

Is China's Rapid Growth Sustainable?

A Theory of Politico-Economic Transition and State Capitalism

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Abstract

Is China's rapid growth sustainable with the current institutions? If not, will the slow-down of growth trigger political changes? To answer these questions, this paper proposes a theory of politico-economic transition. In oligarchy, a political elite extracts surplus from the state sector and taxes the private sector. However, to maintain the power, the elite needs political support from a sufficient number of citizens. A "divide-and-rule" strategy is implemented to guarantee such a support. The elite gives state workers high wages and turns them into supporters, at the cost of the private workers. Moreover, the elite distorts the capital allocation in favor of the state sector to maintain enough state workers. The consequences are: in the short term, the low wage in the private sector helps private firms and the economy grow rapidly. In the long term, the capital market distortion slows down the growth. The theory suggests that the economy develops along an endogenous three-stage transition: *rapid growth* is followed by *state capitalism*, and then in the third stage the economy may follow one of the two paths, *middle-income trap* or *sustained growth*, depending on whether democratization occurs. The theory is consistent with salient aspects of China's recent development and gives predictions on China's future political and economic development. It also offers explanations for some general questions in development, e.g., the challenges to sustainable growth.

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1 Introduction

China has by now been growing at a stellar rate for more than three decades. While this is generally acknowledged to be a great historical achievement, there is major controversy on how far in time and scope the Chinese success story can go. The optimists argue that China can provide a new model for growth as an alternative to the liberal democracy growth model known as the Washington Consensus. For example, in a debate hosted by *The Economist* (see also Musacchio and Lazzarini (2012)), Aldo Musacchio argues that China's hybrid form of capitalism can become a new growth model for the 21st century. In his view, such a model offers attractive features including mitigating recessions, focusing on long-term investment, and producing world champion companies. These features make him optimistic about the sustainability of China's future growth, and even the possibility that China could become a role model for other developing and emerging countries. In contrast, pessimists predict that China's growth will soon slow down. For example, Acemoglu and Robinson (2012) argue that China's current institution is not compatible with innovation and sustainable long-run growth, for the following reason. The extractive institution can lead to rapid growth in the early stage, when economic growth is in line with the interest of the ruling elite. However, in the long-run, the elite fears losing its economic rent to new technology or even losing its political power to groups rising from the growth and does not adopt economic arrangements favoring growth. China's growth process driven by catch-up may continue for a while, but given the current institution, it will come to a halt as soon as China reaches the living standards of a middle-income country.

The pessimistic perspective of Acemoglu and Robinson raises a number of questions. Will slowing growth, which they predict, trigger changes in the political system, with unsatisfied citizens outing the oligarchy, and in turn allowing growth to resume under a more democratic system? Or, alternately, will the oligarchy be able to retain sufficient support even in a low-growth economy? Modernization theory suggests that democratization is likely to occur. But, then, one can argue that it may have been right for China to adopt its hybrid form of state capitalism to achieve high economic growth in the catch-up stage, and then switch to liberal democracy when state capitalism runs out of steam. The Chinese model, in other words, could be a model of transition, albeit not a mode of long-

run growth for mature economies. This view, however, may well be overly optimistic: at the time of transition, the political elite could be unwilling to give up state capitalism, and might seek to maintain political power and control of economic resources, as we see in countries like Venezuela. In the language of Acemoglu et al. (2006), state capitalism may be appropriate to promote growth at an early stage of development, but may be impossible to reform when it becomes a barrier to further economic growth.

To answer the above questions - first, whether China's growth can continue, and second, whether changes in political system will occur - this paper proposes a theory of politico-economic transition. I incorporate political economy into a two-sector growth model and calibrate it to China's economy. The model is consistent with three salient aspects of China's recent developments, including: rapid growth with low wages, large state investments accompanied by financial restriction on private firms, and the support of the middle-class for the government.

In this theory, a political elite runs the government and is able to extract surplus from state firms and tax the private sector. However, it faces a political constraint, that is, support from a sufficient number of citizens. That the government can use the following two policy tools to maintain the support: in the labor market, regulating the state sector wage, and in the capital market, controlling capital allocations in the state and the private sector.¹ How does the government use these tools? First, it sets the state wage sufficiently high to buy support from state workers - higher than the income that a worker expects to get in democracy. A dual labor market is created. State workers receive high wages and in turn support the elite. Private worker's wages are reduced due to the general equilibrium effect, as follows. Given certain capital, the high state wage makes a state firm hire less than it would if the wage were determined by the market. Then more workers are pushed to the private sector, and private sector wages are reduced. This "divide-and-rule" strategy gains support from state workers at the cost of private workers. Second, to keep enough supporters in the state sector, the government needs to balance capital in the state and the private sector. When private firms hold little capital, it is cheap and easy to maintain the support, because workers' expected wages and incomes in democracy are low, and also because the number of workers in the state sector is larger than the number of supporters needed. To extract more tax from the private sector, the elite encourages its growth and helps it to build capital. When private entrepreneurs get richer and private firms hold more capital, a trade-off emerges: a larger private sector contributes more tax,

¹This means that the government can only adopt clientelism to gain the support, as in Robinson et al. (2013). Other tools, for example, using direct lump-sum transfer to buy the support, are assumed away, following Acemoglu (2003). More discussions on this are in section 3.

but it also increases the cost of maintaining supporters, because it increases the wage and also competes for labor. Then the government may choose to financially repress private firms, i.e., to limit their borrowing from banks. This restrains the growth of the private sector capital and its relative size in the economy.

Because the government policy in the capital market changes as the private sector grows, the economy's growth pattern also changes accordingly. More specifically, the economy develops along a three-stage transition as follows. The first stage is *rapid growth*, during which the GDP share of the private sector grows rapidly, triggering reallocation and high productivity growth. Private firms benefit from low wages in the private sector, which are induced by the policy. The government supports privatization because it increases tax revenue. However, as privatization continues and the state employment share declines to the critical level, the economy enters the second stage, *state capitalism*. In this stage, the elite over-invests in the state sector to keep the state employment sufficiently large. The government also imposes gradually increasing financial restriction to limit the growth of private firms. Growth continues to be fast due to large state investment, which overcomes the slowing down effect of the financial restriction on private firms. As the private sector capital keeps growing (largely through self-financing), the economy enters the third stage, and two possible outcomes emerge. The first is *middle-income trap*: over-investment in state firms and financial restriction on private firms continue, while the efficiency loss also grows, due to decreasing return to capital and the capital market distortion.² Eventually, growth stops and the output converges to a relatively low level. This happens if the government keeps the distortions to retain the regime, in the case that the cost of retaining the regime is low, e.g., when the number of supporters needed is small. The other possible outcome is *sustained growth*, following democratization. In this case, the cost of maintaining enough supporters in the state sector is high. As the private sector capital grows, the elite finds it too costly to continue investing in the less efficient state sector, and therefore chooses to democratize, for its own interest. Financial distortion between the state and private firms disappear and the economy keeps growing in democracy and eventually converge to a higher level of output.

The first two stages in the theory are consistent with three salient facts in the recent development of China.³ First, low private sector wages help private firms and the economy grow rapidly. Between 1995 and 2007, the private employment share increased from

²The term *middle-income trap* is used to describe the phenomenon that a country rapidly grows out of poverty and attains a middle-level income but then for a long time fails to keep growing and become rich, in contrast to *sustained growth*, which describes the case that a middle-income country continues to grow rapidly and becomes rich.

³These facts will be discussed in more details in section 2.

40% to 80%. This era of fast privatization implies large efficiency gain and *rapid growth*, as in the first stage of the theory. However, the employment share of private sector subsequently stopped growing. Private firms face tighter financial constraints, and around 60% of investment and the majority of bank loans are allocated to less productive state firms.⁴ This is the second fact, i.e., the capital market misallocation in favor of state firms. It implies that the economy is now entering the *state capitalism* stage. The third fact is that the middle-class, consisting largely of state sector workers and private entrepreneurs, are the beneficiaries and supporters of the regime. The reason is that state workers receive high wages, and entrepreneurs enjoy the cheap and abundant labor in the private sector. Chen and Lu (2011) and Tsai (2007) document that the Chinese middle-class, including state employees and private entrepreneurs are “achieving their material interests without pursuing any real freedom”. The third stage of the transition in the theory provides an answer to questions about China’s future political and economic developments. The model in this paper, calibrated to the Chinese economy, predicts that the economy will enter the *middle-income trap*. The reason is: the government is both economically and politically powerful. First, the government’s cost of retaining enough supporters in the state sector is low. It controls abundant financial resource through state banks and holds abundant financial assets, including the huge foreign reserve. It is capable of keeping investment in the state sector high and maintaining the current level state employment. Second, the current relative low state employment share has been sufficient to provide the support that the government needs and keep the political system relatively stable. This shows that a relatively small supporter base is sufficient and the government is politically powerful. Given these conditions, support for the regime will continue, and policy distortions will persist, which will eventually slow down the growth before China converges to rich countries.

Is China doomed to fall into the *middle-income trap*? Are there possibilities to redirect China towards the other development path, i.e. *sustained growth*? Many economists have proposed reform plans to sustain growth, including political reforms, financial reforms and state sector reforms. However, a crucial but often neglected question is whether the government is willing to implement those reforms. Many reforms which are beneficial for growth can be harmful for the elite’s interest. With the aid of the model, I can study consequences of these reforms, including how they affect the long-run growth, and also the elite’s interest. Studying the latter is important for understanding how likely a reform will be implemented. This analysis is useful for predicting China’s future policies and directions of reforms.

⁴See Brandt and Zhu (2010) on the investment in state sector.

This paper is related to three strands of literature. The first is on China's economic growth with structural transition. Song et al. (2011) construct a two-sector growth model to study how the capital and labor reallocation from the state to the private sector leads to rapid growth. Brandt and Zhu (2000, 2010) document the contribution of private firms to growth and the consequence of the government's strategy of maintaining state sector employment. Cheremukhin et al. (2015) study China's structural changes and the evolution of wedges in the labor and capital markets and their contribution to growth from 1953. These studies capture key features of China's economic growth, including the capital and labor market frictions. However, an important question is not answered yet: why do large labor and capital market frictions persist and how will they evolve in the future? Political constraints can be the root of these frictions, including the financial friction in Song et al. (2011) and the state employment constraint in Brandt and Zhu (2010). This paper provides the micro-foundation for the endogenous evolution of capital and labor market frictions. In contrast to the conventional wisdom which expects these frictions to gradually decline as China's labor market and financial market develop, and consistent with the current trend, this paper predicts that these frictions will be persistent and will even become more severe, within the current political regime.

Second, the theory contributes to the study of *middle-income trap*, i.e., the phenomenon that some middle-income countries, which have rapidly grown out of poverty, experience slow growth and get stuck at the middle-income level for very long time, e.g., Argentina and Indonesia. In contrast, some others continue to grow fast and converge to rich countries, e.g., Korea. The empirical studies on the middle-income trap have been developing and the understandings on this phenomenon have been improving. The literature starts from (incorrectly) claiming that when countries reach certain *absolute* level, e.g. \$2,418 to \$15,220 in 2005 PPP, growth becomes more difficult, and they are likely to stuck in this income range. See Gill and Kharas (2007). Later researchers, e.g., Bulman et al. (2014) and Han and Wei (2015), reject the initial claim of middle-income trap using cross-country panel data. Their results suggest the following alternative patterns. First, it is not the absolute income but the income *relative* to the U.S. may get stuck in a moderate range. Han and Wei (2015) show that the long-run incomes of many MICs relative to the U.S. will stay in the range of 16% to 36% and 36% to 75%, corresponding to the lower-middle-income and the upper-middle-income level in relative term. Second, the middle-income trap is a *conditional* but not a universal phenomenon. Conditional on policies and fundamentals, some MICs can avoid the trap and converge to the U.S., while others fail to do so. Han and Wei (2015) identify important policies and macroeconomic conditions for the growth of MICs, e.g., openness. An important question is: why do some MICs successfully adopt

policies and provide the conditions to escape the middle-income trap while others fail to do so? The discussion has been heated, but there is a lack of theoretical frameworks to guide it. In this paper I try to provide a tractable framework. The model can explain why and how an economy initially grows rapidly but fails to sustain the growth. It can also be used to analyze policies and conditions that determine the development path of a MIC.

The third strand of literature is on the relation between political development and economic development. Acemoglu and Robinson (2012) explain how political institutions affect economic performance. They argue that the *extractive* political institution is detrimental to growth in the long-run. On the other side of the relation, i.e., how economic development affects political development, the modernization theory, originated from Lipset (1959) suggests that the economic development will ultimately lead to political modernization, i.e., liberal democracy. This paper's contribution to this strand of literature is two-folds. First, it combines both sides of the relation between political and economic development. Second, the theory distinguishes the short-term effect of political institutions on economic development from the long-term effect, i.e., institutions that help rapid growth in the catch-up stage can be detrimental to growth in the long run.

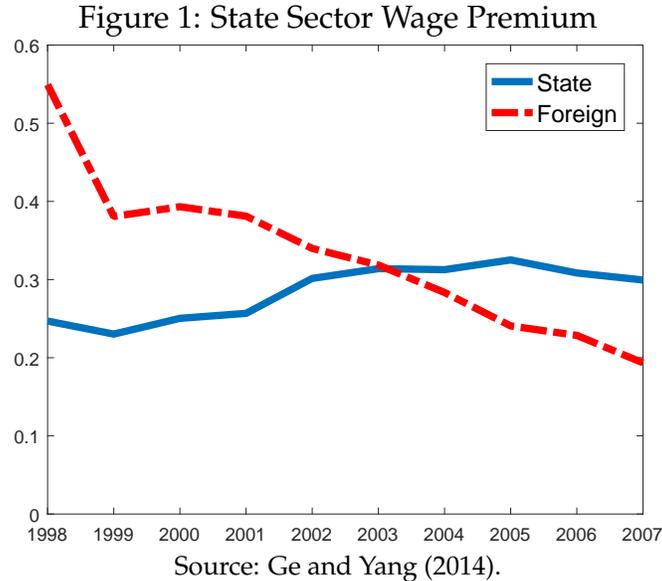
The rest of the paper is organized as follows. Section 2 discusses crucial facts in China's political-economic development which motivate the theory. In section 3, I build a two-sector dynamic growth model with the three-stage political-economic transition. The first two stages are consistent with and useful for understanding important phenomena and puzzles in China's recent development, while the third stage is used to predict future politico-economic development. Section 4 concludes. Extensions and robustness of the model are discussed in the Appendix which is available online.

2 Empirical Facts on China's Recent Development

In this section, a couple of important facts in China's recent development that motivate the theory are documented: (1) large wage gap between the state and the private sector; (2) low support for democracy from the middle-class; (3) the slow-down and stop of privatization; and (4) financial market wedges between the state and the private firms.

2.1 Large State-Private Wage Gap

China's state workers have been enjoying a wage premium of around 20% to 30%, as documented by Ge and Yang (2014). Their finding is based on a Mincer regression controlling for observable characteristics - age, education, industry, region and so on. Their results



are reproduced in Figure 1. The solid line is the state wage premium over domestic private sector wage, showing a persistent labor market friction between the state and the private sector. In contrast, the foreign firm wage premium, shown as the dashed line, has been declining, implying that the labor market has become more efficient, at least in the private sector. In comparison, in other developed economies, such as Canada, Germany and the U.S. the wage premium of state workers is estimated to be lower than 5% or even insignificant after the 1990s. See Melly (2002), Mueller (1998) and Poterba and Rueben (1994).

2.2 Middle-class Support for the Regime

Given that the state workers earn high wages, it is not surprising to see that they are more supportive of the current political system than non-state workers, as documented by Chen and Lu (2011). The authors use survey data of 2810 individuals, collected in three Chinese cities in late 2006 and 2007 to estimate how the individual's political opinions depend on his or her characteristics, especially the social group identity. They find that state sector workers and the middle class are less supportive for democratic values. For example, only 24.9% of the middle class support multi-party competition, while 38.7% of the lower-class do. Only 22.9% of the middle class agree that demonstrations should be allowed, while this number is 35.6% for the lower class. Similar patterns apply for other questions related to democratic values and institutions. To formally show the difference between the middle class and the lower class, the authors construct an index of support for democratic values and institutions combining the answers to all the questions. Then

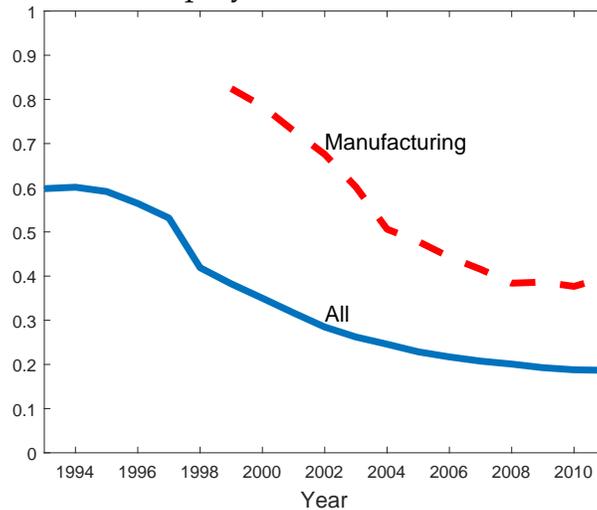
they run a regression of this index on individual characteristics, including a dummy for middle-class membership and a dummy for state employment. The coefficients of dummies for middle-class and state employment are both negative (-1.23 and -0.54) and significant at 1% level. In contrast, party membership does not affect the political opinion too much, after controlling for other variables. These results suggest that economic interest plays a more important role than ideology. Tsai (2007) documents that the Chinese entrepreneurs are “achieving their material interests without pursuing any real freedom,” in contrast to “the business classes in historical England, France and the United States” who “have risen up against the government to defend material interests.”

In short, the Chinese middle class, relatively affluent and consisting largely of state workers and private entrepreneurs, do not support democracy, contrary to the European history and the conventional wisdom that the middle-class are the driving force for democratization and reforms (see Chen and Suen (2015)).

2.3 The Slow-Down and Stop of Privatization

If state workers support the government but many state firms are not efficient, will the government allow state firms to exit the market and the state employment to decline? The answer is mixed: initially the government allows it, to improve the efficiency of the economy, but the government will not tolerate the state employment - the important supporter base - becoming too small. The decline of state employment and the privatization of state firms was initially very rapid after the fifteenth national congress in 1997, which initiated the state firm reform. As the solid line in Figure 2 shows, the employment share of state sector in urban areas declined rapidly from 53% in 1997 to 23% in 2005. Then, the privatization slows down and the state employment share stagnated at around 20%. If we focus only on the manufacturing, mining, and construction, represented by the dashed line, the trend is similar while the state employment share stops declining at a higher level around 40%, and even slightly increases after 2011. This trend is the so-called “the state advances as the private sector retreats”. It suggests that the privatization and the decline of state employment has come to a halt. Moreover, there is more direct evidence that the government strategically maintains the state sector as the backbone of the economy. For example, the closing announcement of the Third Plenary Session of 18th Chinese Communist Party Central Committee in 2013 stated that “China will stick to the dominant role of public ownership, playing the leading role of the state-owned economy, while encouraging, supporting, and guiding the non-public sector.”

Figure 2: State Employment Share in the Urban Sector



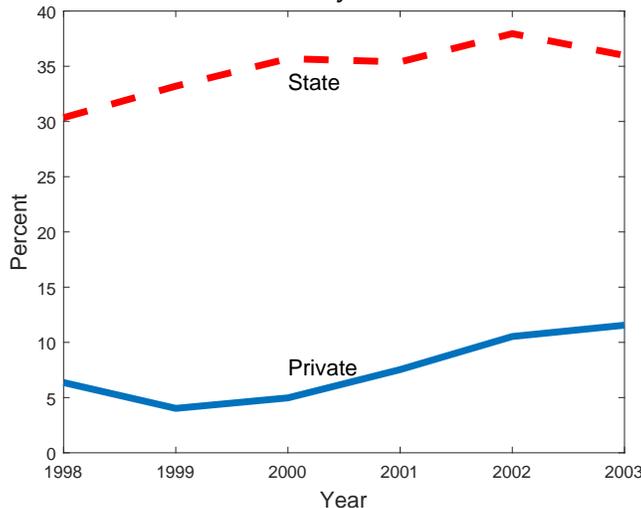
Source: Statistical Yearbook 2012 and Storesletten and Zilibotti (2014).

2.4 Capital Market Wedge between the State and the Private Sector

How do state firms survive and still employ a significant share of workers, given they pay higher wages and are less efficient, as documented by Hsieh and Klenow (2009)? State firms survive because they are in a more advanced position in the financial market compared to private firms. It is easier and cheaper for state firms to get loans from state banks compared to private firms. Song et al. (2011) document, as shown in Figure 3, that state firms finance more than 30% of their investment through bank loans and government budget, compared to less than 10% for private firms. Their result is reproduced in Figure 3. Also, Brandt et al. (2013) estimate that the capital wedge, i.e. the ratio of costs per unit of capital between state and private firms, has increased in all the provinces, on average from 4.2 in 1996 to 6.8 in 2007.

Why is the capital market friction so large, and even increasing? Is it driven purely by market force, or strategically maintained by the government, to keep the state sector large enough? If it is the latter, will this trend continue in the future? In the next section, I study the above questions with the help of a general equilibrium growth model, incorporating political economy. The model is also used to explain other facts discussed in this section, including the state wage premium, middle class's support for the regime, and the decline of state employment share. Furthermore, the model is used predict the future trend of these phenomena, as well as economic growth, political transition.

Figure 3: Share of Investment Financed by Bank Loans and Government Budget



Source: Song et al. (2011).

3 The Model

This section presents a theory of politico-economic transition. Here I first briefly describe the key elements of the model. Then in the rest of this section, I present the details of the model and the solution.

A two-sector dynamic general equilibrium growth model is built. The two sectors are the state (S) and the private (P) sector. In the state sector, there are a continuum of state firms and in the private sector, private firms. They are standard neoclassic firms: they produce the same final goods using capital and labor, and they maximize profits taking prices as given. There are three groups of infinitely many agents: the elite (e), private entrepreneurs (p) and workers (w). The population size of workers is normalized to 1, while the sizes of the elite and the entrepreneurs are relatively small and have measure 0. The elite supplies capital to state firms, and entrepreneurs supplies capital to private firms. They receive incomes from capital returns, consume and save. Workers supply labor and receive wages. A worker may work in a S firm or a P firm. For some groups of agents including firms, entrepreneurs and elite members, we can use a representative agent to describe the behavior of the group of agents, and call the representative agents as “the S firm”, “the P firm”, “the elite” and “the entrepreneur”, respectively.⁵

Political economy is incorporated into the model. In oligarchy, the elite controls the government but faces a political constraint: it needs sufficiently many workers to support

⁵We do not introduce the representative worker, because it is more convenient to treat workers as individuals in the political game described later.

the regime. Each worker decides whether to support the regime. If the number of supporters is large enough, the regime survives; otherwise it collapses and democratization occurs. To maintain enough support, the elite-controlled government can strategically influence incomes of workers by changing the state sector wage and capital allocation in the two sectors.

3.1 Preferences, Technology and Markets

Agents live for infinite periods. The preferences of the elite and the entrepreneur are standard:

$$U = \sum_{t=0}^{\infty} \beta^t \log(c_t).$$

They discount the future at the rate β and the rate of relative risk aversion equals one. The elite and the entrepreneur receive incomes from the capital that they supply to the state (S) sector and the private (P) sector, respectively. After the redistribution of the government, they consume and save the rest to the next period. The savings will affect their capital supply and capital income in the next period.⁶ A worker supplies a fixed one unit of labor to a firm. Workers are assumed to be myopic and to live hand-to-mouth: they care only about the current period income and consume all the income every period.⁷

The two representative firms - the state (S) firm and the private (P) firm - are different in two aspects: the access to the capital market and productivities. First, the S firm rents capital in the S sector, which is supplied by the elite, and the P firm can only rent capital in the P sector, supplied by the entrepreneur. They can hire labor from the same pool of workers. Second, the S firm is less productive than the P firm. The technologies of the representative S and P firms are described by Cobb-Douglas production functions:

$$\begin{aligned} Y_S &= (z_S K_S)^\alpha L_S^{1-\alpha}, \\ Y_P &= (z_P K_P)^\alpha L_P^{1-\alpha}, \end{aligned}$$

where $z_S = z < z_P = 1$.⁸ K_S and K_P are S and P sector capital, and L_S and L_P are S and P sector labor, respectively. Capital depreciates at the rate δ . Firms are neoclassical,

⁶Details about the government and the capital market will be explained later in this section.

⁷The myopicity assumption allows for concise analytical expressions. It is not crucial for the theory - as long as a high enough wage buys the support of a work. In the Appendix, I discuss the consequence of changing this assumption.

⁸Notice that the productivities are constant. There is no exogenous technological progress and the economy will converge to a steady state with a constant output, like in a Solow model. Productivity growth is orthogonal to the key discussions in this paper, so it is omitted.

so wages and capital returns, though potentially different in the two sectors, should be equal to the marginal productivities. So wages can be expressed as:

$$w_S = (1 - \alpha) (z_S K_S)^\alpha L_S^{-\alpha}, \quad (1)$$

$$w_P = (1 - \alpha) K_P^\alpha L_P^{-\alpha}, \quad (2)$$

Similarly the gross returns of capital, without depreciation and taxes, are:

$$\begin{aligned} r_S &= \alpha (z_S K_S)^{\alpha-1} L_S^{-\alpha}, \\ r_P &= \alpha K_P^{\alpha-1} L_P^{-\alpha}. \end{aligned} \quad (3)$$

For convenience, we can also denote the net return to an agent i - either the elite or the entrepreneur - who supplies capital to sector j depends on the tax rate for her τ_i and the depreciation as ρ_i :

$$\rho_i = (1 - \tau_i) r_j - \delta,$$

where $j = S$ if $i = e$, and $j = P$ if $i = p$, representing net return of the S sector capital to the elite, and P sector capital to the private entrepreneur, respectively. The net return directly influence the decision of capital supply from this agent.

Now we describe the financial markets. There is a representative competitive bank, as in the neoclassical models. It can save in and borrow from the international bond market at an exogenous interest rate r . So the bank serves as an intermediary to pin down the interest rate. We can safely ignore the bank and simply think that the agents in the model can save and borrow in safe bond at the interest rate r . Denote $R = 1 + r$. The elite can get the bank loan, combines it with savings she has accumulated, and then invests in the S sector. There is no financial constraint on how much the elite can borrow.⁹ The entrepreneur can also borrow from banks, but she faces a borrowing constraint: the bank loans she gets can not exceed $\eta - 1$ fraction of her savings. In other words, the capital she can supply to the P sector is bounded above by the leverage η , as follows:

$$k_P \leq \eta a_p,$$

where a_p is the entrepreneur's asset, and k_P is her capital supply in the P sector. In the equilibrium, the aggregate variables - capital in the P sector and asset of entrepreneurs -

⁹Alternatively, one can think this setting as the state firms borrow as much as they want from the state bank. In oligarchy, the elite is connected to the government, and it is in fact the owner of state firms, so it is equivalent.

is the same as the variables of the representative entrepreneur, so

$$K_p = k_p,$$

$$A_p = a_p.$$

3.2 Political Support for Oligarchy

There are two political systems: oligarchy (O) and democracy (D). Democracy serves mainly as a reference point and will be discussed later in this section. The focus of the model is oligarchy. In oligarchy, the government is controlled by the elite. A representative elite sits in the office, makes government policies to maximize her utility, which is the equivalent to the utility of all elite members. Meanwhile, the government faces the political constraint: it needs support from sufficiently many workers to keep the regime stable. In each period, each worker i , decides whether to support the oligarchic regime ($m_i = 1$) or not ($m_i = 0$).¹⁰ The aggregate mass of supportive workers is $M_w = \int_0^1 m_i di$. If it is larger than a crucial threshold \underline{L} , the regime survives this period, otherwise democratization occurs.¹¹

The political decision of a worker is made after she gets employed - either by the S firm or the P firm - and before she receives her wage and final income. The expected final income of a worker from sector $j \in \{S, P\}$ in oligarchy is denoted as y_{wj} , and after democratization y_{wj}^D . They are endogenously determined by economic factors, which will be explained in this section.¹² So the payoffs can be summarized as in Table 1.

Table 1: Payoffs of Workers

	$M_w \geq \underline{L}$	$M_w < \underline{L}$
$j = S$	y_{wS}	y_{wS}^D
$j = P$	y_{wP}	y_{wP}^D

Obviously, for the myopic worker i in sector S , one (weakly) dominating pure strategy is to support oligarchy if and only if the expected income is higher than in democracy, i.e., $m_i = 1$ if $y_{wS} \geq y_{wS}^D$ and $m_i = 0$ if $y_{wS} < y_{wS}^D$. Similar for P sector

¹⁰When there is no confusion, I omit the time subscript t and also superscript O for oligarchy.

¹¹The setting that only the support from workers counts is without loss of generality, for two reasons. First, the population size of the workers are much larger than the other two groups, so they should count. Second, as we will see later, the elite and entrepreneurs generally prefer oligarchy in most cases, so the model is robust to whether considering their political support or not.

In the extreme case that the elite holds dominating political power, and needs little support from workers - e.g., it uses mostly military force to control the citizens - \underline{L} can be close to 0. In the case that the elite needs to win a majority vote, \underline{L} can be 50% if the voting system is fair, or smaller than 50% if the voting system is manipulated in favor of the elite. A regime with a voting system may not necessarily be democracy. It can still be an oligarchy, and the government serves the interests of the elite.

¹²Workers are ex-ante identical, so there is no need for subscript i to denote worker i .

workers. This strategy expresses the voters' sincere preferences. Without loss of generality, I assume that workers choose this strategy, similar to the sincere voting assumption in the literature.¹³

This political game is a very simple global game of regime switching, in the spirit of Morris and Shin (2000). The game here is simple because there is no heterogeneous information, no cost of being against the regime, and no punishment for the supporters of the regime after the regime collapses. These differences make the workers' optimal actions very simple and intuitive.

The equilibrium outcomes depend on the payoffs, and there can be a couple of different cases. One possible case is the following. The S sector workers expect a high final income: $y_{wS} \geq y_{wS}^D$ and they choose to support the regime. If at the same time the P sector workers expect a low wage: $y_{wP} < y_{wP}^D$, then the regime survives if and only if there are enough workers in the S sector: $L_S \geq \underline{L}$. When does this case happen? It depends on the economic factors, and the government policies which influence the endogenous payoffs.

3.3 The Government

In oligarchy, the government is controlled by the elite. In the beginning of each period, it can choose to voluntarily democratize. If it does so, the regime switches to democracy forever. If not, it can defend oligarchy using the following three sets of policies: (1) tax and transfer; (2) S sector wage; and (3) capital allocation in the S and the P sector.

First, the government can tax the ruled groups, including workers and entrepreneurs, and then it can transfer the tax income to the ruling group, i.e., the elite. Without loss of generality, I assume that the government taxes P sector workers and entrepreneurs at an exogenous rate $\tau > 0$.¹⁴ This assumption about transfer is crucial and formally stated as follows.

Assumption 1. *In oligarchy, the government can only make lump-sum transfer to the ruling group - the elite, but not the ruled group - workers and entrepreneurs.*

¹³Of course, given that there are a continuum of workers, worker i knows that her action does not affect the aggregate political outcome and feels indifferent about what she does. There are other dominating strategies and equilibria with pure or mixed strategies. However, if there are finite workers and there is some small probability that worker i 's choice can be pivotal, then it is wise to follow the sincere strategy described above.

¹⁴The government does not want to tax S sector workers, if it wants to increase their final income to gain their support. If we allow the government to choose the tax rates on P sector workers and entrepreneurs subject to an upper bound, exogenously given, e.g., by the state capacity constraint as in Besley and Persson (2009), then naturally the government wants to extract the private sector as much as possible: it prefers to tax P sector workers at the highest rate if P sector workers are not the supporters, and it taxes entrepreneurs also at the highest rate in most cases. This is equivalent to simply setting the tax rate at an exogenous level.

This assumption is often made in the literature of political economy, because if the government can make lump-sum transfer to any agent, the Coase theorem applies and all types of governments in all political systems should implement efficient policies to maximize the aggregate output, and then simply use transfers to maintain the support. In other words, the political system does not affect the efficiency of the economy and there is little need for political economy. See this discussion in, e.g., Acemoglu (2003). This assumption, in this paper, implies that the government can not simply make lump-sum transfer to some workers to buy their support. It has to implement other distortive policies to gain the support, as we will see later. The budget constraint of the government is balanced in every period, implying that the transfer to the elite is equivalent to the tax income from the private sector:

$$\begin{aligned}
T &= \tau w_P L_P + \tau (r_P + \delta) K_P \\
&= \tau (1 - \alpha) K_P^\alpha L_P^{1-\alpha} + \tau \alpha K_P^\alpha L_P^{1-\alpha} \\
&= \tau Y_P,
\end{aligned}$$

where T is the transfer. Notice that the tax is set as on P workers' labor income and entrepreneurs' raw capital return (before deducting the depreciation). The total tax income is simplify τ fraction of the private sector output.¹⁵

Second, the government can influence the income of workers, in the following way:

Assumption 2. *In oligarchy, the government can set a minimal wage in the S sector.*

Then S firms have to hire workers at a wage which is not lower than than the minimal wage set by the government. If the minimal wage constraint is binding, it increases the wage and the final income of S sector workers, compared to the wage determined in a competitive market. Basically, the government can set the wage in the S sector at a level greater or equal to the wage determined by a competitive market. In the rest of the paper, we simply consider that the government directly set the S sector wage w_S , subject to $w_S \geq w^D$, where w^D denote the wage determined by a competitive market.¹⁶ This policy also changes the employment decision of S firms, and in turn affects the labor and the equilibrium wage in the P sector.¹⁷

¹⁵This setting is not crucial but makes expressions much more concise.

¹⁶As we will see later, the competitive market wage is in fact the wage in democracy. That is why I use the superscript D .

¹⁷Notice that in this model setting a high minimal wage in the P sector to increase the income of P sector workers is not allowed. This setting fits the situation in China, where the government has better control over how the state firms pay to their workers, but much less control over private firms. For example, the government makes the rule that firms should pay for unemployment insurance for their workers, and while

Third, the government, controlled by the representative elite, can influence the capital allocations in the S and the P sector. The government can set the aggregate capital in the S sector. It can also set the level of borrowing constraint that entrepreneurs face: η . Notice that an individual elite member may not prefer to invest in the S sector to the level that the government finds optimal, because an individual elite member does not internalize how her decision affects the equilibrium, while the government does so. This setting implies that in oligarchy the elite is able to solve the collective action problem: the representative elite can decide on the capital in the S sector, optimally for the elite as a group. One way to interpret this setting is that the government sets a level of investment, and each elite member follows that, otherwise she will be punished.¹⁸ The setting that the government can influence the leverage in the P sector means that it can create barriers to entrepreneurs, or give administrative instructions to the bank on how much loan is allocated to entrepreneurs (see Brandt and Zhu (2000)). Moreover, I assume that the influence of the government on η is limited, subject to $\eta \in [\underline{\eta}, \bar{\eta}]$. Then $\underline{\eta}$ is the lower bound of η , and the highest level of financial restriction that the government can set.¹⁹ $\bar{\eta}$ is the highest level of leverage that the entrepreneurs can use, given that the government does not restrict private sector financing at all.²⁰ The setting on the financial market is similar to Song et al. (2011), while an important difference is that here state sector capital and private sector credit constraint are endogenously determined by the government, which allows us to understand and study financial market friction and its future trend.

3.4 Timing

After introducing all agents, we can now set up the detailed timing and complete the model setting in oligarchy. Notice that the active agents in the model include the elite, entrepreneurs, and workers, while the behavior of others - the neoclassic bank and firms

state firms generally follow the rule, many private firms ignore it.

¹⁸There are other ways to interpret this setting. One is that the government directly invests in the S sector, and the cost and benefit of the investment eventually enters the transfer to the elite, through the government budget constraint. One example is the Chinese government's four trillion yuan stimulus package implemented in 2008. Most of the investment goes to the state sector. Another way is that the government plans a level of capital and the corresponding bank loan in the S sector, and then provides subsidy (positive or negative) to incentivize elite members to take the loan and invest in the S sector. Similarly, the cost of the subsidy is paid by all the elite members. Mathematically, these two are equivalent to the first one, where we can ignore the government's expenditures in controlling the S sector capital. More details are in the Appendix.

¹⁹For example, if $\underline{\eta}$ equals 1, then the most stringent policy that the government can set is lending nothing to entrepreneurs. Then entrepreneurs can still finance their investment with their own savings.

²⁰In this case, the bank still prefers to put a limit on the lending to private entrepreneurs, because of a moral hazard problem: if a private entrepreneur gets a too large loan compared to her savings, she may run away with the debt, while this is not a big problem for the state firms.

- can be safely ignored. The government is controlled by the elite, so its decision is simply the decision of the elite. In the following, I summarize the timing of the events in each period, which include 8 steps and are classified as three stages.

Stage 1: determination of capital allocation.

1. In the beginning of a period, the elite decides whether to voluntarily democratize or not. If yes, the political system switches to democracy forever and it is the end of game in oligarchy. If no, the following events happen, subsequently.

2. Capital allocation: the elite sets K_S and η , and then the representative entrepreneur chooses K_P .

Stage 2: the equilibrium given the capital allocation.

3. The elite sets the S sector wage w_S .

4. S and P firms rent capital, and hire workers. Workers are randomly selected by S firms.

5. S and P workers decide whether to support the regime. If there are not enough supporters, the political system switches to democracy. The game in oligarchy ends and the economy enters the equilibrium of democracy given the capital allocation. If there are enough supporters, the oligarchy survives and the following events happen.

6. Firms produce. Labor and capital incomes are distributed.

7. The elite collects taxes for entrepreneurs and P sector workers. Then the tax income is transferred to the elite.

Stage 3: savings to the next period.

8. The elite and entrepreneurs consume and save. The economy enters the next period.

Three stages correspond to step 1 to 2, step 3 to 7 and step 8, respectively. This classification is useful, because stage 2 is a static problem. It allows for more clear analytical solution, and it will be analyzed first. Stage 1 and 3 are the dynamic part of the problem, which are solved mainly numerically, after obtaining the solution to the problem in stage 2.

Notice that the game in oligarchy may end in either step 1 or 5, and then the regime switches to democracy. The exit payoffs can be considered as exogenously given in the game in oligarchy, while they should be determined by the equilibrium in democracy. The payoffs can be different in step 1 and 5, because in step 1, capital has not been allocated into the two sectors, while in step 5, capital has been fixed. Next, we describe the equilibrium in democracy, to pin down the payoffs, and also to serve as a benchmark for later discussion on economic growth in oligarchy.

3.5 Democracy

Though the focus of this paper is oligarchy, the equilibrium outcomes in democracy, including the expected incomes after democratization, are important outside options for workers and the elite when they make political decisions, so it is necessary to use a model of democracy to endogenously generate these variables. On the other hand, these variables are taken as exogenously given in the model of oligarchy, and a different model of democracy which generate similar income levels has similar implication for the model of oligarchy, so it is not necessary to discuss too much in details of the model of democracy.²¹ Here I briefly discuss the settings and outcomes of democracy, and leave the details in the Appendix.

The key difference in democracy, compared to in oligarchy, is that the government is run by the representative worker forever, because their dominating population guarantees that they win the majority voting. The government then taxes the elite and entrepreneurs at the exogenous rate $\tau^D > 0$. The economy is simply a competitive equilibrium given taxes. The model is similar to the classic two sector growth models with competitive equilibria in literature, e.g., Lewis (1954), and it is more closely linked to a more recent paper Song et al. (2011).

The labor market is competitive. This means that wages in the S and the P sector are the same, determined by the marginal productivity of the labor.

$$w^D = (1 - \alpha) (z_S K_S)^\alpha \left(L_S^D\right)^{-\alpha} = (1 - \alpha) (K_P)^\alpha \left(L_P^D\right)^{-\alpha}.$$

A worker's income consists of the wage and the transfer which equals the tax collected from entrepreneurs and the elite:

$$\begin{aligned} y_w^D &= w^D + \tau^D \left(\alpha (z_S K_S)^\alpha \left(L_S^D\right)^{1-\alpha} + K_P^\alpha \left(L_P^D\right)^{1-\alpha} \right) \\ &= \left(1 + \tau^D \frac{\alpha}{1 - \alpha} \right) w^D. \end{aligned} \quad (4)$$

A worker's final income in democracy - y_w^D , given the capital allocation, is the reference point for her political decision in oligarchy.

The capital market is also competitive. Given that the elite can borrow from the international market at the interest rate r without financial constraint, the rate of return for the

²¹For example, one can model democracy differently and introduce the possibility of failed democracy, non-consolidated democracy, or other inefficiencies in democracy, but as long as the expected income of workers and of the elite are not so different from the incomes used here, the main results should still hold.

elite's investment, i.e., the capital in S sector is also r . So the elite's income is simply equal to the return from the saving at the rate r . The entrepreneur faces financial constraint, so potentially the rate of returns to her savings and the P sector capital can be higher than r . In this case, the entrepreneur wants to borrow more but her supply of capital is constrained by $K_P \leq \eta a_p$. The government, controlled by the workers, prefers to set η high, i.e., $\eta = \bar{\eta}$ to maximize the capital in the economy and the wage of workers.

The dynamic equilibrium in democracy is similar to the papers mentioned above: the more productive sector (the industrial sector in Lewis (1954) and the E-sector in Song et al. (2011)) grows and eventually takes over the less productive sector. Here, P sector grows as the entrepreneur accumulates capital, and eventually all workers move to P firms. Before that, S firms exist and hire the surplus labor that P firms do not need. The aggregate TFP grows as the resources are reallocated into the P sector. The economy grows because entrepreneurs accumulate savings and capital.

The equilibrium outcomes in democracy which are important for the model of oligarchy, is summarized in the proposition below. See more details in the Appendix.

Proposition 1 (Equilibrium in democracy). *In democracy, a worker's final income includes the wage and the transfer which is $\tau^D \frac{\alpha}{1-\alpha}$ fraction of wage: $y_w^D = (1 + \tau^D \frac{\alpha}{1-\alpha}) w^D$.*

Income of a elite member comes from the return to her asset at rate r . Her income only comes from her asset and income net of asset return is 0: $y_e^D = 0$. Her lifetime income net of the initial asset is also 0: $V^D = 0$.

The entrepreneur saves β fraction of her total resource - asset plus asset return - at the end of each period. If β is large enough, entrepreneur assets increase over time, and gradually the relative size of S sector over P sector, measured by $\frac{K_S}{K_P}$, decreases to 0.

3.6 Equilibrium in Oligarchy

Variables in the model of oligarchy can be classified into three groups. First, the exogenous variables. They include: firm productivities (z_S and z_P), minimal number of supporters needed (\underline{L}), world interest rate (r), tax rate (τ), the upper and the low bound of the credit constraint (η and $\bar{\eta}$). The exit payoffs, e.g., the worker's expected income after democratization: y_w^D , are also taken as exogenous variables in the model of oligarchy, though these variables are endogenously generated by the model of democracy. The second group includes variables endogenously chosen by the agents - including the elite and the workers - which influence the political outcomes. They include the elite's decision on whether to stay in oligarchy or to voluntarily democratize, credit constraint (η), P sector capital (K_S), and S sector wage (w_S); and each worker's decision on whether to support

the regime (m_i) and its aggregate (M_w). Notice that even if the elite chooses to stay in oligarchy here, the political system is still not decided yet, because later the elite may set policies that attract enough support and sustain oligarchy, or it can set the policies that lead to revolution and democratization. So in fact, the elite can choose three types of political outcome, i.e., $M_e \in \{O, D, R\}$ where O stands for sustaining oligarchy, D for voluntary democratization, and R for revolution. The third group include usual economic choices by individuals: consumption, saving, etc., as in a standard growth model. For example, private entrepreneur asset (a_p) and the elite's asset (a_e) and the corresponding aggregate variables A_p and A_e . An additional variable is the type of the political system: $M \in \{O, D\}$. Of course, in oligarchy, M always takes value O , but it evolves and may turn to D , in two cases: either $M_e = D$, or $M_e = R$ which means that the elite chooses to stay in oligarchy but does not get enough support - $M_w < \underline{L}$. Then the game in oligarchy ends and the equilibrium in democracy starts.

Given these variables, the key events in oligarchy in each period can be summarized as the following, corresponding to the three stages of the game described in timing table. First, given (A_e, A_p) , the elite chooses M_e . If $M_e \neq D$, then it chooses (η, K_S) , rationally anticipating the impact of these choices on the actions of other agents and the equilibrium outcomes. The entrepreneur chooses K_P and the capital allocation is determined.²² Second, in oligarchy, given $(\eta, K_S; A_e, A_p)$, which determine the capital allocation, the elite chooses w_S , and then each worker i , either employed by a S and or a P firm, chooses whether to support oligarchy or not: m_i . The mass of supportive workers is M_w . It determines the political outcome: whether the regime can survive. Finally, after production and distribution of output, the elite and entrepreneurs, decide on the consumptions and savings. The savings (A'_e, A'_p) become the initial states in next period.

Next, we can analyze and solve for the equilibrium in oligarchy. We can first focus on stage 2 of each period - the political and the economic outcome given capital allocation. Then I discuss how capital is allocated and savings are determined.

3.6.1 Equilibrium Given Capital

The following analysis of the equilibrium outcome given capital allocation can be considered as part of the dynamic problem, or a simple static model. Given the capital, then as in step 3 of the timing table the government first chooses S sector wage to influence the

²²The determination of K_P given η is simple. Under the condition that the net return of P sector capital to the elite is larger than r , the choice of the entrepreneur is simply borrow as much as possible, i.e., $k_P = \eta a_p$, and $K_P = \eta A_p$. I leave the proof to the solution to the representative entrepreneur's problem, and verify that this is the case when solving for the model numerically.

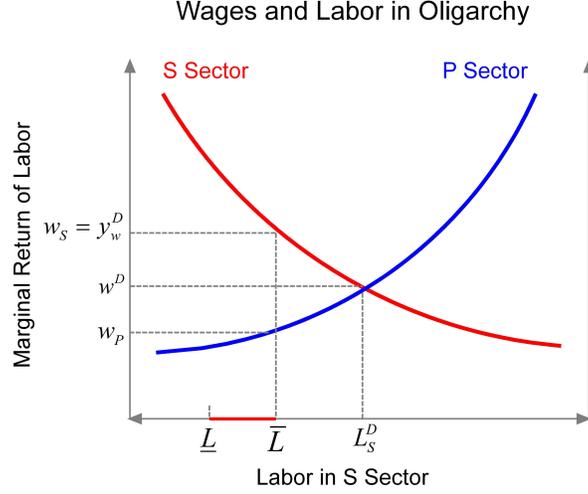


Figure 4: Labor allocation and marginal returns.

labor market outcome and the economic benefits to S and P workers. Then the FOC of S firms, as stated in equation (1), pins down the employment in the S sector: L_S . We can see this in Figure 4. The two lines are the marginal productivities of S sector and P sector labor, respectively. The intersection of the two lines pins down the equilibrium in democracy. The S sector labor, wage and workers' income in democracy are denoted as L_S^D , w^D and y_w^D . In oligarchy, a level of w_S corresponds to a level of S sector labor and its marginal productivity. The rest of labor goes to the P sector and it pins down a P sector wage w_P .

Second, we analyze how wages affect the workers' political support. Remember that the oligarchic government cannot use direct transfer to buy political support, so the final income of S workers is simply $y_{wS} = w_S$. As argued above, in order to get the support of S workers, the S sector wage need to be greater than y_w^D . From the S firm's FOC w.r.t. L_S - equation (1) - we can see that the declining marginal productivity of S sector labor implies that the S sector labor is less than a corresponding level, denoted as \bar{L} . This statement can be formalized as the following equations:

$$w_S \geq y_w^D \Rightarrow \quad (5)$$

$$(1 - \alpha) (z_S K_S)^\alpha L_S^{-\alpha} \geq \left(1 + \tau^D \frac{\alpha}{1 - \alpha}\right) (1 - \alpha) (z_S K_S)^\alpha (L_S^D)^{-\alpha} \Rightarrow$$

$$L_S \leq \nu L_S^D = \nu \frac{z_S K_S}{z_S K_S + K_P} \doteq \bar{L}, \quad (6)$$

where $\nu = \left(1 + \tau^D \frac{\alpha}{1 - \alpha}\right)^{-\frac{1}{\alpha}}$. Furthermore, a relatively small L_S , given capital, implies

a relatively large amount of labor is pushed to the P sector, which reduces the P sector equilibrium wage, and then a P sector worker always gets lower income in oligarchy than in democracy. So P sector workers never supports oligarchy. This logic can be shown as follows.

$$\begin{aligned} w_S &\geq w^D \Rightarrow \\ (1 - \alpha) K_S^\alpha L_S^{-\alpha} &\geq (1 - \alpha) K_S^\alpha (L_S^D)^{-\alpha} \Rightarrow \\ L_S &\leq L_S^D. \end{aligned}$$

Then $L_P = 1 - L_S \geq L_P^D$, and $w_P = (1 - \alpha) (z_S K_S)^\alpha L_S^{-\alpha} \leq w^D$. Given that the government can not make transfer to workers, we have $y_{wP} = (1 - \tau) w_P < w^D < y_w^D \Rightarrow x_P = 0$. This also implies that in oligarchy, there is a wage premium in the S sector: $w_S \geq w^D \geq w_P$ and the inequalities are strict in the case $w_S \geq y_w^D > w^D$.

Third, the number of supporters is also crucial. Under the condition that $w^S \geq y_w^D$, S sector workers - and only S sector workers - support oligarchy. Then if and only if the number of S workers is sufficiently large, i.e.,

$$L_S \geq \underline{L}, \tag{7}$$

the regime receives enough support and survives.

The important message from the analysis above is summarized in the following.

Lemma 1. *The oligarchic regime gets enough political support and survives if and only if two economic constraints are satisfied: $w_S \geq y_w^D$, and $L_S \geq \underline{L}$.*

Now we can see that the political constraint that the government faces - sufficiently many supporters - is equivalent to two economic constraints. The first is the *high state wage constraint*, i.e. $w_S \geq y_w^D$, which is again equivalent to low enough state employment share:

$$L_S \leq \bar{L}. \tag{8}$$

The second is the *minimal support constraint*, i.e., $L_S \geq \underline{L}$.

The government faces a critical trade-off in the labor market: a high enough w_S guarantees the high state wage constraint to be satisfied, but at the same time, it implies a relatively low level of S sector employment L_S , so the minimal support constraint may be violated. Given that \underline{L} is an exogenous parameter, whether both constraints - equation (7) and (8) - can be satisfied depend on the endogenous variable \bar{L} . \bar{L} is governed by the capital allocation K_S and K_P , as stated in equation (6). The larger K_S , relative to K_P , the

larger \bar{L} . So if S sector capital is large enough, then $\bar{L} \geq \underline{L}$, and there exists an $L_S \in [\underline{L}, \bar{L}]$, such that both the *high state wage constraint* and the *minimal support constraint* are satisfied. Formally speaking,

$$\begin{aligned}\bar{L} = v \frac{zK_S}{zK_S + K_P} &\geq \underline{L} \Rightarrow \\ \frac{K_S}{K_P} &\geq \frac{\underline{L}}{z(v - \underline{L})}.\end{aligned}\tag{9}$$

In other words, sustaining oligarchy requires that S sector is equipped with enough capital, relative to the P sector capital. Intuitively, when there is a high level of S sector capital, even given a high w_S , the S sector can hire enough workers.

The analysis above can be summarized in the following proposition.

Proposition 2 (Equilibrium given capital allocation). *If S sector capital is relatively large compared to the P sector capital, i.e., $\frac{K_S}{K_P} \geq \frac{\underline{L}}{z(v - \underline{L})}$, oligarchy can survive. In this case, the wage premium in the S sector buys support from S sector workers: $w_S \geq y_w^D > w_P$, and the sufficient capital in the S sector guarantees that there are enough workers in the S sector: $L_S \geq \underline{L}$. If $\frac{K_S}{K_P} < \frac{\underline{L}}{z(v - \underline{L})}$, democratization happens.*

The proof of the proposition follows the logic discussed above. More details of the proof, and some additional properties of the equilibrium given capital are in the Appendix.

In the case that oligarchy survives given w_S , the one period income of the elite, can be written as:

$$\begin{aligned}y_e &= (zK_S)^\alpha L_S^{1-\alpha} - w_S L_S - (r + \delta) K_S + \tau (K_P)^\alpha L_P^{1-\alpha} \\ &= \alpha (zK_S)^\alpha L_S^{1-\alpha} - (r + \delta) K_S + \tau (K_P)^\alpha L_P^{1-\alpha}.\end{aligned}\tag{10}$$

y_e is the income net of asset return. It includes the profit from the S sector (the first two terms) and the tax collected from the P sector (the last term). We can also denote the total income, including the asset return as $y_e^{tot} = ra_e + y_e$.²³ Similarly, the income of an entrepreneur with capital k_P is

$$\begin{aligned}y_p &= ((1 - \tau) r_P - \delta) k_P - r k_P \\ &= (1 - \tau) \alpha K_P^{\alpha-1} L_P^{1-\alpha} k_P - (r + \delta) k_P,\end{aligned}\tag{11}$$

²³Notice that there is no need to think about how much of asset is invested into the S sector and how much is saved in the bank, because interest rates in both places are the same: r .

where y_P is the entrepreneur's income. Of course, in the equilibrium $K_P = k_P$ and we have

$$y_P = (1 - \tau) \alpha K_P^\alpha L_P^{1-\alpha} - (r + \delta) K_P.$$

Notice that r_P is the gross return of capital in the P sector, and the net return to the entrepreneur - $(1 - \tau) r_P - \delta$ - takes into account tax and depreciation. The first term in equation (11) is the return from investment, and the second term is the cost. The entrepreneur takes the return to capital r_P as given, which depends on and can be expressed as a function of aggregate variables including (K_P, w_P, L_P) , which are again determined by (K_P, K_S, w_S) . If we also count the return from asset, the total income is $y_P^{tot} = r a_P + y_P$.

The analysis of the equilibrium given capital shows that the economic power determines the political outcome: the sizes of capital in the state and in the private sector shape political and economic outcomes, including the survival of the regime. Then, how capital is allocated into the two sectors and how agents accumulate assets - which affect the capital formation - are crucial for the elite. Next, we study these dynamic decisions on how capital is allocated and accumulated.

3.6.2 The Dynamic Equilibrium

Given the solution to stage 2 of the game (outcomes given capital) above, we can study stage 1 (capital allocation) and stage 3 (asset accumulation) of the game in each period.

Let us first look at the representative entrepreneur's problem. The entrepreneur, in stage 1 chooses capital supply to the P sector (k_P) and in stage 3 chooses the saving (a_P). What simplifies this problem significantly is that an entrepreneur is infinitesimal and takes aggregate variables including the dynamics of prices and the political system as given. So the problem can be formed as:

$$\begin{aligned} \max_{\{k_{Pt}, a_{Pt}\}} \quad & \sum_{t=0}^{\infty} \beta^t \log c_{Pt} & (12) \\ \text{s.t.} \quad & c_{Pt} = R a_{Pt} + y_{Pt} - a_{P,t+1}, \\ & y_{Pt} = ((1 - \tau) r_{Pt} - \delta) k_{Pt} - r k_{Pt}, \\ & k_{Pt} \leq \eta_t a_{Pt}. \end{aligned}$$

We can denote the net return of capital to the entrepreneur as $\rho_{Pt} = (1 - \tau) r_{Pt} - \delta$.²⁴

²⁴Introducing ρ simplifies the notation, but given that there is a one-to-one mapping between ρ and r_P , we can simply think r_P as the return to capital that we need to track in the model and define in the equilibrium.

When choosing k_{pt} , the entrepreneur anticipates r_{pt} and ρ_{pt} rationally and takes them as given. Intuitively, the decision of k_{pt} is simple: if r_{pt} is large enough and the net return of investment ρ_{pt} is larger than the cost, i.e., $\rho_{pt} = (1 - \tau) r_{pt} - \delta > r$, she wants to invest as much as possible, i.e., $k_{pt} = \eta_t a_{pt}$. Intuitively, the return from P sector capital will be high enough if P sector productivity z_P is high enough, and given that we normalize z_P to 1, this is equivalent to say that z_P relative to z_S , i.e., $1/z_S$, is large enough. In fact, this is always the case throughout this paper given calibrated parameter values. We can safely ignore other cases and only consider $k_{pt} = \eta_t a_{pt}$. Moreover, the saving decision of the entrepreneur is also simple: given the log utility and that income is proportional to asset, she constantly saves β fraction of her total resource in hand - asset and income - into next period to smooth consumption. Moll (2014) solves a similar problem of entrepreneur's decisions. The solution to this problem is summarized below and formally proved in the Appendix.

Lemma 2. *In oligarchy, if the P firm capital return r_P is sufficiently high, the entrepreneur's optimal choice can be decomposed into two steps. First, she borrows as much as possible and invests all into P firms $k_P = \eta a_p$, to maximize her current period income; and then in the second step, she saves a constant fraction of this period's total wealth and consumes the rest, to maximize her lifetime utility.*

Of course, we can also write the entrepreneur's problem recursively. The entrepreneur, at the beginning of each period, solves the following problem: ²⁵

$$\begin{aligned}
 W_p(a_p; A_e, A_p) &= \max_{k_p, a'_p} \log c_{pi} + \beta W_p^{M'}(a'_p; A'_e, A'_p) & (13) \\
 \text{s.t. } c_p &= R a_p + y_p - a'_p, \\
 y_p &= ((1 - \tau) r_P - \delta) k_P - r k_P, \\
 k_P &\leq \eta a_p,
 \end{aligned}$$

where k_P and a_p are variables chosen by the representative entrepreneur as an individual, while A_p is the aggregate variable for all entrepreneurs which the representative entrepreneur takes as given. Also, the entrepreneur takes other aggregate variables as given and rationally expect them determined by the state variables by equilibrium functions and laws of motion. These variables including current period price and policy r_P and η , and future aggregate variables: the next period political system $M' \in \{D, O\}$, elite asset A'_e and aggregate entrepreneur asset A'_p . We can denote the equilibrium functions

²⁵The details on how to write the entrepreneur's problem in this recursive form is left in the Appendix.

for r_p and η simply as $r_p(A_e, A_p)$ and $\eta(A_e, A_p)$. Future variables are expected to follow laws of motion: $M' = \mathcal{L}_M(A_e, A_p)$, $A_e' = \mathcal{L}_{A_e}(A_e, A_p)$, $A_p' = \mathcal{L}_{A_p}(A_e, A_p)$, where \mathcal{L}_X denotes the law of motion of variable X . Because all entrepreneurs are identical, in the equilibrium, $A_p = a_p$. Still, in the recursive form, it is necessary to also use a_p , to avoid the case that an individual entrepreneur believes her choice of a_p can affect the aggregate variables.

Next, we define the representative elite's dynamic problem recursively. This problem contains two stages. First, choice of the political system, in step 1 in the timing of events. The elite solves

$$W(A_e, A_p) = \max \left\{ W^O(A_e, A_p), W^D(A_e, A_p), W^R(A_e, A_p) \right\}, \quad (14)$$

where W is the value function representing the elite's lifetime utility, and W^O is the value function conditional on choosing to sustain oligarchy, while W^D and W^R are the value functions conditional on choosing to voluntarily democratize and to stay in oligarchy without getting enough support, or equivalently, to create revolution. The last two value functions are determined by the equilibrium in democracy and can be considered as exogenously given in the game of oligarchy. Second, if she chooses W^O , then she stays in power and picks government policies (η, K_S, w_S) to make sure that the political constraint is satisfied. She also decides consumption and saving to maximize her lifetime utility.

$$\begin{aligned} W^O(A_e, A_p) &= \max_{\eta, K_S, w_S, C_e, A_e'} \log C_e + \beta W(A_e', A_p') & (15) \\ \text{s.t. } w_S &\geq y_w^D(\eta, K_S, w_S, A_p), \\ L_S(K_S, w_S) &\geq \underline{L}, \\ A_e' &= RA_e + y_e(\eta, K_S, w_S, A_e) - C_e, \\ A_p' &= \beta(RA_p + y_p(\eta, K_S, w_S, A_p)). \end{aligned}$$

From the equilibrium given capital allocation, we know how incomes (y_w^D, y_e, y_p) are determined by (K_S, K_P, L_S) , as in equations 4, 10 and 11. From the entrepreneur's problem, we know that K_P is determined by (r_p, η, A_p) and r_p is determined additionally by (K_S, w_S) . So eventually, the incomes can be expressed as functions of (η, K_S, w_S, A_e) .²⁶

²⁶We can simply substitute K_P and L_S expressed by η, K_S, w_S, A_p into equations (10) and (11). The exact forms of these functions are in the Appendix.

The function determining variable x given the set of state variables \mathcal{S} is denoted as $x(\mathcal{S})$, or simply $x(\cdot)$, e.g., $y_e(\eta, K_S, w_S, A_e)$. The function is simply denoted using the same letter of the variable. In the rest of the paper, when there is no confusion, I use this type of notation instead of creating a new letter for the function.

Notice that in the constraints, we substitute into the workers' optimal choice - supporting the regime as long as the wage is high enough - and the entrepreneur's optimal choice - saving β fraction of her total resource.

An crucial feature of this problem is that the elite faces no financial constraint. In other words, she can choose K_S and other variables that are crucial for the equilibrium and other agents, including also M, w_S, η , no matter which level A_e is at. The only contribution of A_e is RA_e in the elite's budget constraint, and it is only used for consumption smoothing. Therefore, the representative elite's problem, similar to an entrepreneur's problem, can be separated into two sub-problems. This result is stated as the following lemma and the proof is in the Appendix.

Lemma 3. *In oligarchy, the representative elite's optimal choices can be separated into two sub-problems. First, maximization of the lifetime income with discounting rate $\frac{1}{R}$ by choosing government policies (η, K_S, w_S) . Second, maximization of the lifetime utility by choosing A_e to smooth consumption.*

In the first subproblem, the lifetime income is defined as the discounted future income net of the return from the elite's asset, RA_e , and its state variable only includes A_p but not A_e .

The first sub-problem is:

$$\begin{aligned}
 V(A_p) &= \max \left\{ V^O(A_p), V^D(A_p), V^R(A_p) \right\}, \\
 V^O(A_p) &= \max_{\eta, K_S, w_S} y_e(\eta, K_S, w_S, A_p) + \frac{1}{R} V(A'_p) \\
 \text{s.t. } w_S &\geq y_w^D(K_S, \eta, A_p), \\
 L_S &\geq \underline{L}, \\
 A'_p &= \beta (RA_p + y_p(\eta, K_S, w_S, A_p)),
 \end{aligned} \tag{16}$$

where V is the value function representing the discounted lifetime income of the elite excluding the return to its initial asset RA_e , and V^O, V^D, V^R stand for the case of sustaining oligarchy, voluntary democratization and revolution.²⁷ The second subproblem is simply

²⁷In fact, $V^D = 0$. Intuitively, if it is in democracy, the elite enjoys no privileges like taxing and transfer anymore, and each elite member supplies capital to the competitive market and the cost and the return are the same. So the elite's income in each period, net of the return from the asset at rate r , is simply 0.

spending the lifetime income to maximize the lifetime utility:

$$\begin{aligned} & \max_{\{C_{et}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \log C_{et} \\ \text{s.t. } & \sum_{t=0}^{\infty} \frac{C_{et}}{R^t} = RA_{e0} + V(A_{p0}). \end{aligned}$$

Notice that in the elite's problem, differently from the entrepreneur's problem, we do not distinguish the individual elite member's capital supply and asset from the aggregate capital supply and asset. The problem is formed as if the representative elite directly chooses the aggregate capital supply in S sector and savings of all elite members. There are two reasons. First, the aggregate capital supply K_S , as discussed previously in the model setup, can be directly chosen by the government, or equivalently the representative elite, because the government is able to solve the collective action problem.²⁸ Second, for elite saving A_e , it does not matter if the representative elite chooses if for all elite members or each member makes her own choice, because A_e does not affect the important equilibrium outcomes except the elite's consumption. The government has no incentives to make it different from each individual elite member's choice. The standard "small k, big K" problem in the recursive dynamic programming does not exist here.²⁹

Finally, the recursive equilibrium in oligarchy can be defined as follows.³⁰

Definition 1. The recursive equilibrium in oligarchy consists of

sets of prices $w_p(A_e, A_p), r_p(A_e, A_p)$,
laws of motion $\mathcal{L}_M(A_e, A_p), \mathcal{L}_{Ae}(A_e, A_p), \mathcal{L}_{Ap}(A_e, A_p)$,
value functions $W(A_e, A_p), W^O(A_e, A_p), W_p^O(a_p; A_e, A_p)$,
decision rules $M_e(A_e, A_p), \eta(A_e, A_p), K_S(A_e, A_p), w_S(A_e, A_p), A'_e(A_e, A_p), k_p(a_p; A_e, A_p),$
 $a'_p(a_p; A_e, A_p), m_i(w_S, K_S, K_P; A_e, A_p)$ and
aggregate variables $K_P(A_e, A_p), L_S(A_e, A_p), L_P(A_e, A_p), M(A_e, A_p), M_w(A_e, A_p)$, such that

²⁸As discussed in details in the subsection about the government, the government may incentivize each elite member to supply capital at the level using punishment or subsidy, or it may directly make the state sector investment.

²⁹In the entrepreneur's problem, and generally in problems of the recursive dynamic programming, it is necessary to use the "small k, big K" formation to distinguish the individual agent's variable from the aggregate variable because the aggregate entrepreneur asset affect the equilibrium outcome, and it is crucial to make sure that the individual agent knows that her decision does not affect the equilibrium. Introducing a_p or not changes the solution of the recursive problem. In the elite's problem, if we introduce the individual elite's asset a_e , it can be verified that the result is the same.

³⁰Notice that here when defining the equilibrium in oligarchy, the equilibrium outcomes in democracy, e.g., $W^D(A_e, A_p), W^R(A_e, A_p), W_p^D(a_p; A_e, A_p)$, can be considered as exogenously given.

(a) Given laws of motion, the value function $\{W, W^O\}$ and decision rules $\{\eta, K_S, w_S, a'_e\}$ solve the elite's problem (14)(15); the value function W_p^O and decision rules $\{k_p, a'_p\}$ solve the entrepreneur's problem (13); and the decision rule m_i solves workers' problems.

(b) Given $\{w_S, K_S\}$, L_S satisfies the S firm's optimization problem - equation (1). Given $\{w_P, r_P\}$, K_P and L_P satisfy the P firm's optimization problem - equations (2) and (3).

(c) The laws of motion $\{\mathcal{L}_M, \mathcal{L}_{A_e}, \mathcal{L}_{A_p}\}$ are consistent with the equilibrium outcomes $\{M, A'_e, A'_p\}$. Aggregate variables $\{K_P, A'_p, M, M_w\}$ are consistent with individual's choices $\{k_p, a'_p\}$.

3.7 Analytical Properties of the Equilibrium

Now we can solve the equilibrium. In fact, the problems of firms, workers, and the entrepreneur are straight-forward and solved analytically already, e.g., the S firm chooses L_S according to the FOC w.r.t. L_S ; a worker supports the regime if her income is higher in oligarchy; and the entrepreneur borrows as much as possible and saves β fraction of her total resource - income and asset - to the next period. The key problem left is the elite's dynamic programming problem. Here we characterize some crucial properties of the elite's problem analytically, and leave the complete solution in the quantitative exercise.

In the first subproblem, the elite maximizes the lifetime income, as in equation (16), taking only the entrepreneur's asset as the state variable. In the second subproblem, the elite chooses A'_e to maximize her lifetime utility. This shows that for the equilibrium in oligarchy, the entrepreneur's asset is the crucial state variable. It affects the key variables of the economic and political outcomes, including the elite's decisions on whether to democratize, choices of government policies, and its lifetime income. There is an important trade-off of the private entrepreneur asset, or generally speaking, the private sector size. Intuitively, a larger private sector, e.g., a larger K_P , contributes more taxes to the elite, but it may also incur a cost to the elite, as we see from the problem given capital: a too large private sector capital relative to the state sector capital makes it impossible to buy enough supporters, so given a large private sector capital, if the elite wants to protect oligarchy, it has to invest at least proportionally in the state sector, and given the decreasing return to capital, the investment in the state sector may lead to larger cost than return. This means that the elite prefers a median size of K_P . Moreover, K_P depends on A_p , and the elite has some but limited influence on K_P given A_p : it can set η high if it wants K_P to be high, but the upper bound is $\eta \leq \bar{\eta}$, and similarly if it wants to set η low. Given the limited influence on η , a large A_p allows for a potentially large K_P . However, a too large A_p implies that K_P can not be too small, even if the elite prefers a smaller private sector. The same tradeoff

of K_P maps into A_p . When A_p and K_P are too large, maintaining oligarchy requires very large K_P and it can be too costly for the elite, and it may even prefer democratization to oligarchy. This trade-off captures the key properties of the dynamic programming, and leads to the following property to the elite's choices: when A_p and K_P are small enough, the elite prefers them to be larger, and it chooses a high η , and oligarchy over democracy. When A_p and K_P are large enough, the elite prefers them to be smaller, and it chooses a low η , and democratization over oligarchy. This is formally stated in the proposition below.

Proposition 3 (Elite's Problem in Oligarchy). *(1) If the private sector capital is small enough, then the one period income of the elite in oligarchy is larger than in democracy. Moreover, an increase of the private sector capital leads to an increase of the elite's one period income in oligarchy.*

(2) If the private sector capital is large enough and the constraints of sustaining oligarchy - (7) and (5) - are binding, then the one period income of the elite in oligarchy is lower than in democracy. Moreover, an increase of the private sector capital leads to a decrease of the elite's one period income in oligarchy.

(3) The above results still hold if the private sector capital is replaced by the entrepreneur's asset, under the condition that the former increases as the latter increases.

(4) The above results still hold if the elite's one period income is replaced by the elite's lifetime income or utility, under the condition that V_e and $\partial V_e / \partial K_P$ are continuous in the discount rate of future income,

The proof and the more formal mathematical expression of the proposition is in the Appendix. The Proposition 3 formally state the key tradeoff of a large private sector, and the similar tradeoff of the high entrepreneur asset. Result (1) states that that given K_S , if K_P is small, a larger K_P increases the elite's current period income, and moreover, if K_P is small, the elite can achieve higher current period income in oligarchy than in democracy. The logic is the following: when K_P is small, its marginal return is large, because there is always a significant amount of labor pushed out of S sector given the high S sector wage. Then for the elite, a larger K_P means larger current period income. Moreover, the elite can achieve higher income in oligarchy because it is the cost of keeping enough supporters in S sector is low, compared to the return from taxing. Result (2) implies that if K_P becomes very large, the cost of maintaining oligarchy increases with K_P , and dominates its benefit. Because a large K_P requires a large K_S to maintain oligarchy and there is decreasing return to capital. This is true for the marginal cost and benefit in oligarchy, and also true for the cost and benefit in oligarchy compared with in democracy. Result (3) states that if K_P is positively correlated with a_p , naturally the same properties hold for a_p . These previous

results focus on the elite's one period income, while result (4) states that if the elite discounts the future sufficiently, these properties still hold for the elite's lifetime income and lifetime utility. The analytical results hold under certain restrictive conditions, but in the quantitative exercises, these conditions are generally satisfied.

Given these properties, we can see that the elite prefers the private sector to develop to a median size, and we can expect that as the private sector grows, the transition dynamics may be like the following: when the entrepreneur asset is small, the elite chooses to sustain the regime and promote private sector growth; however, when the private sector gradually grows larger, the elite changes the attitude and restrains its growth, and if the entrepreneur asset and private sector capital become too large, the elite prefers peaceful democratization to maintaining oligarchy at the high cost. Whether the entrepreneur asset can grow to the high level or it converges to a steady state level which is lower, depends on the model parameters and determines the long-run outcome. The following quantitative analysis formally shows this.

3.8 Quantitative Analysis

I calibrate the model to the Chinese economy and solve the model. The dynamics is simulated, to understand China's development in the past and to give a prediction about its future. The targets of the calibration are the key facts in China's recent development, including the wage gap, speed of privatization, and the state employment share.

3.8.1 Benchmark Calibration

In the benchmark, the model is calibrated to the Chinese economy. First, the production function is Cobb-Douglas with the capital share $\alpha = 0.5$ (Bai et al. (2006)).³¹ The depreciation rate is set as $\delta = 0.1$ (Song et al. (2011)). The state capital efficiency is half of the private capital, i.e., $z_S = 0.5$. This implies that the TFP of state firms is 71% of the TFP of private firms. This is higher than 59% estimated by Hsieh and Klenow (2009) with data before 2005, but is reasonable considering that the trend of declining TFP gap discussed in Hsieh and Song (2015). Second, the interest rate of bank saving is $r = 5\%$. Third, the discount factor of entrepreneurs, which is also the saving rate of their lifetime income, is

³¹One important feature in China's recent development is that the labor share has been declining. This seems to be inconsistent with the Cobb-Douglas production function with a constant labor share. However, Song et al. (2011) show that the declining labor share can be reconciled in a two sector model with Cobb-Douglas production. They explain the decline of labor share by the labor reallocation from the state sector to the private sector, where the labor share is smaller, not because of a different production functions but the payment to the management. I follow the literature and keep the Cobb-Douglas production function setting.

set to $\beta = 0.9$ to match the rapid private sector employment share growth from around 40% to around 80% in 5 years, as we can see from Figure 2. Finally, the tax rate is set to $\tau^D = \tau = 20\%$ to match the state-private wage gap of 30%, as in Figure 1. The political parameter in this model is \underline{L} , the minimal support needed to sustain oligarchy. I set $\underline{L} = 20\%$, to be consistent with the trend that the state employment share decreases and converges to around 20% as in Figure 2.³²

3.8.2 Numerical Solution

The calibrated model is numerically solved. Moreover, I also rely on numerical solution to study the properties of the dynamic equilibrium, especially the elite's problem, which can be decomposed and explained in three steps: (1) given K_S and K_P , the choices of w_S and the equilibrium values of other variables like L_S, y_e ; (2) given K_P , the choice of K_S ; and (3) the choice of η that affects K_P .

First, given K_S and K_P , we know from the equilibrium solution given capital allocation, that if K_S is large enough, there is some w_S that sustains oligarchy, or equivalently, some L_S that falls into the region $[\underline{L}, \bar{L}(K_S, K_P)]$. Generally, the optimal choice of w_S is y_w^D , or equivalently, $L_S = \bar{L}$. This choice implies the least labor distortion but still satisfies high state wage constraint.³³ In other words, the elite prefers not to distort the labor market more than the necessary.³⁴

Second, how does the government choose K_S , given K_P ? In Figure 5, a numerical example shows how the choice of K_S affects the elite's one period income, and also state sector labor, political outcome, and marginal benefit of state capital for the elite. Given certain K_P , if K_S is above a certain level, $\frac{K_S}{K_P}$ is high enough, and then as discussed in the equilibrium given capital allocation, state firms can hire enough workers (upper-left

³²This number is not large compared to some European countries, while it is indeed larger than East Asian countries like Korea (7.6%) and Japan (7.9%).

In this simple and abstract model, the supporter share equals the state employment share and it is not large. The supporters are economic agents who care only about the economic benefits. In a natural extension, one can model as the government gains support not only using economic benefits but also using its ideological and cultural influence. There can be two groups of citizens: some are economic agents who care only about economic benefits, while the others can be influenced by the government's cultural or ideological policies. So the government can, for example, gain the support from 30% of the latter group by leveraging its ideological policies, and if it still needs an additional 20% support from the first group of economic agents, it behaves exactly as in this model. This extension may make the supporter share reasonably larger, but it does not add much insight, so I keep the simple model.

³³This is true as long as the tax rate τ is not too high. One sufficient condition is $\tau \leq \alpha$, which is a reasonable constraint, considering that α is as high as 0.5. If τ is too large, and the elite can extract more from the private sector than from the state sector, the solution may change.

³⁴As an extension, if I allow the elite to endogenously chooses tax rates on workers and entrepreneurs subject to an upper bound constraint $\bar{\tau}$, the numerical solution shows that it will choose

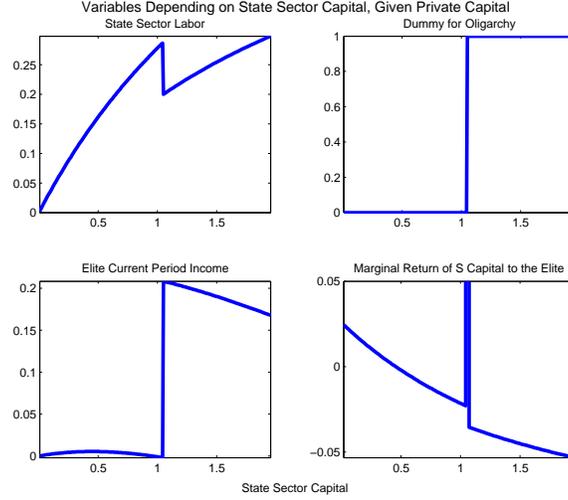


Figure 5: The outcome depending on choice of K_S

panel) - in this case, $\underline{L} = 0.2$ - and therefore oligarchy gets enough support and is sustained (upper-right panel). Then there is a jump in elite income right at the critical level of K_S (lower-left panel) because above that level the elite sustains oligarchy, controls the government and taxes others. For this reason, though as K_S increases, its return declines and even goes below 0, the elite may still prefer to keep investing in the state sector until K_S reaches the critical level, where the marginal return is infinite (right-lower panel).

In the example above, given that particular level of K_P , choosing K_S that just sustains oligarchy gives highest current period income to the elite. But for other levels of K_P , the situation may be different. The upper-left panel of Figure 6 shows the optimal K_S corresponding to different levels of K_P : when K_P (the x-axis) is very small, K_S is negatively correlated with K_P , and L_S is larger than \underline{L} (upper-right panel). In this region, the private sector is not large enough to hire enough workers and make the minimal supporter constraint binding. Then it is safe and optimal for the elite to reduce K_S in response to a larger K_P . However, when K_P is large enough, and S sector labor reaches the minimal level \underline{L} , a larger K_P requires a corresponding larger K_S , if the government wants to sustain oligarchy. We can see from the lower-left panel that while a larger K_P increases the benefit for the elite - the tax income, it also creates higher cost - a larger cost of investment. Given the decreasing return to capital, there is a median level of K_P that maximizes the elite income in the current period (right-lower panel).

How does the government set K_P , given a_p , close to the optimal level? It is through the private firm leverage η : the choice of η affects the entrepreneur's borrowing capacity and capital available for P firms given that $K_P \leq \eta a_p$. When a_p is small and the government prefers a larger K_P , it sets $\eta = \bar{\eta}$: it imposes no additional credit restriction in addition

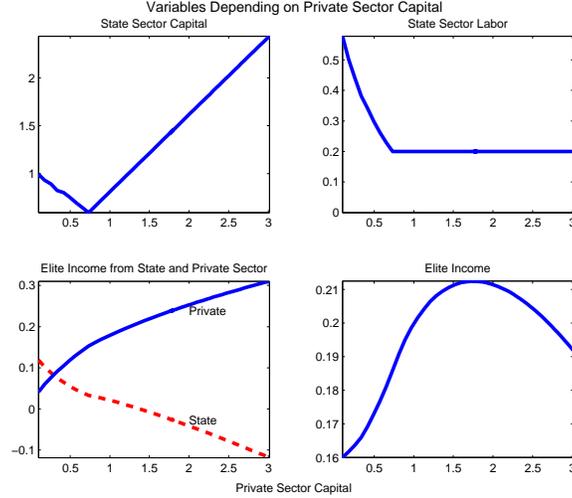


Figure 6: The outcome depending on choice of K_S

to the natural level generated by the moral hazard problem in the market. When a_p is large and the government wants a smaller K_P , it sets $\eta < \bar{\eta}$, possibly $\eta = \underline{\eta}$, and P firms receive a lower level of bank loan than they can in a competitive market without this additional friction. This can be seen from Figure 7. As a_p moves to the right in the x-axis, from a small level to a large level, η goes down gradually (as in the upper-left panel) as the government prefers K_P to be close to its preferred median level. K_S first goes down but then goes up proportionally to K_P (upper-right panel), to guarantee enough S sector workers (lower-left panel). The government's influence on K_P is bounded because the choice of η is bounded by $\underline{\eta}$ and $\bar{\eta}$, so it may not be able to set K_P to the optimal level when a_p is too small or too large. For this reason, for an intermediate level of a_p , the elite lifetime income is maximized (right-lower panel). This is the second trade-off for the elite. This following remark essentially restates this tradeoff, which also exists in the first part of proposition 3, with more details from the numerical solution.

Remark 1 (Trade-off of private sector capital). A larger K_p contributes more tax income to the elite, but it also requires a larger K_S to sustain oligarchy. Both the return and the cost increase with K_P . As K_P increases, starting from a small enough level, the elite's current-period income first increases and then decreases. The elite's lifetime income also follows this pattern. This trade-off also applies to entrepreneur asset level a_p , given that K_P is non-negatively correlated with a_p .

In the above numerical example, for the levels of a_p and K_P shown in Figure 7, which are not too large, the optimal choice of the elite is always to sustain oligarchy by investing in the S sector proportionally. However, this is not always true, when K_P is large enough.

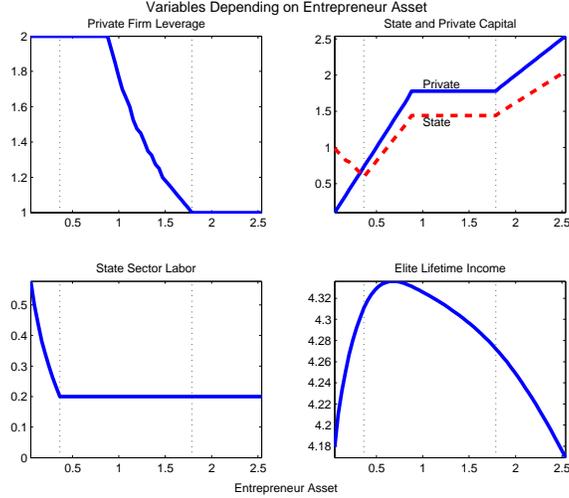


Figure 7: Equilibrium variables, depending on the entrepreneur’s asset.

Of course, the government always has the option to set K_S at a high level that sustains oligarchy, but the cost of the large investment may dominate the return, i.e., the tax income. If K_P and a_p are large enough, sustaining oligarchy leads to a lower lifetime income of the elite compared to democracy - the line for the elite’s income in Figure 6 and (7) may drop below the zero line as K_P and a_p get large enough: $V(a_p) < 0 = V^D(a_p)$. In this case, the elite chooses to democratize.

3.8.3 Transition Dynamics

Given these parameters and the numerical solution, I simulate the economy starting from a low private entrepreneur asset and a small private sector: $a_p = 0.05$, and then study the transition to the steady state. Figure 8 and Figure 9 show the transition in the benchmark calibration where $\underline{L} = 0.2$. The key variables, including capital, labor, P firm leverage and output in oligarchy are the solid lines. In comparison, I also simulate the transition in democracy from the same initial state, following the solution in Proposition 1. The variables in democracy are shown with the dashed lines.

In oligarchy, starting from a small a_p , the private sector is small and hires a low amount of workers. It is not a threat to the supporter base of oligarchy so the elite encourages it to grow, in order to extract more tax income. Therefore, in this stage (from period 1 to 5 in Figure 8) the government sets $\eta = \bar{\eta}$ (as in the lower-right panel of Figure 8) and private firms can borrow at their maximum capacity. Moreover, private firms and entrepreneurs benefit from low wage and abundant labor, so private sector capital grows fast, and even faster than in democracy (in the upper-left panel). State employment and capital decline

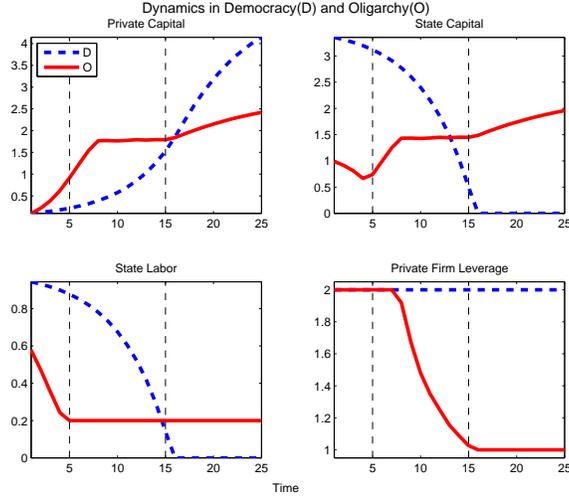


Figure 8: Dynamics in democracy (dashed) and oligarchy (solid) ending in middle-income trap.

accordingly (in the upper-right panel and lower-left panel). Because of this reallocation of capital and labor into the more efficient private sector, the economic growth is rapid (Figure 9). Notice that the output in oligarchy starts from a lower level compared to in democracy, because of the distortion of labor allocation in oligarchy, but the rapid capital accumulation and resource reallocation make the output catch up fast. This is the first stage of the transition, which features and is therefore called *rapid growth*.

As the private sector grows and the state employment share declines to the critical level \underline{L} (as in lower-left panel of Figure 8, from period 5 to 15), the economy enters the second stage. The declining state employment share threatens the supporter base of oligarchy. If no action is taken, the elite cannot keep the political power any more. So in order to sustain oligarchy, it increases the state sector capital K_S (upper-right panel) and later it restricts the lending to private firms by setting $\eta < \bar{\eta}$ (lower-right panel). Because of the policies in favor of state firms, the state sector maintains its relative size and hires \underline{L} workers with high enough wages. The privatization process stops: no more labor and capital reallocation to the more efficient private firms. However, the output can still grow in the beginning of this stage (in Figure 9, from period 5 to 8) driven by the large investment in state sector. Gradually, the growth slows down (period 8 to 15) because the financial restriction on private firms harms the economic efficiency. This stage features large state investment and financial market favoring state firms, so it is a stage of *state capitalism*. Notice that though in the first stage the output is lower in oligarchy, in the second stage it can catch up with the output in democracy because of the large state investment (see Figure 9).

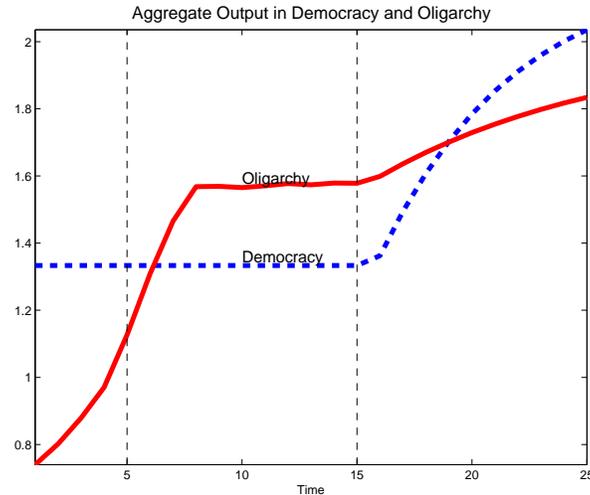


Figure 9: Output in democracy (dashed) and oligarchy (solid) ending in middle-income trap.

What happens in the long-run? Will the state capitalism continue forever or will democratization occur? From the elite's problem, we have learned that the elite's choice on sustaining oligarchy or democratization depends on the cost of buying enough supporter, which in term depends on the size of the entrepreneur asset and private sector capital. So in the long-run, if a_p converges to a sufficiently low steady state level, which is the case in the benchmark calibration, the elite always finds it optimal to sustain oligarchy along the whole transition. It keeps investing in the state sector proportionally to the private sector capital as the latter grows (in the upper panels of Figure 8, from period 15). Employment share stays at \underline{L} . Though the elite has to pay the investment cost, the return from tax extraction is higher, so it prefers oligarchy to democracy. The key patterns in the second stage - including over-investment in state firms and financial discrimination against private firms- continue. Growths of private sector capital and aggregate output are harmed by the capital market friction. Eventually growth gradually slows down and output stops growing at a middle level, which is lower than in democracy (see Figure 9). So in this case, the third stage is called "*middle-income trap*".

This result from the benchmark calibration predicts that China will stay in oligarchy and will fall into the middle-income trap, given the current fundamentals. This is not surprising. The government is powerful, both politically and economically: first, the political power of the government is significant so together with support from a relatively small fraction of workers, it is able to keep the current regime stable; and second, the economic resource that the government holds is also abundant - e.g., the large foreign reserve - which allows the government to increase the investment in the state sector eas-

ily when necessary. For instance, after the 2008 financial crisis, the Chinese government initiates the four trillion yuan stimulus package and most of it flows into the state sector. This shows that the government is able to keep the economy and resource allocation under control. According to this theory, it implies that the government is able to maintain a powerful state sector to guarantee political stability.

The political power of the government, captured by \underline{L} , is the crucial parameter determining the cost of sustaining oligarchy, a larger \underline{L} compared to the level in the benchmark calibration implies the larger cost of maintaining support in oligarchy and may lead to a different political choice of the elite and a different long-run development path. If we change \underline{L} to be large enough, e.g., $\underline{L} = 0.4$, while holding other parameters the same as in the benchmark calibration, democratization will occur during the transition. In this case, sustaining oligarchy requires two times more state workers, so the elite has to invest much more in state sector given certain private sector capital level. As the private sector capital grows large enough, the elite finds the cost of maintaining the state sector too large, and it is optimal to democratize. This development path is different in the long-run, or equivalently, the third stage, compared to the development in the calibration to China, but it is similar in the first two stages. Starting from small a_p and K_p , private sector employment share grows and reaches the critical level for sustaining oligarchy (see the lower-left panel of Figure 10, from period 1 to 21). Then the government over-invests in state sector to maintain enough supporters for oligarchy (the upper-right panel). Finally, the development path differ from the third stage: K_S reaches a very high level and the elite optimally chooses to democratize; after democratization, the over-investment and over-employment in the state sector are removed immediately (upper-right and lower-left panel, from period 22 on), while the private sector investment rises up (upper-left panel) as the extra financial constraint on private firms are removed by the democratic government (lower-right panel). This divergence of two paths is the so-called “critical juncture” in development (see Acemoglu and Robinson (2012)). The output, as shown in Figure 11, slightly drops after democratization due to the rapid decline of the state sector, however, it eventually recovers and converges to a high level. This model offers a theoretical explanation for the empirical findings on the sharp deceleration in growth following democratization and the more stable long-run growth in democracy, e.g., in Pritchett and Summers (2014).

In both cases, the transition includes three stages, and the properties are summarized as follows.

Remark 2 (Three stage transition). The economy, starting with a small enough private sector, follows a three-stage development path.

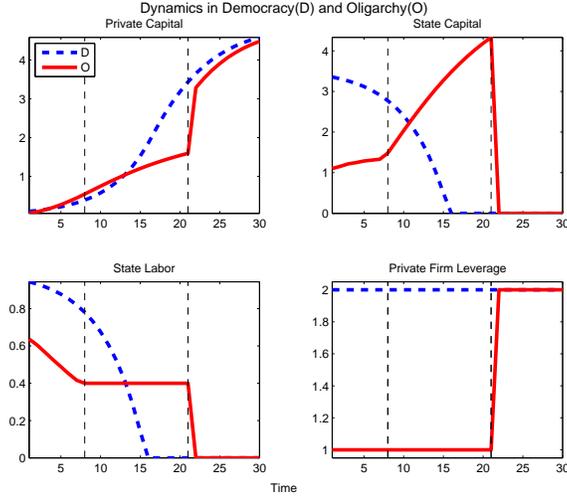


Figure 10: Dynamics in democracy (dashed) and oligarchy (solid) ending in sustained growth.

Stage 1: *rapid growth*. Growth rate is high. Private sector grows rapidly, benefiting from the low wage. The government encourages private sector growth and does not impose financial restriction: $\eta = \bar{\eta}$. Rapid privatization reallocates labor from the state to the private sector.

Stage 2: *state capitalism*. Growth continues. The government over-invests in the state sector, while restricting private firms' access to the financial market: $\eta \leq \bar{\eta}$. Privatization stops and the state employment share stays at the critical level \underline{L} .

Stage 3: Two cases.

Case A: *middle-income trap*. Oligarchy is sustained permanently and growth slows down. State investment keeps growing at the same rate of the private sector capital, to keep state employment share constant at \underline{L} . Repression on private firm reaches the tightest level $\eta = \underline{\eta}$. This happens if \underline{L} is sufficiently small.

Case B: *sustained growth*. Democratization occurs and output growth recovers again. Capital market and labor market distortions are removed. State sector capital and labor drop while the private sector grows. This happens if \underline{L} is sufficiently large.

3.9 Implications of the Model

3.9.1 Static Implications of the Equilibrium Given Capital Allocation

Following how we solve the model - first the equilibrium given capital allocation and then the dynamics of capital - we discuss the model implications in these two steps. First, the implication of the equilibrium given capital allocation, which can be considered as

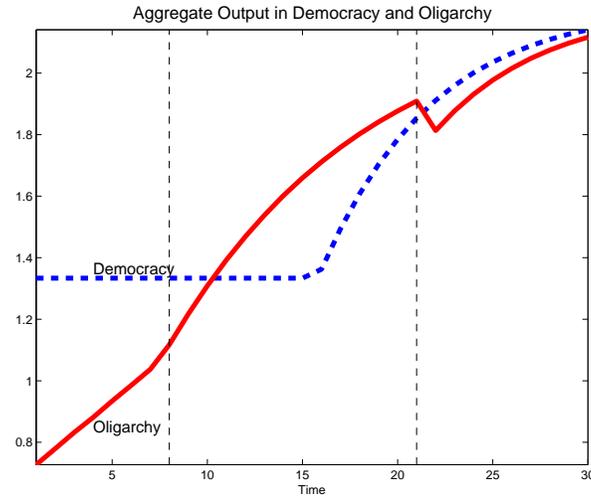


Figure 11: Output in democracy (dashed) and oligarchy (solid) ending in sustained growth.

the static implications for the current China. In the equilibrium, the government creates a dual labor market: state workers get high wages and hence support the government, while private workers get low wages. This is the so-called “divide-and-rule” strategy: breaking workers into two sub-groups, and providing different economic interests to gain support from one group at the cost of the other. This is consistent with three phenomena in current China: (1) large state-private sector wage gap, (2) middle class’s political support for the current regime, and (3) higher capital labor ratio and low capital return in the state sector.

The first fact is discussed in section 2, and is the immediate consequence of proposition 2. High state sector wages are necessary for getting political support from workers, and the general equilibrium effect leads to abundant and cheap labor in the private sector. Entrepreneurs and private firms benefit from the abundant cheap labor, at least in the short-run. This allows potentially faster capital accumulation and growth of the private sector and the whole economy.

Second, the middle class workers - state workers in the model - are supportive to the existing political regime because of the economic benefits they receive.³⁵ This is consistent with the finding of Chen and Lu (2011) discussed in section 2, but contrary to the traditional wisdom that the middle class are the natural driving force of democracy, as in the European history. In the history of democratic movements in Europe, such as the

³⁵Entrepreneurs, as the other group of the middle class in the model, also support oligarchy. Their short-run income in the equilibrium given capital is higher in oligarchy, due to the cheap labor. This is also generally true for their lifetime income in the calibrated dynamic model, which will be discussed in the implications of the dynamic model and details are in the Appendix.

Glorious Revolution and French Revolution, the middle class were against the aristocracy of the kings, whose political power relied on repression. The middle class did not rely on the state but emerged from private businesses. In contemporary China, the state sector is large and a significant fraction of the middle class are created by and rely on the state, so in turn they become supporters of the regime. It is also similar in many other developing countries. This helps to understand why in some emerging markets, economic growth and the burgeoning bourgeoisie do not automatically lead to growing demand for democratization. For example, as reported in *The Economist* 2009, 95% of adult Kuwaitis work for the government, usually in white-collar civil-service jobs which are typical middle class jobs, while its private-sector middle class consists almost entirely of foreigners. There the wage gap between the state and private sector is large. These distortions keep local workers, who are politically important, in the state sector and is an efficient way for the elite to maintain oligarchy.

The third fact is well documented in the literature. Song et al. (2011) show that state sector capital labor ratio is much larger than the private sector, in every industry. Brandt and Zhu (2010) show that the capital return in the state sector is lower than 5% while the number for the private sector is above 50%. The difference in capital returns is partly due to the difference of wages and distorted labor allocations. The other reason is capital allocation, as we will see in the discussion of the dynamic equilibrium below.

In summary, the equilibrium given capital allocation helps to illustrate the interactions of the political and economic systems in oligarchy in each period. On the one hand, the political interests, largely shape the economic frictions and outcomes. Taking into account political considerations, we can explain many economic phenomena and puzzles in China. On the other hand, economic power determines political outcome. Only when the state sector is economically powerful and equipped with enough capital, can the elite keep a large enough supporter base to sustain oligarchy.

3.9.2 Dynamic Implications

Corresponding to the second part of the model solution - the dynamics of capital - here we discuss the dynamic implications. The political and economic development consists of three stages. The first two stages are consistent with China's recent development. From 1997 to around 2003, it is a stage of rapid privatization, as the state employment share declines dramatically (see Figure 2). The private sector, in terms of both employment share and output, grows rapidly, for two reasons. First, the wage is low in the private sector. Compared to state firms which face regulations on wages and other pay, including pension tax, health insurance, unemployment insurance and so on, private firms pay

relatively low wages, which allow for high capital returns. Therefore, private firms accumulate capital rapidly and grow fast. The low wage keeps Chinese private firms competitive. It greatly contributes to the growth of export and the growth of the economy. Second, the government encourages the private sector growth, because a larger private sector contributes more tax while it is still not too large to threaten the supporter base of the government - state employment. So the government encourages various financial resources flowing into the private sector, not only bank loans but also foreign direct investment (FDI), and so on.

At around 2003, as the state employment share approaches the critical level, the privatization slows down and stops dramatically. The direct reason is that investment becomes even more diverted in favor of state firms but not private firms. State sector investment share stays at around 60% though its employment is much smaller (see Brandt and Zhu (2010)). The state over-investment helps to maintain state employment, but it reduces the capital return of state firms. In the private sector, the capital return is high, not only because they are more efficient and pay lower wages, but also because of the credit constraint: private firms cannot get enough bank loans for their high return projects. In fact, the financial constraint on private firms has been getting tighter over time, signaling growing repression on them. This is formally documented as the growing state-private capital wedge in Brandt and Zhu (2010). The phenomenon that state firms receive more and more financial support while private firms face tighter and tighter constraints on their growth has gained attention in the public and are called "the state advances as the private sector retreats". For example, in the passenger airline industry, by 2006, eight private carriers had grown rapidly and had challenged the three state-controlled majors, thanks to the previous government policies encouraging private investors to enter. However, afterward, the government policies switch to favoring the state airlines and keeping them alive, including direct investment and purchase of stocks from the central government. The state airlines not only survive and also successfully defend their dominance position from the challenge of private carriers. This theory explains why this is happening: in the second stage - *state capitalism* - the government policy changes to defending the relative share of state firms.

4 Conclusion

This paper proposes a political-economic theory to understand China's recent development and to predict China's future economic and political transition. The political constraint interacts with economic policies, which lead to a three-stage transition. The first

two stages are *rapid growth* and *state capitalism*, which are consistent with several salient aspects of China's development, including (1) rapid growth with repressed wage in the private sector; (2) political support from the middle class, including state sector workers and private entrepreneurs; and (3) financial constraints on private firms and support for state firms. In the future, i.e., the third stage of development, China is likely to enter a *middle-income trap* given the current conditions, especially the economically and politically powerful state. To switch to the other development path that leads to *sustained growth*, necessary reforms have to be taken, though such reforms may face resistance from the elite.³⁶

Even though the focus of this paper is on China, it is also useful for understanding the development of many other emerging countries and some developed countries with similar patterns. The key political constraint in the theory also exist in some other countries such as Kuwait where the political elite need to gain support from public workers, and Korea before 1980s, when politicians need support from citizens from industries tightly connected to them. Similar stories have occurred in Korea. Before 1980, the large conglomerates (chaebol) in Korea were granted privileged access to low-cost credit and the employment share of small and medium enterprises (SME) had stagnated. The difference is that after 1980, democratic movements and financial reforms happened together, and the employment share of SMEs increased from 50% to 68% in 1990 and continued in the early 1990s after democracy was consolidated. This political and economic development path is consistent with the second case in the theory: the *sustained growth*.³⁷ the

Moreover, the theory is also useful to think on a important question in development: should other developing countries apply the "China model", i.e., the combination of authoritarian politics and state-guided capitalism, to promote economic growth? Some suggestions in favor of adopting this model are based on its past success, but the long-run outcome should be carefully examined and distinguished from the short-run effect. This paper provides a quantitative framework to evaluate the economic and political consequences of "China model", and also "Korea model".

In this paper, the model is made simple and abstract, in order to keep it a tractable framework while preserving the main elements in the theory. Of course, in the future research, more elements can be incorporated into this model, to make it richer, more realistic, and useful to study more questions. Other approaches to gain support, including propaganda, repression, and transfer, can be introduced, to discuss different forms of

³⁶In the Appendix, I discuss the consequences of financial reforms, political reforms and so on.

³⁷See Song et al. (2011) for more details. They also discuss the case of Taiwan and the drop of SME employment share before 1971 and the increase until 1991.

non-democratic societies and the possible switches between the different forms along the transition. When the theory is used to discuss other countries, the distinction between state and private firms can be dropped, and more generally, political connected firms which the government has better control over, and other firms which rely mainly on the market, can be used.

Further empirical work can be done to examine the theory, especially the three-stage political-economic transition. Which conditions determine the transition to democracy and long-run growth? Is it consistent with the theory? The theory predicts that if a country can easily build a large state sector, for instance due to rich natural resource, is more likely to sustain the oligarchy; in contrast, if efficiency is very important, for example in an high-technology oriented country, democratization is more likely to occur. Anecdotal evidence from Gulf countries in comparison to Korea seem to support the theory. Still, more systematic evidence will be useful to check and improve the theory.

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5 Appendix (Available Online)

First, I present some extensions of the benchmark model including reforms changing the fundamentals of the model, the entrepreneur's political attitude, and so on. Then I discuss the setting and assumptions of the benchmark model in the main text and the consequences of changing them. Finally, I provide mathematical proofs to the propositions and lemmas, and also provide more mathematical details for some of the discussions in the main text.

5.1 Reforms and Comparative Statics

Is China doomed to fall into the middle-income trap? Not necessary. If the underlining conditions change, the policies and the development path can change accordingly. Mapping into the model, if the parameters such as $\underline{L}, \underline{\eta}, z_S$ change, the government policies and the dynamics, including the third stage, will change. Many policy suggestions on how to switch China's development to a more sustainable path have been made by economists and China watchers. For example, Gary Becker suggested that financial reform should be implemented, in order to allocation more resource to private firms, and rural immigrants should be given more rights. Will the government take the suggestions and implement all the policies and reforms that sustain growth? We need to notice that policies or reforms that benefit economic growth may not benefit the elite, who is very influential in the government.

Suppose the government takes a reform that gives more political rights to workers, especially the immigrant workers. This implies that the government has to buy support from a larger fraction of the population. We know that if \underline{L} increases from 0.2 to 0.5 leads to democratization and sustained growth. But does the elite like that? Its income goes down to 0 if democratization occurs, so obviously this reform will encounter strong resistance from the elite.

In the above model, I assume that the government is completely under the control of the elite. Some may believe that, in some cases, some technocrats become powerful in the government, and they care only about the output growth in the long-run, but not the economic benefit of the elite. In this case, they can initiate reforms which correspond to changing the key parameters of the model, such as $\underline{L}, \underline{\eta}, z_S$. To which extend they can push the reform to depends on their political power in the government. P can be one of the key parameters $\underline{L}, \underline{\eta}, z_S$. Notice that I consider reform as changing parameters but not the endogenous policy variables such as K_S, η . This implies that technocrats get a chance to push for a big change of the society and the political and economic system,

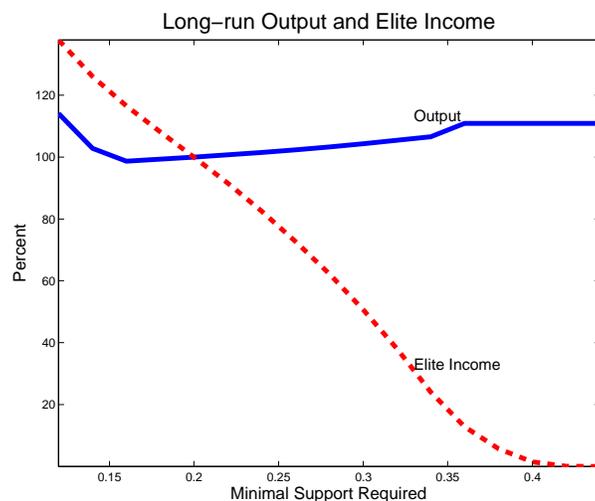


Figure 12: Elite income and long-run output depending on \underline{L} .

and afterwards, the government decisions will be made by the elite. The consequences of reforms are then essentially the comparative statics of the model.

Figure 12 shows how the elite's lifetime income V , and the long-run output Y_∞ which the technocrats care respond to different levels of \underline{L} . Technocrats would like to increase workers' political rights and increase \underline{L} from the current level $\underline{L} = 0.2$, because this makes the government invest more in the state sector, or even choose to democratize. Both of them lead to larger output levels. However, the reform as the result of the bargaining can only push \underline{L} to the right limited by α . If α is small, the increase of workers' political rights won't be large.

Similarly, financial reform, which reduces the financial restriction on private firms can be considered as increasing $\underline{\eta}$. It again increases output, because the private firms can grow larger, and it may even lead to democratization. But again, it harms the elite interests and is hard to be implemented.

One exception is the state firm reform. If a successful reform is taken to increase state firm productivity and reduce the TFP gap between the private and state firms, it increases the output potential. More than that, under the condition that oligarchy is sustained, a more efficient state sector implies that the government can allow the private sector to grow more without worrying about their supporter base - state workers. Less repression on private firm is needed and higher economic efficiency can be achieved. This reform also increases elite income because of higher total output. Figure 13 plots how the long-run efficiency, measured as the long-run output in oligarchy over democracy, can be improved by a more efficient state sector (in the region $z_S \in [0.6, 0.75]$), while the elite income always increases with that. This reform is more likely to be implemented the gov-

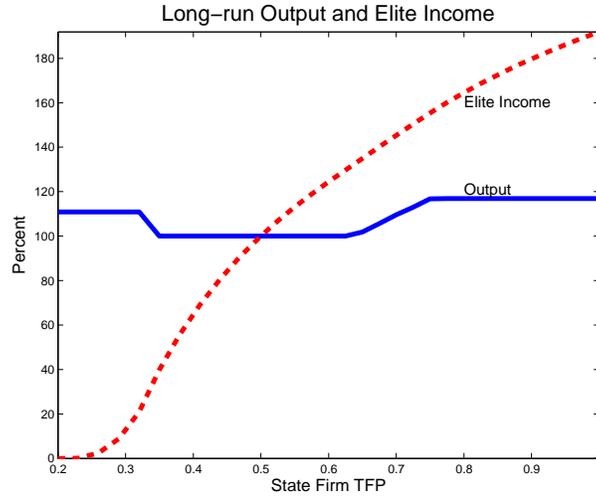


Figure 13: Elite income and long-run output depending on z_S .

ernment. In fact, this is happening right now in China. Hsieh and Song (2015) document that the state-private TFP gap is declining. The so-called “industrial upgrading”, which aims at building high-tech state firms, is at the top of the agenda for China’s further economic reforms. However, it is also very difficult to completely close the gap between the state and private firms, because they are less flexible and provide less economic incentives for the managers, compared to private firms.

5.2 Political Roles of Entrepreneurs

In the benchmark model, entrepreneurs, given the small population, are assumed to have no important political power. Moreover, each entrepreneur runs a small firm and expects her behavior has no influence on the politico-economic development. What can happen if these assumptions change?

Suppose the political power of entrepreneurs is not 0 but ω_p , then the oligarchic government may want to not only buy the support from workers but also from entrepreneurs. However, this does not have a large effect on the equilibrium, because even in the benchmark model where the elite does not care the political support from entrepreneurs, entrepreneurs do prefer oligarchy to democracy, in almost all stages of development. As Figure 14 shows, the lifetime utility of entrepreneurs in oligarchy (solid line) is much higher than in democracy (dashed line) in the early stage of development when entrepreneur asset is low. This is quite intuitive: entrepreneurs benefit from the abundant and cheap labor in the private sector due to the labor market distortion and the government imposes no extra financial constraint on them. Shouldn’t entrepreneurs expect financial restriction

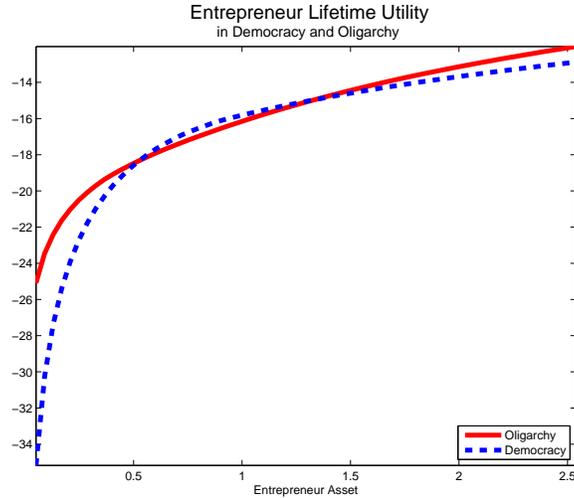


Figure 14: Entrepreneur Lifetime Utilities in Democracy and Oligarchy

in the future and prefer democracy? Not really. Each entrepreneur may prefer removing the restriction on her so that she can borrow more. However, entrepreneurs as a group may actually benefit from the financial restriction, because it lowers the capital stock in the private sector from the competitive equilibrium level, and makes it closer to the monopolistic level. Entrepreneurs get higher return on their asset in oligarchy. That is why, as we can see from the right part of the figure, that even when high entrepreneur asset is large, which means there is financial restriction, the entrepreneurs' lifetime utility is still higher in oligarchy. Only in a small region of entrepreneur asset, entrepreneurs' lifetime utility is lower in oligarchy than in democracy, but the difference is small. This implies that the oligarchic government gets the support from entrepreneurs almost for free, and it does not have to change the policies too much if entrepreneurs become politically powerful. This is consistent with findings from Chen and Lu (2011).

If entrepreneurs solve the collective problem and can as a group strategically decide the aggregate asset in the private sector, will they save more to promote democratization? Given that entrepreneurs generally prefer oligarchy to democracy, this is unlikely to happen.

5.3 Discussion of Model Settings and Assumptions

In this subsection, I explain the reasons for the crucial assumptions and the important settings in the model. Some of them are specific to current China, for example, the assumption that the government cannot directly set state firm labor but has to influence it in a more indirect way through prices. Modeling China 30 years ago or other countries

requires changes on these assumptions, but they are reasonable for China now. Some other assumptions, e.g., no transfer to the ruled, are more general phenomena which are frequently seen in other studies for other countries.

Assumption 1 is that the government cannot make direct transfer to the ruled groups to buy their political support. Though direct transfer seems to be cheap and attractive, in the political economy literature it is considered as difficult to implement, for two reasons. One is the credibility and commitment problem. Acemoglu (2003) and Acemoglu and Robinson (2005) argue that even if the state promises to make a transfer to the ruled group, the latter, without political power, gets no guarantee that it will eventually receive the transfer. The promise of transfer is not credible, and transfer cannot be used to solve all the political conflicts. The other reason why direct transfer is difficult to implement in reality is the high cost due to local capture. Reinikka and Svensson (2004) document that 87% of the transfer from the central government to local schools in Uganda was not received during 1991-1995 due to local capture. This means that the cost of 1 dollar of transfer is as high as 7.7 dollars. For these two reasons, the government usually builds inefficient “white elephant” projects (see Robinson and Torvik (2005)) to guarantee the economic benefits for certain groups. In this paper, the government has to inefficiently distort the prices and allocations in the labor and capital markets to buy the support.

The second assumption that the government can only control S sector wage but cannot directly set S sector labor, and generally in this model, the government can only influence agents’ decisions through manipulation of prices and policies, means that the oligarchic government is not totalitarian. Each S firm is free to make its own employment decision. Moreover, it is responsible for the decision and pays the wage bill on its own - the government provides no labor subsidy. This is consistent with the current situation of state firms in China. After a series of state-owned enterprises (SOE) reforms, especially after the 15th Party Congress in 1997, management decisions have been gradually delegated from central and local governments to the firm level, in order to improve their efficiency. The official slogan is to turn SOEs into modern firms which “operate independently and assume sole responsibility for its profits and losses” (*zizhu jingying, zifu yingkui*). The first part of this slogan maps into the setting in this model that S firms decide employment by themselves. The second part essentially means that S firms maximize profits and there is no subsidy to firms. Of course, the government still maintains its influence over state firms, but indirectly, through the regulations on state sector wages, and the allocation of loans.

The following settings in the model are also important and worthy of explanation and clarification. First, the logic for the minimal number of supporters \underline{L} to maintain

the regime is similar to Morris et al. (2002) where the regime switches if more than a number of agents attack the regime. Alternatively, \underline{L} can also be micro-founded based on Acemoglu et al. (2012). If the elite and their supporters form a coalition which has large enough political power, oligarchy is sustained. More specifically, a coalition of a set of agents holds a corresponding level of political power. If the political power of a certain coalition is large enough, it can choose the political system. In this paper, under oligarchy, the elite as the ruling group is granted political power ω_e . Each worker has political power ω_w . The aggregate political power of entrepreneurs is assumed to be 0, given its size of 0. Workers can change the political regime from oligarchy to democracy if and only if they form a coalition of size L_r whose power is larger than α , namely $\omega_w L_r / (\omega_w + \omega_e) > \alpha \Leftrightarrow L_r > \alpha (\omega_w + \omega_e) / \omega_w$, where α is exogenous. In other words, to sustain the oligarchy, there must be at least $1 - \alpha (\omega_w + \omega_e) / \omega_w$ workers supporting it. This size is denoted as \underline{L} . Notice that \underline{L} captures the relative political power of the elite compared to workers. If the elite is very powerful, it needs only a small fraction of workers as supporters to form a winning coalition. If workers are well-organized and politically motivated, \underline{L} becomes large.

Second, in the model, the government can set the minimal wage in S sector but not in P sector. This is natural in China, and also in other countries. The government has better control over the state sector, so if it sets a minimal wage, state firms have to obey it. However, the government cannot set a binding minimal wage in the private sector, because its ability to monitor private firms is weaker. Private firms can find ways to walk around the wage regulation if the government forces them to pay higher wages. In some countries, where the public sector is very small, clientelism is often in the form of subsidizing a subgroup of private firms - those well-connected to the elite. This model can be easily modified to capture that.

Next, I will discuss what happen if some of the assumptions are relaxed and some settings are modified. These modifications are useful if the theory is extended to discuss the development of other countries which may have different environment compared to China.

5.4 Transfer to the Ruled

In the benchmark model, I assume away the possibility that the oligarchic government can use lump-sum transfer to buy the support, following the political economy literature and to be consistent with the observation in China. However, it is still interesting to think on what should happen if lump-sum transfer is allowed. Moreover, perhaps in some

countries and certain special circumstances, lump-sum transfer can be used as a credit way to buy political support. Below I discuss three alternative settings on how transfer is made and their consequences.

First, suppose that in oligarchy transfer can be made to state workers, but its size is constraint by an upper bound which is tight and smaller than the transfer that workers receive in democracy. Then only using the transfer is not sufficient to buy the support, so the government still chooses to distort the labor market to increase the wage of state workers. Results will be qualitatively similar to the benchmark model.

The second setting is the following: there is no constraint on the size of transfer, but it can only be made to state workers. Then the government can use transfer to get support from state workers, and it can reduce labor market distortion, though it does not have to remove the distortion. However, capital market may still be distorted because the government needs to maintain enough workers in the state sector. Moreover, a very large private sector pushes up the wage and the cost of maintaining oligarchy. Compared to the benchmark model, in this setting, the static equilibrium given capital allocation becomes less distorted, but the properties of the dynamic equilibrium should be still the same. The cost of maintaining oligarchy become smaller, which makes democratization less likely to happen, while qualitatively the dynamics should be still similar.

Third, if there is no restriction on how the transfer can be made in oligarchy, then the government can simply pick a group of private workers, pay them transfer and turn them into supporters. Then it is not necessary to keep the state sector in the long-run. One choice for the elite is to pick policies that maximize the discounted output for all periods and use transfer to redistribute and sustain the regime. Actually, the elite can achieve higher utility than that. It can even strategically reduce the capital stock, which reduces workers' expected income after revolution, and makes it cheaper to hire labor and to buy supporters. In any case, there is no cost of maintaining oligarchy and the elite never chooses to democratize.

5.5 More Government Control

Previously, state firms are modeled as small firms which take prices and policies as given while employment decisions are made by firms to maximize their profits. This is consistent with the outcome of China's SOE reforms. Moreover, the government has no control over the wage in the private sector. This means that given capital, the government can only use state sector minimal wage to influence labor allocation and workers' income. What happen if we change these settings? I consider the following different scenarios.

First, suppose that the government can at the same time decide state employment. In this case, the government can increase public sector wage without worrying about reducing public employment. Then it is equivalent to the case that the government can pay lump-sum transfer to state workers. An interesting outcome of this setting - the opposite of the benchmark model - is that there may be “over-employment” in the state sector. Even given relative small state capital, there can be at the same time over-payment and over-employment in the state sector, while in the benchmark model, over-payment implies under-employment. In economies where state firms are under tight control of the government, this phenomenon is likely to be observed.

The second thought experiment is to allow the government to set wages of some private firms. Korea before democratization maps into this case. The large local conglomerates (chaebol) are private firms but they provide support for the government. The government’s supporter base can be built on these firms instead of less efficient state firms. This of course is cheaper than building the supporter base only in the state sector. However, labor market distortions and capital market distortions are still similar to the benchmark - just replace state firms with these connected firms. Moreover, it is reasonable to think that these private firms should become less efficient over time, because they rely on the capital market advantages and are less exposed to competition. If this is the case, it is equivalent to the benchmark model.

5.6 Endogenize Tax Decisions

Tax decisions, including the target groups and the rate can be endogenized. The same results stay. The setting in Acemoglu (2008) and Besley and Persson (2009) can be used. Suppose that the government decides which groups to tax and tax rates, and tax payers can hide their income at some cost. In democracy, the cost is τ^D fraction of the tax base. Then the government sets tax rate at the highest level which does not trigger tax hiding - τ^D . In oligarchy, if the elite is allowed to choose the target groups, it optimally chooses not to tax S workers, because this makes it easier to buy their support. The elite may or may not set the tax rate on entrepreneurs to the highest possible level, depending on how much asset entrepreneurs hold. In the calibrated model, the numerical solution with endogenous tax rate decision is that the elite always optimally chooses to tax entrepreneurs and P workers at the highest rate τ , so it is equivalent to the simple setting of exogenous tax decisions.

5.7 Forward-Looking Workers

If workers are forward-looking, when they decide whether to support the regime or not, they take into account their future income. From the conclusion that in the long-run democracy brings higher output, one natural intuition is that then the forward-looking workers prefer democracy more than the myopic workers, and the oligarchic government needs to provide higher current period wage premium for them. However, it is not necessarily true. As shown in Figure 9, in the medium-run the output is higher in oligarchy than in democracy. Depending on how patient workers are, they may find the discounted future income in oligarchy more attractive and it can be cheaper to buy their current support. Of course, there can be some interest dynamics, i.e., in the first and second stage, when the growth in oligarchy is promising, state workers requires not so high wage premium; and when the growth starts to slow down, state workers become more willing to switch to democracy and they require more compensation to support the regime.

5.8 Cost of Democratization

There can be some costs associated with democratization, including revolution, and even peaceful democratization. They can be exogenous - the natural costs of changing regimes, or endogenous - created by the government which tries to prevent democratization.

First, the cost of revolution changes the workers' expected income in democracy and the wage premium, but this cost never gets paid, because revolution will be off equilibrium in the quantitative analysis. The intuition is the following: even if there is no cost of revolution, for the elite, choosing policies that leads to revolution is dominated by either sustaining oligarchy when the private sector is small or peaceful democratization when the private sector is very large. The intuition for the first part is simple: oligarchy implies paying no tax and getting transfer for the elite, and when the private sector is small, oligarchy is cheap to sustain. When the private sector capital gets large and staying in oligarchy is not attractive anymore, the elite has two choices - peaceful democratization and choosing policies that lead to revolution. After peaceful democratization, all privileges disappear and the elite only gets the return r on its asset. If it chooses not to democratize, it can set the state and the private sector capital such that there are not enough supporters for oligarchy. However, in the calibrated model, before the elite gives up oligarchy, the private sector capital is already so high that then state sector capital return is already lower than r , and setting state sector capital to any positive level means lower return than saving the asset in banks and get the return r . So revolution is worse than

peaceful democratization for the elite. ³⁸If there is positive cost of revolution, revolution becomes even less attractive, so the result that revolution never occurs should be robust. However, the cost of revolution reduces the workers' expected income in democracy. If it is not very large, it shifts the state wage premium down without changing the economy qualitatively. If it is large enough, workers, either in the state sector or the private sector, are not willing to pay the cost to revolt. The elite maintains oligarchy forever, without paying any state wage premium. More interestingly, if the elite can pay lower state sector wages than in the private sector and force workers to stay in the state sector, it prefers to do that. Because for the elite, the marginal return of labor in the state sector is higher than the marginal return of labor in the private sector, provided that the social returns are the same. The elite extracts a fraction of marginal return of the private sector labor using taxes, while the whole marginal return of the state sector labor goes to the elite, through state firms. In such a totalitarian regime, the government keeps more workers in the state sector than the socially efficient level, at the cost of state workers. Those who can escape to the private sector are the lucky ones with higher labor productivities, though they may not get higher income, because the government is also likely to impose other costs to the private sector workers, in order to prevent the escape.

If the cost of democratization affects the cost of maintaining oligarchy as stated above, it is reasonable to consider an extension that the elite can strategically influence the expected cost of democratization to workers. The cost may be real - investing military can increase the cost of revolution, or it may simply be a bias of the workers' expectation due to incomplete information and propaganda. In either case, the oligarchic government can invest in the technology that increases the expected cost of democratization. The investment can be interpreted as control from the government, which is costly for the government but also increases the expected cost of revolution. If the cost of control increases with the cost of democratization, their dynamics should be the following: in the early stage of development where maintaining oligarchy is relatively easy, the control is loose - the elite does not bother to pay high cost of control to reduce the cost of maintaining oligarchy only a little; when the private sector gets big and it becomes more costly to maintain the regime, the elite tightens the control. The political environment is expected to become tighter, same as the financial environment. This is contrary to the conventional wisdom which expects the environment to be more open and freer, but consistent with empirical findings of Cantoni et al. (2014) on China's textbook reform between 2004 and

³⁸Notice that this is true when there is no uncertainty - the elite expects the future perfectly and plans rationally. If there are shocks, the political process may run out of control of the elite - e.g., an unexpected loss of state capital or increase of workers' power.

2010 that led to “more positive views of China’s governance, changed views on democracy, and increased skepticism toward free markets”.

5.9 Proof of Proposition 2

Most of the properties in the proposition - the equilibrium outcomes in oligarchy given capital allocation - have been discussed in the main text. Here I provide a more complete proof of all equilibrium outcomes. In the main text, I show that given capital, a high enough wage to buy support from S workers $w_S \geq y_w^D$ implies a relatively low S sector labor $L_S \leq \bar{L}$. Meanwhile, having enough supporters requires $L_S \geq \underline{L}$. If $\bar{L} \geq \underline{L}$, or equivalently, $vzK_S / (zK_S + K_P) \geq \underline{L} \Leftrightarrow K_S / K_P \geq z\underline{L} / (v - \underline{L})$, the political constraint can be satisfied by setting w_S such that $L_S \in [\underline{L}, \bar{L}]$, and oligarchy can be sustained.

If oligarchy is sustained, given that $w_S \geq y_w^D > w_P$, shown in the main text, we know that the wage is higher in S sector than in P sector. Moreover,

$$(1 - \alpha) (z_S K_S)^\alpha L_S^{-\alpha} > (1 - \alpha) (K_P)^\alpha L_P^{-\alpha} \Rightarrow \\ \frac{K_S}{L_S} > \frac{1}{z_S} \frac{K_P}{L_P} > \frac{K_P}{L_P},$$

which states that the capital labor ratio is higher in S sector. Given the larger capital labor ratio in S sector, capital return is obviously lower:

$$\alpha z_S^\alpha K_S^{\alpha-1} L_S^{1-\alpha} < \alpha K_S^{\alpha-1} L_S^{1-\alpha} \\ < \alpha K_P^{\alpha-1} L_P^{1-\alpha}.$$

Compared to in democracy, entrepreneurs get cheap and abundant labor in P sector. As shown in the main text, $L_P > L_P^D$ and this implies higher P sector capital return

$$\alpha K_P^{\alpha-1} L_P^{1-\alpha} > \alpha K_P^{\alpha-1} (L_P^D)^{1-\alpha},$$

and higher entrepreneur income, which is simply the after-tax capital return, minus depreciation.

5.10 Details on the Elite Income Given K_S Chosen by the Government

In the main text, the government can directly decide the S sector capital. The simple way to interpret this setting is that the government sets K_S and each elite member follows. I argue that this setting can also be interpreted as that the government use subsidy to

incentivize the elite members or that the government makes investment. Here I show that they are equivalent, given that the government surplus becomes the transfer to the elite members. In each period of oligarchy, the elite receives capital return of S firms, transfer from the government and pays the cost of interest payment for bank loans. This implies,

$$y_e^{tot} = \pi_S - \delta K_S + T_e - r_e B_e,$$

where π_S, T_e, B_e, r_e denote raw capital return from the S sector, transfer to the elite, the bank loan to the elite and the corresponding interest rate. In the main text, the setting is interpreted as that the government directly decides K_S and each elite member follows the government's decision and borrow from the international market at rate $r_e = r$ to finance the investment. Alternatively, the government can cover part of cost of investment in S sector by providing subsidized interest rate $r_e < r$ to the elite, or directly finance part of the investment $B_g = K_S - B_e - a_e$. The cost of subsidy and direct investment comes from the government balanced budget in each period:

$$(r - r_e) B_e + r B_g + T_e = \tau w_P L_P + \tau \pi_P.$$

Substitute the expressions of T_e and B_e , we get

$$\begin{aligned} y_e^{tot} &= \pi_S - \delta K_S + \tau w_P L_P + \tau \pi_P - (r - r_e) B_e - r B_g - r_e B_e \\ &= \pi_S - \delta K_S + \tau w_P L_P + \tau \pi_P - r (B_e + B_g) \\ &= \pi_S - \delta K_S + \tau w_P L_P + \tau \pi_P - r (K_S - a_e) \\ &= \pi_S - (r + \delta) K_S + \tau w_P L_P + \tau \pi_P + r a_e. \end{aligned}$$

This expression is independent of r_e and B_g , implying that no matter how much cost of investment in S sector is covered by the government, the elite's final income is the same, because the government transfers all the surplus to the elite eventually. The expression of the final income is quite intuitive, if we think about the resource constraint. It includes the total capital return from S sector, asset return from the international market, and the extraction from the private sector. If we call income y_e the part without asset return, as usually seen in growth models, we get:

$$\begin{aligned} y_e &= y_e^{tot} - r a_e \\ &= \pi_S - (r + \delta) K_S + \tau w_P L_P + \tau \pi_P. \end{aligned}$$

5.11 Proof of Proposition 1

This proof is similar to Song et al. (2011). The main idea is the following. The competitive labor market implies that wages are the same in both S and P sector. The competitive capital market in S sector and the elite's infinity borrowing capacity imply that the return of capital to the elite is equal to the borrowing rate r , which pins down the rate of return to S sector capital and the S sector capital labor ratio. In comparison, the credit constraint of the entrepreneur and the higher productivity of P firms imply that P sector capital return is higher than in the S sector but the entrepreneur's capital supply is bounded by the credit constraint. Given a large enough β , the entrepreneur keeps accumulating asset and the P sector keeps growing until all workers move to the P sector while S firms no longer produce. Next, I explain the details.

First, in democracy, if S firms still exist, the return to S firm capital has to be r . If it is greater than r , each elite member wants to supply infinite capital and S sector capital becomes positive infinity; if the return is lower than r , the elite does not want to get any loan or supply any capital to S sector. In other words, competition of S sector capital supply implies that the net rate of return to the elite should equal the marginal cost:

$$\rho_e^D = (1 - \tau^D) \alpha z_S^\alpha K_S^{\alpha-1} (L_S^D)^{1-\alpha} - \delta = r.$$

This determines S sector capital labor ratio and wage:

$$\begin{aligned} \frac{K_S}{L_S^D} &= \left(\frac{r + \delta}{(1 - \tau^D) \alpha z_S^\alpha} \right)^{\frac{1}{\alpha-1}} \Rightarrow \\ w^D &= (1 - \alpha) \left(\frac{z_S K_S}{L_S^D} \right)^\alpha. \end{aligned}$$

The wage pins down the private sector labor, given capital:

$$\begin{aligned} w^D &= (1 - \alpha) \left(\frac{K_P}{L_P^D} \right)^\alpha \Rightarrow \\ L_P^D &= \left(\frac{w^D}{1 - \alpha} \right)^{-\frac{1}{\alpha}} K_P. \end{aligned} \tag{17}$$

Moreover, an elite member in democracy gets no transfer, so she only relies on asset return and her income from other sources is simply 0.

The entrepreneur's problem is to maximize lifetime utility by optimally choose each

period's capital supply and savings to the next period, taking capital returns and credit constraint as given.

$$\begin{aligned} \max_{\{K_{pt}, a_{pt+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \log c_{pt} \\ \text{s.t.} \quad & K_{pt} \leq \eta_t a_{pt}, \\ & y_{pt} = \rho_{pt} K_{pt} - r K_{pt} \\ & a_{pt+1} = R a_{pt} + y_{pt} - c_{pt}. \end{aligned}$$

A similar entrepreneur's problem is solved in Moll (2014), where he shows that a recursive formulation of the problem with value function in the log form leads to a simple solution of the problem: the entrepreneur maximizes each period's income and saves a constant fraction of income to the next period, due to the log utility. Here given the time varying η_t and ρ_{pt} , it is hard to write the problem recursively, so we have to solve the sequential problem directly but the logic is the the same.

Suppose that the sequence $\{K_{pt}^*, a_{pt+1}^*\}_{t=0}^{\infty}$ is the optimal solution to the sequential problem. First, K_{pt}^* must maximize y_{pt} , given a_{pt+1}^* . Otherwise $\exists t'$ and a feasible $\hat{K}_{pt'}$ such that $\rho_{pt'} \hat{K}_{pt'} - r \hat{K}_{pt'} > \rho_{pt'} K_{pt'}^* - r K_{pt'}^*$, we can simply construct a new sequence of $\{K_{pt}^*, a_{pt+1}^*\}$ by replacing $K_{pt'}^*$ by $\hat{K}_{pt'}$ while keeping all other K_{pt}^* for all $t \neq t'$ and all other a_{pt+1}^* . The new sequence is feasible and implies $c_{pt'} > c_{pt'}^*$ and $\forall t \neq t', c_{pt} = c_{pt}^*$. The lifetime utility of the new sequence is higher. Second, the optimal K_{pt}^* to maximize y_{pt} given a_{pt}^* is simple:

$$K_{pt}^* \begin{cases} = \eta_t a_{pt}^* & \text{if } \rho_{pt} > r, \\ \in [0, \eta_t a_{pt}^*] & \text{if } \rho_{pt} = r, \\ = 0 & \text{if } \rho_{pt} < r. \end{cases}$$

As we mentioned above, in the calibrated model, it is always the first case, so we know $K_{pt}^* = \eta_t a_{pt}^*$ and $y_{pt}^* = (\rho_{pt} - r) \eta_t a_{pt}^*$. Adding $r a_{pt}$, we get the total income, which all come from the return of asset: $y_{pt}^{tot} = R a_{pt}^* + y_{pt}^* = (R + (\rho_{pt} - r) \eta_t) a_{pt}^*$. So $\rho_{pt}^{tot} = R + (\rho_{pt} - r) \eta_t$ can be considered as the total return to the entrepreneur's saving and the entrepreneur lives on the asset return.³⁹ Finally, the only problem left is the choice of

³⁹If we consider the other two cases $\rho_{pt} \leq r$, it is not difficult: then the total return to the entrepreneur's asset is simply r .

a_{pt+1} , taking the total return to asset ρ_{pt}^{tot} as given:

$$\begin{aligned} & \max_{\{a_{pt+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_{pt} \\ & \text{s.t. } a_{pt+1} = \rho_{pt}^{tot} a_{pt} - c_{pt}. \end{aligned}$$

Given log-utility, the substitution effect of the return to saving exactly cancels the wealth effect, and in each period, the agent saves β fraction of the total resource to the next period, i.e., $a_{pt+1}^* = \beta \rho_{pt}^{tot} a_{pt}^*$, no matter how high or low ρ_{pt+1}^{tot} is.

If $\beta \rho_{pt}^{tot} > 1$, then a_{pt} keeps growing until $\beta \rho_{pt}^{tot} = 1$. Meanwhile, L_P^D keeps growing according to (17), and L_S^D declines gradually. When a_{pt} reaches $1/\bar{\eta} (w^D/1-\alpha)^{\frac{1}{\alpha}}$, L_P^D reaches 1, and S sector becomes 0. Afterwards, entrepreneurs keep accumulating assets, and the economy behaves like a neoclassic growth model.

5.12 Proof of Lemma 2

This is in fact already proved in the proof of proposition 1. In oligarchy, same as in democracy, the entrepreneur takes the return to capital in P sector, leverage η and eventually the return to her asset as given, so her behavior is basically the same: maximizing the current period income and then saves β fraction of total resource to the next period.

5.13 Expressions of Incomes in Different Forms

In the main text, in different places, we need to express the expected incomes of the worker, the elite, and the entrepreneur in different ways. The reason is that in different steps of the timing, the expected incomes need to be computed using different variables that are given at that point of time. For example, the income of a worker in democracy, after the production and redistribution, has a simple and direct expression as in equation (4), i.e., $y_w^D = (1 + \tau^D \alpha / (1 - \alpha)) w^D$. When the expected income of the worker in democracy is calculated by workers when they decide whether to support the regime, i.e., at step 5, workers calculate it given capital allocation, so y_w^D can be expressed as a function of K_S and K_P by substituting the expression of w^D and L_S^D :

$$\begin{aligned} y_w^D &= \left(1 + \tau^D \frac{\alpha}{1 - \alpha}\right) (1 - \alpha) (z_S K_S)^\alpha \left(L_S^D\right)^{-\alpha} \\ &= \left(1 + \tau^D \frac{\alpha}{1 - \alpha}\right) (1 - \alpha) (z_S K_S + K_P)^\alpha. \end{aligned}$$

If we go further, when the elite plans policies at step 1, it needs to expect y_w^D and K_P using the state variable at that point of time, i.e., A_p , and its choice variables: (η, K_S, w_S) . So by substituting the expressions of $K_P = \eta A_p$, we can write y_w^D as a function of these variables:

$$y_w^D = \left(1 + \tau^D \frac{\alpha}{1 - \alpha}\right) (1 - \alpha) (z_S K_S + \eta A_p)^\alpha,$$

which is used in the elite's problem - equation (15). Similarly, in the elite's problem, the expected incomes of the elite and the entrepreneurs can also be expressed as functions of (η, K_S, w_S, A_p) .

5.14 Recursive Formation of the Entrepreneur and the Elite's Problem

The sequential formation of the entrepreneur's problem is as in equation (12). If the entrepreneur is asked to plan the future choices, at the beginning of period 0 given A_e and A_p , the recursive formation simply becomes equation (13). In the sequential form, the entrepreneur rationally expects $\{\eta_t, r_{Pt}\}_{t=0}^\infty$; while in the recursive form, the entrepreneur expects the current period (η, r_P) using the current period state variables (A_e, A_p) and predicts the evolution of future (η, r_P) using the laws of motion of state variables. This is standard, as in classic neoclassic growth models and the household's problem.

It is worth mentioning that in this simple recursive form the problem is as if the entrepreneur chooses k_P and a_p at the beginning of each period using her predictions of η and r_P . This simplification is fine for two reasons: first, we only look at solution given all other agents behave as in the equilibrium; and second, an entrepreneur is infinitesimal and does not think that her behavior can affect the aggregate variables and the equilibrium. This recursive form is more restrictive compared to the sequential form which allows for the solution given any sequence of $\{\eta_t, r_{Pt}\}_{t=0}^\infty$ which can be off-equilibrium. However, this restriction is standard, like in the recursive formation of the classic neoclassic growth model.

The elite's problem is formed with the similar logic. We write down the recursive problem and the value functions - equation (14) and (15) - as if the elite chooses $\{M_e, \eta, K_S, w_S, A_e'\}$ at the beginning of each period, i.e., in step 1 of timing of events, instead of choosing M_e in step 1, η and K_S in step 2, w_S in step 3 and A_e' in step 8. Similar to the entrepreneur's problem, the elite expects other agents reacting optimally according to the equilibrium solution. This problem is a bit more complicated than the entrepreneur's problem, because the elite takes into account that its choice affects other agents' behaviors and the

equilibrium outcome. For example, the elite does not take the entrepreneur's capital supply K_P , the worker's income in democracy y_w^D , entrepreneur income y_p and entrepreneur saving A'_p as given, but understands that they react to choices of (η, K_S, w_S) , as we can see in the constraints of problem (15). Though the elite is not infinitesimal, it is the only one: all other agents - entrepreneurs, workers, and firms - are infinitesimal and take aggregate variables as given. So it is correct to form the problem as if the elite chooses all variables at the same time while internalizing the reactions of other agents, given that we are only interested in the equilibrium solution.⁴⁰ To show the equivalence, we can write the recursive problem reflecting the elite's choices in each step using more value functions. In step 1, the elite chooses whether to sustain oligarchy, to democratize or to create revolution. Value functions are the same as in equation (14). Next, let us only consider the case when $M_e = O$. In step 2, the elite chooses K_S and η , expecting that afterwards entrepreneurs choose k_P which shapes K_P accordingly:

$$W^O(A_e, A_p) = \max_{\eta, K_S} \beta W^3(K_S, K_P; A'_e, A'_p)$$

$$\text{s.t. } \frac{K_S}{K_P} \geq \frac{\underline{L}}{z(v - \underline{L})},$$

where K_P is determined by the entrepreneur's optimal choice of k_P given η , K_S and the expected r_P , denoted as a function $K_P(\eta, K_S, A_p)$ and is simply $K_P = \eta A_p$ throughout this paper. The constraint is the necessary condition of the capital allocation in the case that oligarchy survives. It is discussed in the equilibrium given capital. In step 3, the elite chooses w_S :

$$W^3(K_S, K_P; A_e, A_p) = \max_{w_S} W^8(y_e, y_p; A_e, A_p)$$

$$\text{s.t. } w_S \geq y_w^D(K_S, K_P),$$

$$L_S(w_S, K_S) \geq \underline{L}.$$

In the first constraint, function $y_w^D(\cdot)$ is determined by the equilibrium in democracy, as in equation (4); in the second constraint, function $L_S(\cdot)$ comes from the FOC of the S firm, as in equation (1). Then, y_e and y_p can be computed using equation (10) and (11), and in the next step, the elite makes decisions conditional on them. In step 8, the elite chooses

⁴⁰If there are two larger enough players making decisions sequentially and understanding that their individual choices affect the equilibrium outcome, usually the problem can not be formed as if one makes all choices simultaneously.

A'_e :

$$\begin{aligned}
W^8(y_e, y_p; A_e, A_p) &= \max_{A'_e} \log c_e + \beta W(A'_e, A'_p) \\
\text{s.t. } A'_e &= RA_e + y_e - C_e, \\
A'_p &= \beta (RA_p + y_p).
\end{aligned}$$

Compared to the simple Bellman equation in the main text, i.e., (15), using more value functions allow for more complete characterization of the problem sequentially, which allows for analyzing the elite's decisions given off-equilibrium variables. For example, W^3 can be used to study the elite's optimal decision if K_p deviates from the equilibrium, i.e., $K_p \neq \eta A_p$; and W^8 can still be solved if y_p is different from the equilibrium level given state variables and the elite's choices. However, if we only care about the equilibrium solution and substitute the equilibrium optimal choices of K_p, y_p, L_S , then we can merge W^8, W^3 into W^O and get the expression of W^O as in the main text, i.e., equation (15). Moreover, given that all other agents are infinitesimal, they have not incentives to deviate from the optimal choice of k_p, y_p, L_S , etc..

We can also write down the Bellman equation for W^R :

$$\begin{aligned}
W^R(A_e, A_p) &= \max_{\eta, K_S, w_S, C_e, A'_e} \log c_e + \beta W^D(A'_e, A'_p) \\
\text{s.t. } w_S &< y_w^D(\eta, K_S, w_S, A_p) \text{ or } L_S(w_S, K_S) < \underline{L}, \\
A'_e &= RA_e + y_e^D(K_S, \eta, A_e) - C_e, \\
A'_p &= \beta (RA_p + y_p^D(K_S, \eta, A_p)).
\end{aligned}$$

The first constraint means that if the elite chooses to go for policies that lead to revolution, then either the minimal wage constraint or minimal supporter constraint is violated. Then the incomes of the elite and the entrepreneur are determined by the equilibrium in democracy. Notice that in the function of y_p^D , there is no w_S but η . This is because that if in step 3 the elite chooses a w_S that does not buy enough supporters, then the wage is determined by the equilibrium in democracy; while the private sector capital has already been determined by η in step 2, which is before step 3 and before the revolution. In other words, when the revolution happens, the economy enters the equilibrium of democracy given capital allocation in this period, as stated in step 5. One can also decompose this Bellman equation as if the elite chooses variables sequentially, but it will be equivalent.

5.15 Proof of Lemma 3

The main idea is that because the elite faces no borrowing constraint, its choices on all other variables, in the current period and in all future periods, are not constrained by its current period asset A_e which always gives return r . So the elite can always choose the same sequence of policies which gives the highest income net of asset return, which is independent of the asset choices.

More formally, the proof is the following. Denote the lifetime utility achieved by solving the two sub-problems - first maximizing lifetime income and then maximizing lifetime utility - as U . Remember that the solution to the original one complete problem gives lifetime utility W . In this proof, we use the sequential form of the dynamic problem, which is equivalent to but simpler than directly looking at the recursive form.

First, $U \leq W$. Let us look at the two subproblems. The solution to the first subproblem achieving V can be denoted as $\{\hat{M}_t, \hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t\}_{t=0}^{\infty}$ in the sequential form. The corresponding consumption and saving decisions obtaining U are denoted as $\{\hat{C}_{et}, \hat{A}_{et+1}\}_{t=0}^{\infty}$. Combine them together, the choice $\{\hat{M}_t, \hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t, \hat{C}_{et}, \hat{A}_{et+1}\}_{t=0}^{\infty}$ achieving U is a feasible choice of the original problem, given that in every period the choice set for $M_t, w_{St}, K_{St}, \eta_t$ is independent of A_{et} . So the optimal solution for the original problem should be at least as good as this candidate choice, i.e., $W \geq U$.

Second, $U \geq W$. Denote the choice that solves the original problem and achieves W with stars, as $\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*, C_{et}^*, A_{et}^*\}_{t=0}^{\infty}$. Let us compare $\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*\}_{t=0}^{\infty}$ with the solution of the first subproblem achieving V : $\{\hat{M}_t, \hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t\}$. Obviously, $\hat{V} = \sum \hat{y}_{et}/R^t \geq \sum y_{et}^*/R^t = V^*$, given that $\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*\}_{t=0}^{\infty}$ is also a candidate solution to the first subproblem, again because of the independence of policies from the elite asset. This implies that the lifetime income from solving the first subproblem is as high as the solution of the original problem. Then, in the second sub-problem, choosing $\hat{C}_{e0} = C_{e0}^* + \hat{V} - V^*$, $\{\hat{C}_{et}, \hat{A}_{et}\}_{t=1}^{\infty} = \{C_{et}^*, A_{et}^*\}_{t=1}^{\infty}$ gives at least as high lifetime utility as $\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*, C_{et}^*, A_{et}^*\}_{t=0}^{\infty}$. In other words, consuming the potential extra lifetime income in the first period and following the same strategy of the solution to the original problem can do as good as the optimal solution to the original problem.

Combing these two results, we have $U = W$, i.e., Solving the original lifetime utility maximization problem is the same as solving the two sub-problems.

5.16 Proof and Details of Proposition 3

The main idea is that when K_p is small, the marginal return of K_p is very large, and the elite's current period income and lifetime income can both benefit from having a larger

K_P . These properties also apply to A_P because it is non-negatively related to K_P . The formal mathematical form for the proposition is the following:

(1) If constraint 5 - high state wage constraint - is binding, then $\forall K_S, \exists \varepsilon_1 > 0, \forall K_P < \varepsilon_1$, such that $\frac{\partial y_e}{\partial K_P} > 0$; and $\exists \varepsilon_2 > 0, \forall K_P > \varepsilon_2, \exists K_S$, such that $y_e > y_e^D$.

(2) If $\exists \sigma, \forall K_P > \sigma$, both constraint 7 and 5 - minimal support constraint and high state wage constraint - are binding, then $\exists \varepsilon_3 > 0, \forall K_P > \varepsilon_3$, such that $\partial y_e / \partial K_P < 0$ and $y_e < y_e^D$.

(3) If K_P is increasing in A_P , the above results hold when K_P is replaced by A_P .

(4) If V_e and $\partial V_e / \partial K_P$ are continuous in the inter-temporal discount parameter, i.e., $1/R$, then $\exists \varepsilon_4, \forall r > \varepsilon_4$, the above results hold when y_e is replaced by V_e .

Next, I prove the proposition. First, let us look at y_e when K_P is small, given the conditions in the proposition. Using equation (10) and the condition that high state wage constraint is binding, i.e., $K_P < \delta$, $w_S = y_w^D$ and $L_S = \nu z K_S / (z K_S + K_P)$, we can write down y_e as

$$\begin{aligned} y_e &= \pi_e - (r + \delta) K_S + \tau Y_P \\ &= \alpha (z K_S)^\alpha \left(\nu \frac{z K_S}{z K_S + K_P} \right)^{1-\alpha} - (r + \delta) K_S + \tau (K_P)^\alpha \left(1 - \nu \frac{z K_S}{z K_S + K_P} \right)^{1-\alpha}. \end{aligned}$$

$\partial y_e / \partial K_P$ contains two parts: $\partial \pi_e / \partial K_P$ and $\partial Y_P / \partial K_P$. When K_P converges to 0, the first part converges to a negative but finite number:

$$\begin{aligned} \lim_{K_P \rightarrow 0} \frac{\partial \pi_e}{\partial K_P} &= \lim_{K_P \rightarrow 0} \alpha (\alpha - 1) \nu^{1-\alpha} (z K_S)^\alpha (z K_S + K_P)^{\alpha-2} \\ &= \alpha (\alpha - 1) \nu^{1-\alpha} (z K_S)^{2(\alpha-1)}. \end{aligned}$$

The second part converges to positive infinity:

$$\begin{aligned} \lim_{K_P \rightarrow 0} \frac{\partial Y_P}{\partial K_P} &= \lim_{K_P \rightarrow 0} \alpha (K_P)^{\alpha-1} \left(1 - \nu \frac{z K_S}{z K_S + K_P} \right)^{1-\alpha} \\ &\quad + (1 - \alpha) (K_P)^\alpha \left(1 - \nu \frac{z K_S}{z K_S + K_P} \right)^{-\alpha} \left(\nu z K_S (z K_S + K_P)^{-2} \right) \\ &= +\infty + 0 \\ &= +\infty. \end{aligned}$$

This means that $\lim_{K_P \rightarrow 0} \partial y_e / \partial K_P = +\infty$ and $\exists \varepsilon, \forall K_P < \varepsilon, \partial y_e / \partial K_P > 0$. When K_P is small, the marginal return of K_P to the elite is infinity, because the binding high state wage constraint pushes a significant amount of labor to the P sector.

The second result can be proved using the decreasing return to capital. Under the condition that $K_P > \sigma$, i.e., both minimal support constraint and high state wage constraint are binding, we have

$$\begin{aligned} L_S &= v \frac{zK_S}{zK_S + K_P} = \underline{L}, \\ zK_S &= \frac{\underline{L}}{v - \underline{L}} K_P. \end{aligned}$$

This shows that if K_P becomes larger, K_S has to be proportionally larger if the elite decides to sustain oligarchy. Now the elite's income becomes:

$$y_e = \alpha \left(\frac{\underline{L}}{v - \underline{L}} K_P \right)^\alpha (\underline{L})^{1-\alpha} - (r + \delta) \frac{\underline{L}}{z(v - \underline{L})} K_P + \tau (K_P)^\alpha (1 - \underline{L})^{1-\alpha},$$

and

$$\begin{aligned} \lim_{K_P \rightarrow +\infty} \frac{\partial y_e}{\partial K_P} &= -(r + \delta) < 0, \\ \lim_{K_P \rightarrow +\infty} y_e &= -\infty. \end{aligned}$$

The decreasing return to capital guarantees that when K_P is large enough, y_e decreases to $-\infty$. Then it is easy to find $\varepsilon > \sigma$ such that $\forall K_P > \varepsilon, \partial y_e / \partial K_P < 0$ and $y_e < 0 = y_e^D$.

Given the conditions in result (3), i.e., K_P is increasing in A_p , the sign of $\partial y_e / \partial K_P$ is the same as $\partial y_e / \partial A_p$. Moreover, one can find some ε_a such that $K_P < \varepsilon$ is equivalent to $A_p < \varepsilon_a$, and similarly for $K_P > \varepsilon$. So the properties in (1) and (2) are still valid.

To prove result (4), we can first show that when the discount rate is 0, the above properties for y_e are also true for V_e . This is obvious, because $V_e = y_e$. Under the condition that V_e and $\partial V_e / \partial K_P$ are continuous in the discount rate, we have that $\exists \varepsilon, \forall 1/R < \varepsilon$, these properties for V_e are still true. $1/R$ being sufficiently small is equivalent to R being sufficiently large.