Assignment 8

EE 553 Power System Economics

Due May 31th, 2017 at 8pm. Email to ywang11@uw.edu

Problem 1. Consider the 3 bus network in Fig. 1 with data given in Tables 1 and 2.

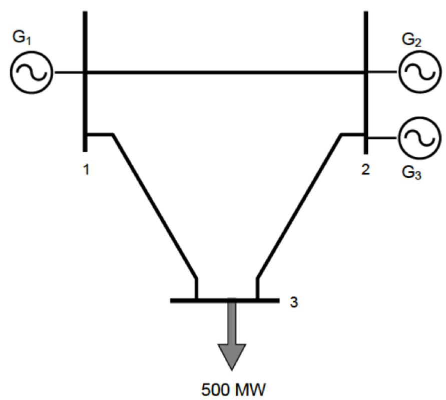

Figure 1: 3 bus network for problem 1.

Generator	Capacity (MW)	Marginal Cost (price) ($/MWh)
G_1	300	10
G_2	300	5
G_3	300	6

Table 1: Generation data for problem 3.

Branch	Reactance	Capacity (MW)
1-2	0.1	280
1-3	0.2	280
2-3	0.2	280

Table 2: Branch data for problem 3.

1. Find the optimal economic dispatch **without considering the line limits**. Calculate the marginal cost of energy. Calculate the flows on the lines.

2. Find the optimal economic dispatch **that satisfies all the line flow limits**. Find the nodal price at each node in this network.

Solutions.

1. Without congestions, since all the generators have linear cost curves, the generator with the lowest marginal cost should therefore be loaded first. So $P_2 = 300$ MW and $P_3 = 200$ MW. The marginal cost is the marginal cost of generator 3, which is \$/MW. Since P_1 generate 0 MW, all of the 500 MW has to flow from 2 to 3 through two parallel paths. From the reactances, for every MW injected at 2 and extracted at 3, 0.6 MW flows on the line 2-3 and 0.4 MW flows 2-1-3. Therefore $F_{2-3} = 300MW$ and $F_{2-1} = F_{1-3} = 200MW$.

2. The flow in line 2-3 exceeds its limit by 20 MW. This flow has to be corrected by generating some energy from G_1. We solve the following equations

$$\Delta P_1 + \Delta P_3 = 0$$
$$\Delta F_{2-3} = 0.4\Delta P_1 + 0.6\Delta P_3 = -20$$

where the first equation comes from demand=supply (so 0 net change in generation) and the second equation is to reduce the flow on the congested line. Solving them gives $\Delta P_1 = 100MW$ and $\Delta P_3 = -100MW$. Then $P_1 = 100$, $P_2 = 300$ and $P_3 = 100$. The nodal price at bus 1 is $10\$/MW$ since it cannot get more power from bus 2 without violating the flow constraint, and the nodal price at bus 2 is $6\$/MW$ which is the cheapest generation. For bus 3, to get one more unit of power, we need to solve

$$\Delta P_1 + \Delta P_3 = 1$$
$$0.4\Delta P_1 + 0.6\Delta P_3 = 0$$

which gives $\Delta P_1 = 3$ and $\Delta P_3 = -2$. Therefore the nodal price is $3 \cdot 10 - 2 \cdot 6 = 18\$/MW$.

Gen	Marginal Cost (\$/MW)	$P_{\max}(MW)$	$R_{\max}$ (MW)
1	2	100	50
2	5	50	50
3	10	200	100

Table 3: Data for problem 2.

Problem 2. Consider a system that has 3 generators with information given in Table 3. The load in the system is 180 MW, and the reserve requirement is 150. Find the least cost solution of meeting the load while maintaining reserve. What is marginal cost of energy? What is the marginal cost of reserve?

Solution. The optimization problem is

$$\begin{aligned}
\min\ & 2P_1 + 5P_2 + 10P_3 \\
\text{s.t.}\ & P_1 + P_2 + P_3 = 180 \\
& 0 \leq R_1 + P_1 \leq 100 \\
& 0 \leq R_2 + P_2 \leq 50 \\
& 0 \leq R_3 + P_3 \leq 200 \\
& R_1 + R_2 + R_3 \geq 150 \\
& 0 \leq R_1 \leq 50,\ 0 \leq R_2 \leq 50,\ 0 \leq R_3 \leq 100
\end{aligned}$$

This problem can be solved via a computer program. If solved by hand, observe that generator 1 should be used all for power. Then to provide 150MW of reserve, $R_2 = 50$ and $R_3 = 100$. Therefore $P_3 = 80$ and $P_2 = 0$. The cost of energy is the cost of generator 3 at $10\$/MW$. To get one more unit of reserve, generator 1 must provide this. So the cost is $10 - 2 = 8\$/MW$.