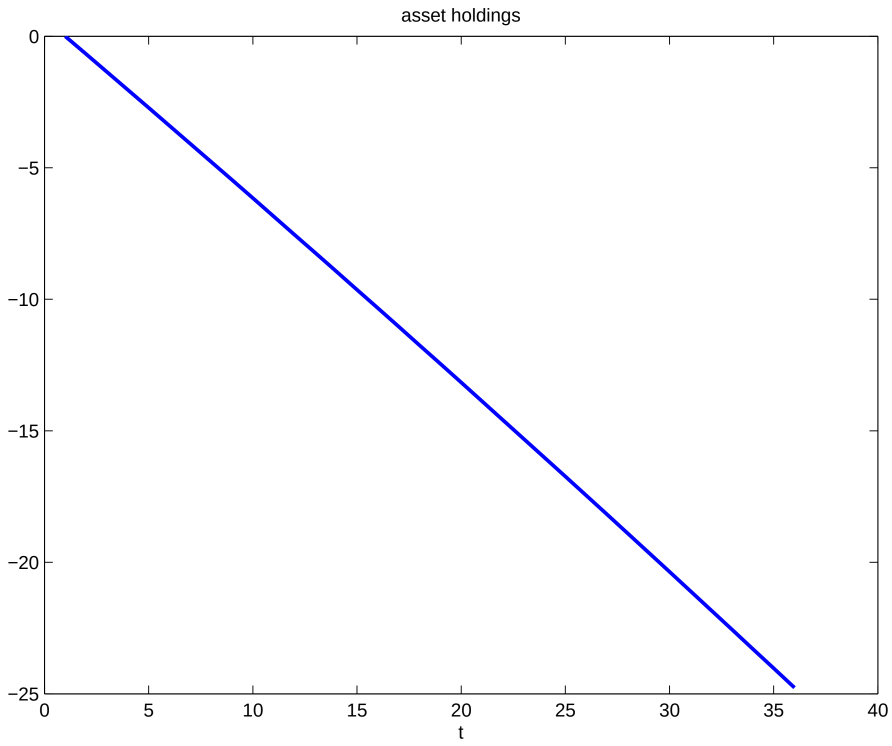

Figure 2: UISA balance when unemployed

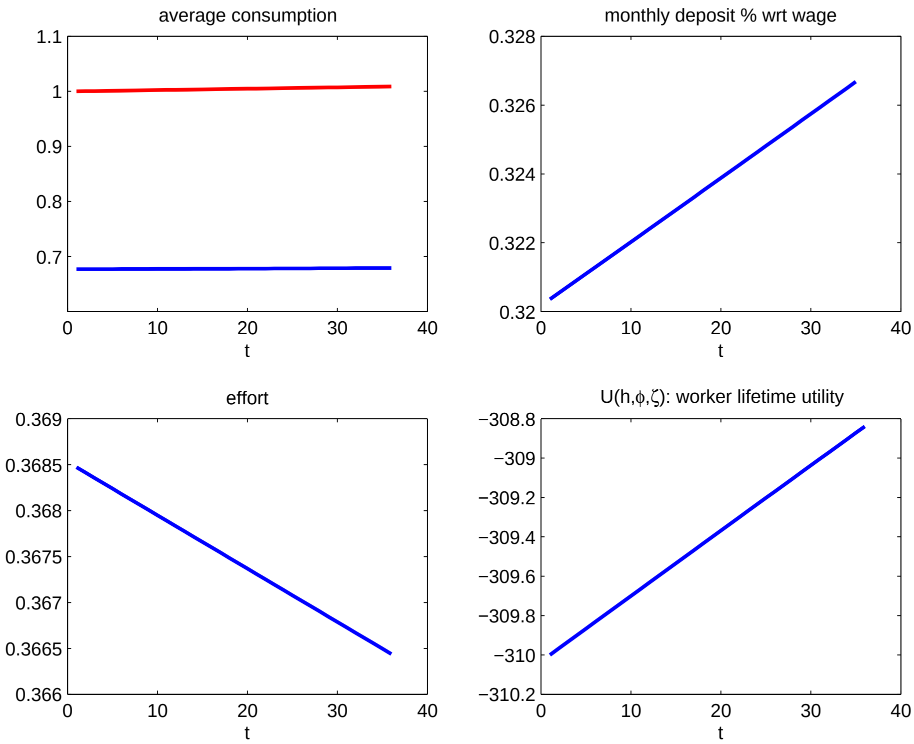

Figure 3: Consumption, monthly deposit, effort and lifetime utility for employed. Note: red solid line in the NW panel is human capital

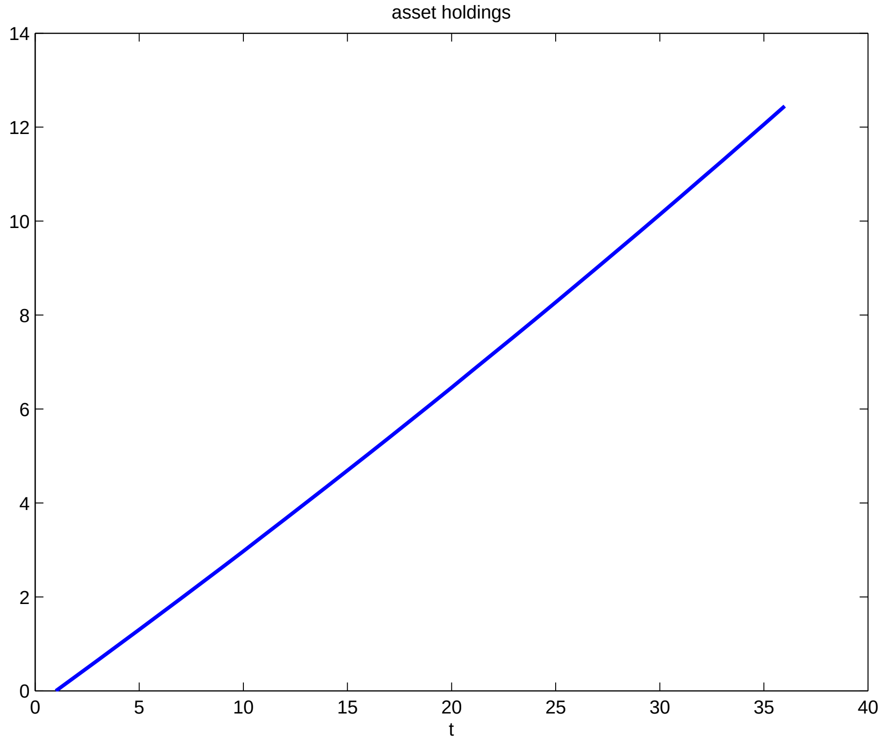

Figure 4: UISA balance when employed

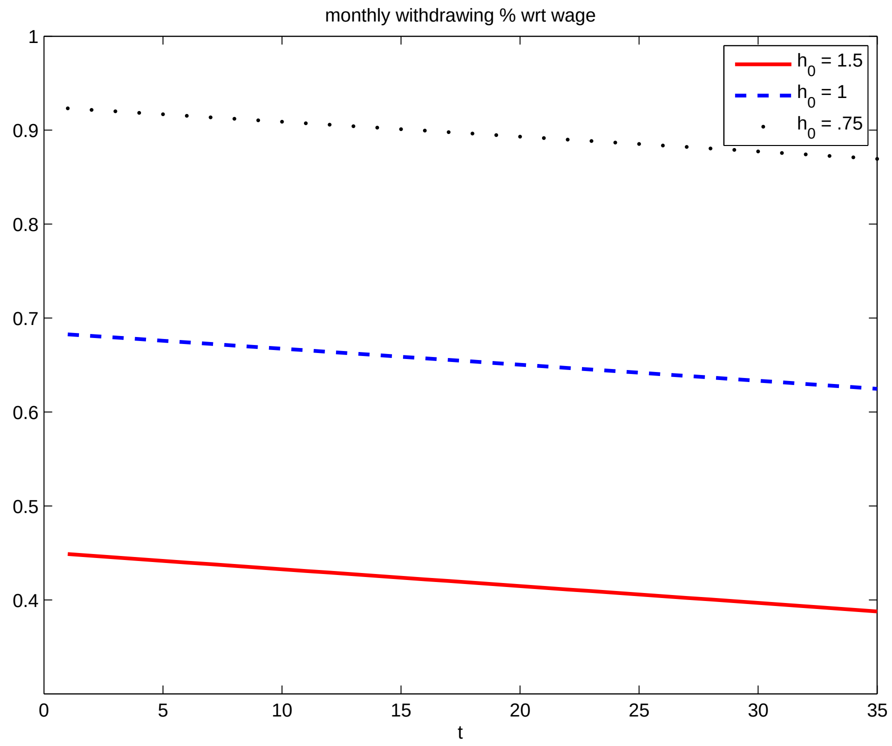

Figure 5: Withdraw rate for different initial human capital

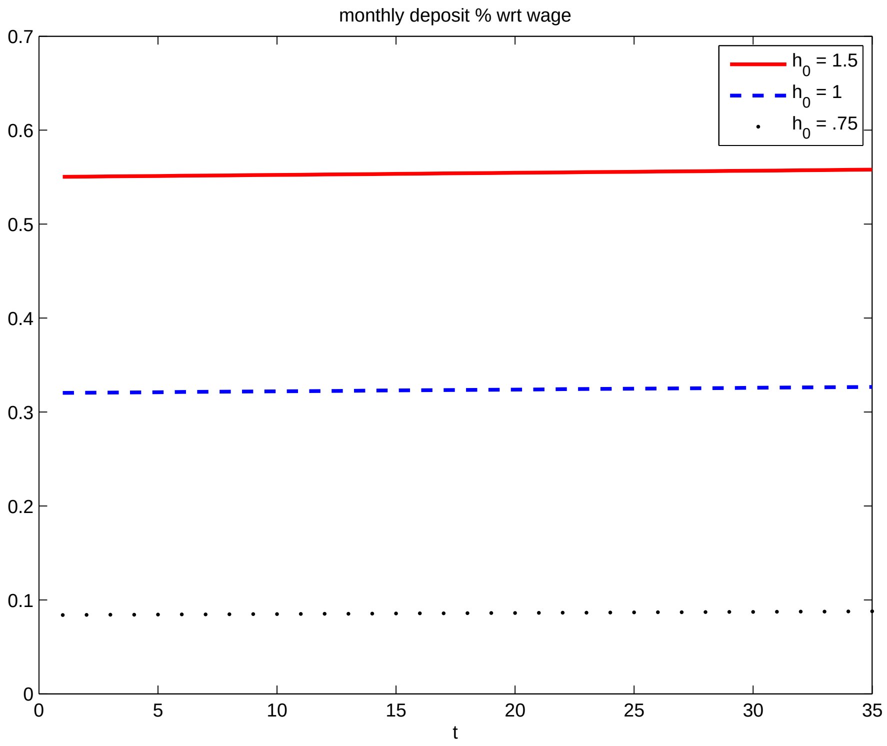

Figure 6: Deposit rate for different initial human capital

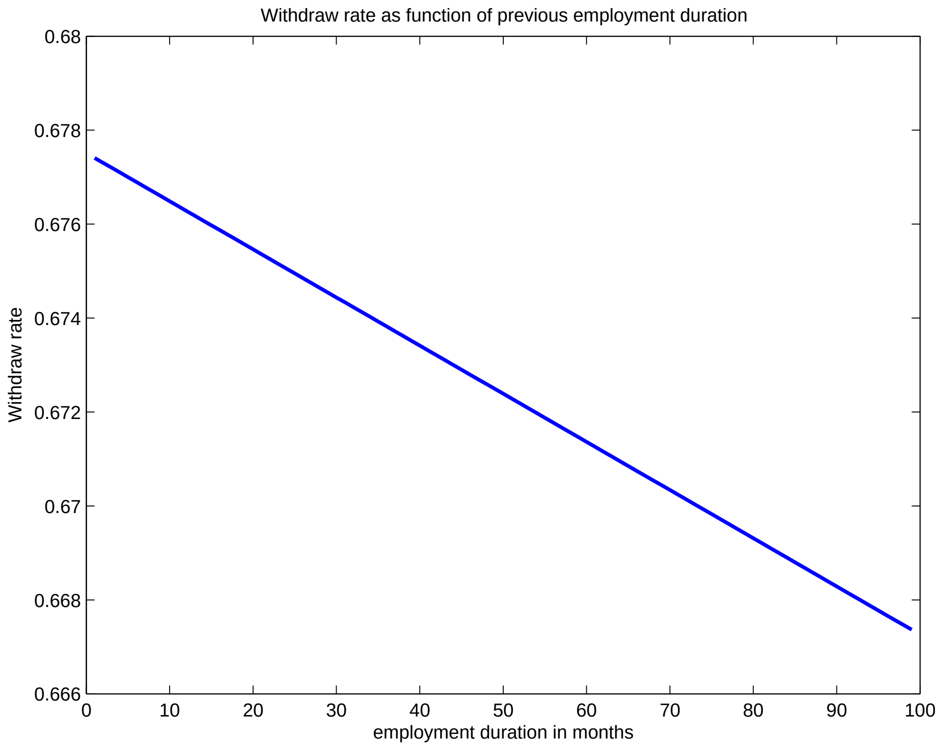

Figure 7: Withdrawal rate as function of previous employment duration