Micro to Macro: Optimal Trade Policy with Firm Heterogeneity*

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Abstract

The empirical observation that "large firms tend to export, whereas small firms do not" has transformed the way economists think about the determinants of international trade. Yet, it has had surprisingly little impact about how economists think about trade policy. In this paper, we characterize optimal trade policy in a generalized version of the trade model with monopolistic competition and firm-level heterogeneity developed by Melitz (2003). At the micro-level, we find that optimal import taxes discriminate against the most profitable foreign exporters, while optimal export taxes are uniform across domestic exporters. At the macro-level, we demonstrate that the selection of heterogeneous firms into exporting tends to create aggregate nonconvexities that dampen the incentives for terms-of-trade manipulation, and in turn, the overall level of trade protection.

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1 Introduction

There are large firms and small firms. The former tend to export whereas the latter do not. What are the policy implications of that empirical observation?

Models of firm heterogeneity have transformed the way economists think about the determinants of international trade. Yet, the same models have had surprisingly little impact about how economists think about trade policy.[1] The goal of this paper is to fill this large gap on the normative side of the literature and uncover the general principles that should guide the design of optimal trade policy when heterogeneous firms select into exporting.

Our basic environment is a generalization of the model of intra-industry trade with monopolistic competition and firm-level heterogeneity developed by Melitz (2003). On the supply side, we let firms be heterogeneous in terms of both their variable costs and their fixed costs. We impose no restrictions on the joint distribution of these costs across firms and markets. On the demand side, we maintain the assumption that the elasticity of substitution between all varieties from a given country is constant, but we impose no restrictions on the substitutability between domestic and foreign goods.

The first part of our analysis studies the ad-valorem taxes that maximize domestic welfare, which we label unilaterally optimal taxes, when governments are free to impose different taxes on different firms. At the micro-level, we find that optimal trade policy requires firm-level import taxes that discriminate against the most profitable foreign exporters. In contrast, export taxes that discriminate against or in favor of the most profitable domestic exporters can be dispensed with. The fact that optimal import taxes discriminate against the most profitable exporters from abroad is reminiscent of an anti-dumping duty.[2] The rationale, however, is very different. Here, discriminatory taxes do not reflect a desire to deter the entry of the most profitable exporters. They reflect instead a desire to promote the entry of the marginally unprofitable exporters who, if they were to face the same tariff, would prefer not to export at all.

At the macro-level, standard terms-of-trade considerations pin down the overall level of trade taxes. Specifically, the only reason why a welfare-maximizing government would like to implement aggregate imports and exports that differ from those in the decentral-

[1] The last handbook of international economics, Gopinath, Helpman and Rogoff, eds (2014), is a case in point. In their chapter on heterogeneous firms, Melitz and Redding (2014) have only one trade policy paper to cite. In his chapter on trade policy, Maggi (2014) has no paper with firm heterogeneity to review.

[2] According to U.S. law, dumping occurs when foreigners' export prices, adjusted for transportation costs, are lower than their domestic prices. The latter prices, however, are rarely available in practice, especially for non-market economies like China. Hence, as noted by Ruhl (2014), anti-dumping duties tend to be imposed on the most productive foreign exporters. This is the feature that we emphasize here as well.

ized equilibrium is because it internalizes the impact of both quantities on the price of the infra-marginal units that it buys and sells on the world markets. Like in a Walrasian economy, the more Home's terms of trade deteriorate with increases in exports or imports, the larger the trade restriction that it optimally imposes.

The second part of our analysis focuses on optimal taxation under the polar assumption that governments are constrained to impose the same tax on all firms from the same country selling in a given market. Our main finding in this environment is a new optimal tariff formula that generalizes existing results in the literature. Under monopolistic competition with homogeneous firms, Gros (1987) has shown that optimal tariffs are determined by the elasticity of substitution between domestic and foreign goods and the share of expenditure on local goods abroad. Our new formula establishes that, conditional on these two statistics, firm heterogeneity lowers the overall level of trade protection if and only if it creates aggregate nonconvexities abroad.[3] When strong enough, these aggregate nonconvexities may even turn the optimal import tariff into a subsidy. In such circumstances, a government may *lower* the price of its imports by *raising* their volume, an example of the Lerner paradox.

The final part of our analysis extends our basic environment to incorporate intra- and inter-industry trade. In this case, sector-level increasing returns to scale, the so-called home market effects, can also shape optimal trade policy. The common wisdom in the literature (Helpman and Krugman, 1989) is that such effects provide a very different rationale for trade protection. Our last set of results suggests a different interpretation, one according to which home-market effects matter to the extent that they shape terms of trade elasticities, but not beyond.

While both the positive and normative implications of imperfectly competitive markets for international trade have been studied extensively, the same cannot be said of the heterogeneous firms operating in these markets. On the positive side, the pioneering work of Melitz (2003) has led numerous researchers to revisit various results of Helpman and Krugman (1985) under the assumption that firms are heterogeneous and select into exporting. On the normative side, however, much less energy has been devoted to revisit the classical results of Helpman and Krugman (1989).

To the best of our knowledge, only three papers—Demidova and Rodríguez-Clare (2009), Felbermayr, Jung and Larch (2013), and Haaland and Venables (2014)—have used the work of Melitz (2003) to explore the implications of firm heterogeneity for optimal

[3]With homogeneous firms (and the standard assumption in that context that there are no fixed costs of trade), aggregate production possibility frontiers are necessarily linear. With heterogeneous firms and fixed trade costs, nonconvexities are likely to arise, as the mild sufficient conditions of Section 5.4 establish.

trade policy. All three papers are restricted to environments where utility functions are CES; fixed costs of exporting are constant across firms; distributions of firm-level productivity are Pareto; and, importantly, trade taxes are uniform across firms.[4] In this paper, we relax all of these assumptions, we derive new results about optimal trade taxes at the micro-level, and we generalize prior results about optimal trade taxes at the macro-level. Beside greater generality, these results uncover a novel connection between firm heterogeneity, aggregate nonconvexities, and lower levels of trade protection.

In terms of methodology, our analysis builds on the work of Costinot, Lorenzoni and Werning (2014) and Costinot, Donaldson, Vogel and Werning (2015) who characterize the structure of optimal trade taxes in a dynamic endowment economy and a static Ricardian economy, respectively. Like in the two previous papers, we use a primal approach and general Lagrange multiplier methods to characterize optimal wedges rather than explicit policy instruments. The novel aspect of our analysis is to break down the problem of finding optimal wedges into a series of micro subproblems, where we study how to choose quantities across varieties conditional on aggregate quantities, and a macro problem, where we solve for the optimal aggregate quantities. The solutions to the micro and macro problems then determine the structure of optimal micro and macro taxes described above. This decomposition helps to highlight the deep connection between standard terms-of-trade arguments, as in Baldwin (1948) and Dixit (1985), and the design of optimal trade policy in models of monopolistic competition.

In spite of their common rationale, i.e., terms-of-trade manipulation, the specific policy prescriptions derived under perfect and monopolistic competition differ sharply. In Costinot, Donaldson, Vogel and Werning (2015), optimal export taxes should be heterogeneous, whereas optimal import tariffs should be uniform. This is the exact opposite of what we find under monopolistic competition. In a Ricardian economy, goods exported by domestic firms could also be produced by foreign firms. This threat of foreign entry limits the ability of the domestic government to manipulate world prices and leads to lower export taxes on goods for which its firms have a weaker comparative advantage. Since the previous threat is absent under monopolistic competition, optimal export taxes are uniform instead. Conversely, lower import tariffs on the least profitable foreign firms under monopolistic competition derive from the existence of fixed exporting costs, which

[4]A fourth paper by Demidova (2015) analyzes optimal trade policy under the assumption of quadratic utility functions, similar to those in Melitz and Ottaviano (2008). All other assumptions are the same as in the aforementioned papers. In this environment, markups vary across firms, which leads to domestic distortions even within the same industry and opens up the possibility of terms-of-trade manipulation even at the firm-level. Our baseline analysis abstracts from these issues and instead focuses on the implication of the self-selection of heterogeneous firms into export markets, as in Melitz (2003). We come back to this point in our concluding remarks.

are necessarily absent under perfect competition.

The previous discussion is related to recent results by Ossa (2011) and Bagwell and Staiger (2012b,a, 2015) on whether imperfectly competitive markets create a new rationale for the design of trade agreements. We hope that our analysis can contribute to the application of models with firm heterogeneity to study this question as well as other related trade policy issues. Bagwell and Lee (2015) offer an interesting first step in that direction. They study trade policy in a symmetric version of the Melitz and Ottaviano (2008) model that also features the selection of heterogeneous firms into exporting. They show that this model provides a rationale for the treatment of export subsidies within the World Trade Organization.

The rest of the paper is organized as follows. Section 2 describes our basic environment. Section 3 sets up and solves the micro and macro planning problems of a welfare-maximizing country manipulating its terms-of-trade. Section 4 shows how to decentralize the solution to the planning problems through micro and macro trade taxes when governments are free to discriminate across firms. Section 5 studies the polar case where governments can only impose uniform taxes. Section 6 explores the sensitivity of our results to the introduction of multiple industries. Section 7 offers some concluding remarks.

2 Basic Environment

2.1 Technology, Preferences, and Market Structure

Consider a world economy with two countries, indexed by $i = H, F$; one factor of production, labor; and a continuum of differentiated goods or varieties. Labor is immobile across countries. w_i and L_i denote the wage and the inelastic supply of labor in country i, respectively.[5]

Technology. Producing any variety in country i requires an overhead fixed entry cost, $f_i^e > 0$, in terms of domestic labor. Once the overhead fixed cost has been paid, firms randomly draw a blueprint $\varphi \in \Phi$. N_i denotes the measures of entrants in country i and G_i denotes the distribution of blueprints φ across firms in that country. Each blueprint describes how to produce and deliver a firm's differentiated variety to any country. $l_{ij}(q, \varphi)$ denotes the total amount of labor needed by a firm from country i with blueprint φ in

[5]The two-country assumption will help us relate our results to those from the existing literature in Section 5. With more than two countries, all the micro-level predictions described in Section 4 would remain unchanged. We discuss how macro-level predictions would change in Section 6.

order to produce and deliver $q \geq 0$ units in country j. We assume that

$$l_{ij}(q,\varphi) = a_{ij}(\varphi)q + f_{ij}(\varphi), \text{ if } q > 0,$$
$$l_{ij}(q,\varphi) = 0, \text{ if } q = 0.$$

Technology in Krugman (1980) corresponds to the special case in which G_i has all its mass at a single blueprint with zero fixed costs of selling in the two markets, $f_{ij} = 0$ for all i, j. Technology in Melitz (2003) corresponds to the special case in which firms have heterogeneous productivity, but face homogenous iceberg trade costs, $a_{ij}(\varphi) \equiv \tau_{ij}/\varphi$, and homogenous fixed costs, $f_{ij}(\varphi) \equiv f_{ij}$ for all φ.

Preferences. In each country there is a representative agent with a two-level utility function,

$$U_j = U_j(Q_{Hj}, Q_{Fj}),$$
$$Q_{ij} = [\int_{\Phi} N_i(q_{ij}(\varphi))^{1/\mu_i} dG_i(\varphi)]^{\mu_i}.$$

where Q_{ij} denotes the subutility aggregator from consuming varieties from country i in country j, $q_{ij}(\varphi)$ denotes country j's consumption of a variety with blueprint φ produced in country i, and $\mu_i \equiv \sigma_i/(\sigma_i - 1)$, with $\sigma_i > 1$ the elasticity of substitution between varieties from country i. Krugman (1980) and Melitz (2003) correspond to the case where $\mu_H = \mu_F \equiv \mu$ and $U_j(Q_{Hj}, Q_{Fj}) \equiv [Q_{Hj}^{1/\mu} + Q_{Fj}^{1/\mu}]^{\mu}$. Here, we do not restrict the elasticities of substitution, σ_H and σ_F, to be the same, nor do we restrict the shape of the upper-level utility function, U_j. The only assumptions that we maintain are that U_j is homothetic and that the upper-level elasticity of substitution between Q_{Hj} and Q_{Fj} is always strictly greater than one.[6]

Market Structure. All goods markets are monopolistically competitive with free entry. All labor markets are perfectly competitive. Foreign labor is our numeraire, $w_F = 1$.

[6] In Section 5.3, we will exploit the fact that lower-level and upper-level elasticities of substitution are allowed to differ in order to disentangle terms-of-trade motives for protection from second-best arguments related to markup distortions. Matsuyama (1992) provides a detailed analysis of how the predictions of monopolistically models may vary depending on the ranking of these two elasticities.

2.2 Decentralized Equilibrium with Taxes

Our focus is on a scenario where governments have access to a full set of ad-valorem consumption and production taxes. That is, we let taxes vary across markets *and* across firms. Section 5 considers a more restricted case, where taxes can vary across markets but are required to be uniform across firms.

We view the availability of a rich set of taxes as a useful benchmark for our analysis. In theory, there is a priori no reason within the model that we consider why different goods should face the same taxes. In an Arrow-Debreu economy, imposing the same taxes on arbitrary subsets of goods would be ad-hoc. Changing the market structure from perfect to monopolistic competition does not make it less so. In practice, perhaps more importantly, different firms do face different trade taxes, even within the same narrowly defined industry. Anti-dumping duties often act as import tariffs imposed on the most productive firms. Loan subsidies provided to small exporters in many countries can also be thought of as export subsidies that vary with firms' productivity.[7]

Formally, we let $t_{ij}(\varphi)$ denote the tax charged by country j on the consumption in country j of a variety with blueprint φ produced in country i. Let $s_{ij}(\varphi)$ denote the subsidy paid by country i on the production by a domestic firm of a variety with blueprint φ sold in country j. For $i \neq j$, $t_{ij}(\varphi) > 0$ corresponds to an import tariff while $t_{ij}(\varphi) < 0$ corresponds to an import subsidy. Similarly, $s_{ij}(\varphi) > 0$ corresponds to an export subsidy while $s_{ij}(\varphi) < 0$ corresponds to an export tax. Tax revenues are rebated to domestic consumers through a lump-sum transfer, T_i.[8]

In a decentralized equilibrium with taxes, consumers choose consumption to maximize their utility subject to their budget constraint; firms choose their output to maximize profits, taking their residual demand curves as given; firms enter up to the point at which expected profits are zero; markets clear; and the government's budget is balanced in each country. Let $\bar{p}_{ij}(\varphi) \equiv \mu_i w_i a_{ij}(\varphi)/(1+s_{ij}(\varphi))$ and $\bar{q}_{ij}(\varphi) \equiv [(1+t_{ij}(\varphi))\bar{p}_{ij}(\varphi)/P_{ij}]^{-\sigma_i} Q_{ij}$. Using the previous notation, we can characterize a decentralized equilibrium with taxes as schedules of output, $\mathbf{q_{ij}} \equiv \{q_{ij}(\varphi)\}$, schedules of prices, $\mathbf{p_{ij}} \equiv \{p_{ij}(\varphi)\}$, aggregate output levels, Q_{ij}, aggregate price indices, P_{ij}, wages, w_i, and measures of entrants, N_i, such

[7]Firms operating in the same industry, say "Cotton, not carded or combed" (HS8 520100), may also face different tariffs because they are producing different varieties. As Kim (2016) notes, the most favoured nation (MFN) tariff rate applied by the United States for "Cotton, not carded or combed, having staple length of 28.575 mm or more but under 34.925 mm (HS8 52010038)" is 14%, as of 2013, whereas "Cotton, not carded or combed, having a staple length under 19.05mm (3/4 inch), harsh or rough (HS8 52010005)" is duty free.

[8]When firm-level taxes are allowed, as in our baseline analysis, our focus on ad-valorem rather than specific taxes is without loss of generality. Ruling out non-linear taxes, like two part-tariffs, is not. We discuss how the introduction of such instruments would affect our results in Section 4.4.

that

$$q_{ij}(\varphi)=\begin{cases}\bar{q}_{ij}(\varphi) & \text{if } \mu_i a_{ij}(\varphi)\bar{q}_{ij}(\varphi)\geq l_{ij}(\bar{q}_{ij}(\varphi),\varphi),\\ 0 & \text{otherwise,}\end{cases} \tag{1}$$

$$p_{ij}(\varphi)=\begin{cases}\bar{p}_{ij}(\varphi) & \text{if } \mu_i a_{ij}(\varphi)q_{ij}(\varphi)\geq l_{ij}(q_{ij}(\varphi),\varphi),\\ \infty & \text{otherwise,}\end{cases} \tag{2}$$

$$Q_{Hj},Q_{Fj}\in\arg\max_{\tilde{Q}_{Hj},\tilde{Q}_{Fj}}\{U_j(\tilde{Q}_{Hj},\tilde{Q}_{Fj})|\textstyle\sum_{i=H,F}P_{ij}\tilde{Q}_{ij}=w_jL_j+T_j\}, \tag{3}$$

$$P_{ij}^{1-\sigma_j}=\int_\Phi N_i[(1+t_{ij}(\varphi))p_{ij}(\varphi)]^{1-\sigma_i}dG_i(\varphi), \tag{4}$$

$$f_i^e=\textstyle\sum_{j=H,F}\int_\Phi[\mu_i a_{ij}(\varphi)q_{ij}(\varphi)-l_{ij}(q_{ij}(\varphi),\varphi)]dG_i(\varphi), \tag{5}$$

$$L_i=N_i[\textstyle\sum_{j=H,F}\int_\Phi l_{ij}(q_{ij}(\varphi),\varphi)dG_i(\varphi)+f_i^e], \tag{6}$$

$$T_i=\textstyle\sum_{j=H,F}[\int_\Phi N_j t_{ji}(\varphi)p_{ji}(\varphi)q_{ji}(\varphi)dG_j(\varphi)-\int_\Phi N_i s_{ij}(\varphi)p_{ij}(\varphi)q_{ij}(\varphi)dG_i(\varphi)]. \tag{7}$$

Conditions (1) and (2) assume that firms that are indifferent between producing and not producing, produce. This is without loss of generality since, to simplify, we assume indifference is measure zero. We do so by adopting the sufficient condition that conditional on a positive value for the fixed cost $f_{ij}>0$ the distribution over the variable cost a_{ij} is continuous.[9] Throughout our analysis, we restrict attention to the interesting nontrivial cases where preferences and trade costs are such that the utility maximization problem (3) admits an interior solution. This rules out equilibria without trade ($Q_{FH}=Q_{HF}=0$) or without domestic production ($Q_{HH}=Q_{FF}=0$).

2.3 Unilaterally Optimal Taxation

We assume that the government of country H, which we refer to as the home government, is strategic, whereas the government of country F, which we refer to as the foreign government, is passive. Namely, the home government sets ad-valorem taxes, $\mathbf{t_{HH}}\equiv\{t_{HH}(\varphi)\}$, $\mathbf{t_{FH}}\equiv\{t_{FH}(\varphi)\}$, $\mathbf{s_{HH}}\equiv\{s_{HH}(\varphi)\}$, and $\mathbf{s_{HF}}\equiv\{s_{HF}(\varphi)\}$, and a lump-sum transfer T_H in order to maximize home welfare, whereas foreign taxes are all equal to zero. We conjecture that our qualitative results would remain unchanged in the presence of taxes

[9]Note that this condition rules out mass points but only for strictly positive fixed costs. The distribution conditional on $f_{ij}=0$ for a_{ij} may have mass points. Thus, the model in Krugman (1980), which has no heterogeneity but no fixed costs of exporting, satisfies our requirement.

abroad.[10] This leads to the following definition of the home government's problem.

Definition 1. *The home government's problem is*

$$\max_{T_H, \{\mathbf{t_{jH}}, \mathbf{s_{Hj}}\}_{j=H,F}, w_H, \{\mathbf{q_{ij}}, Q_{ij}, \mathbf{p_{ij}}, P_{ij}, N_i\}_{i,j=H,F}} U_H(Q_{HH}, Q_{FH})$$

subject to conditions (1)-(7).

The goal of the next two sections is to characterize unilaterally optimal taxes, i.e., taxes that prevail at a solution to the domestic government's problem. To do so we follow the public finance literature and use the primal approach. Namely, we will first approach the optimal policy problem of the domestic government in terms of a relaxed planning problem in which domestic consumption, output, and the measure of entrants can be chosen directly (Section 3). We will then establish that the optimal allocation can be implemented through linear taxes and characterize the structure of these taxes (Section 4).

3 Micro and Macro Planning Problems

In this section, we focus on a relaxed version of the home government's problem that abstracts from all constraints in which Home's tax instruments, T_H, $\{\mathbf{t_{jH}}, \mathbf{s_{Hj}}\}_{j=H,F}$, and Home's prices, w_H, $\{\mathbf{p_{Hj}}\}_{j=H,F}$, appear. This relaxed problem can be interpreted as the problem of a fictitious planner who directly controls the quantities demanded by home consumers, $\mathbf{q_{HH}} \equiv \{q_{HH}(\varphi)\}$ and $\mathbf{q_{FH}} \equiv \{q_{FH}(\varphi)\}$, as well as the quantities exported by home firms, $\mathbf{q_{HF}} \equiv \{q_{HF}(\varphi)\}$, and the measure of home entrants, N_H. Specifically, we drop conditions (2), (5), and (7) for $i = H$; we drop condition (3) for $j = H$; and we relax conditions (1) and (4) for $i = H$ or $j = H$ by imposing instead

$$\int_{\Phi} N_i (q_{ij}(\varphi))^{1/\mu_i} dG_i(\varphi) \geq Q_{ij}^{1/\mu_i}, \text{ for } i = H \text{ or } j = H. \tag{8}$$

We refer to this new problem as Home's relaxed planning problem; see Appendix A.

In order to solve this relaxed problem, we take advantage of the nested structure of preferences in this economy and follow a three-step approach. First, we take Home's local output, Q_{HH}, and exports, Q_{HF} as given and solve for the domestic micro quantities, $\{\mathbf{q_{Hj}}\}_{j=H,F}$, as well as the measure of domestic entrants, N_H, that deliver these macro quantities at the lowest possible cost. Second, we solve for the foreign micro quantities,

[10] Accordingly, our results should also hold in the Nash equilibrium of a simultaneous game in which both countries behave strategically.

$\{\mathbf{q_{Fj}}\}_{j=H,F}$, as well as the measure of foreign entrants, N_F, and local output, Q_{FF}, that maximize Home's imports conditional on its exports. Third, we solve for the optimal macro quantities, Q_{HH}, Q_{FH}, and Q_{HF}.. The solution to these micro and macro problems will determine the optimal micro and macro taxes, respectively, in Section 4.[11]

3.1 First Micro Problem: Home's Production Possibility Frontier

Consider the problem of minimizing the labor cost of producing Q_{HH} units of aggregate consumption for Home and Q_{HF} units of aggregate consumption for Foreign subject to condition (8) for $i = H$ and $j = H, F$. This can be expressed as

$$L_H(Q_{HH}, Q_{HF}) \equiv \min_{\mathbf{q_{HH}}, \mathbf{q_{HF}}, N_H} N_H[\sum_{j=H,F} \int_\Phi l_{Hj}(q_{Hj}(\varphi), \varphi) dG_H(\varphi) + f_H^e] \tag{9a}$$

$$\int_\Phi N_H (q_{Hj}(\varphi))^{1/\mu_H} dG_H(\varphi) \geq Q_{Hj}^{1/\mu_H}, \text{ for } j = H, F. \tag{9b}$$

Together with Home's resource constraint, i.e., condition (6) for $i = H$, the previous value function will characterize Home's production possibility frontier.

This minimization problem is infinite dimensional and non-smooth. More precisely, since there are fixed costs, the objective function is neither continuous nor convex around $q_{Hj}(\varphi) = 0$ for any φ such that $f_{Hj}(\varphi) > 0$. To deal with the previous issues and derive necessary properties that any solution to (9) must satisfy, we adopt the following strategy.

First, we consider a planning problem that extends (9) by allowing for randomization: conditional on φ, we let the planner select a distribution of output levels.[12] Since this problem is convex, we can invoke Lagrangian necessity theorems. We then show that randomization is not employed at any solution to the extended problem, so that the planner effectively solves (9). It follows that any solution to (9) must minimize the associated Lagrangian, given by $\mathcal{L}_H = N_H \ell_H$ where

$$\ell_H \equiv \sum_{j=H,F} \int_\Phi \left(l_{Hj}(q_{Hj}(\varphi), \varphi) - \lambda_{Hj} (q_{Hj}(\varphi))^{1/\mu_H} \right) dG_H(\varphi) + f_H^e,$$

for some Lagrange multipliers, $\lambda_{Hj} > 0$. The complete argument can be found in Ap-

[11] Together with the foreign equilibrium conditions, the previous variables determine all foreign prices at the solution of Home's relaxed planning problem. For expositional purposes, we omit the description of these variables from the main text and present them in Appendix A.2.

[12] There are two interpretations of this randomization. In the first, a firm with a blueprint φ is randomly assigned a q according to this conditional distribution; in the second, there is a continuum of firms for a given φ and each firm is assigned a different q so that the population is distributed according to the conditional distribution.

pendix A.1.

Second, we use the additive separability of the Lagrangian $\mathcal{L}_H$ in $\{q_{Hj}(\varphi)\}$ to minimize it variety-by-variety and market-by-market, as in Everett (1963), Costinot, Lorenzoni and Werning (2014), and Costinot, Donaldson, Vogel and Werning (2015). Although the discontinuity at zero remains, it is just a series of one-dimensional minimization problems that can be solved by hand. Namely, for a given variety φ and a market j, consider the one-dimensional subproblem

$$\min_q l_{Hj}(q, \varphi) - \lambda_{Hj} q^{1/\mu_H}.$$

The solution to this problem follows a simple cut-off rule, which must then apply to any solution, $q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H)$, to the original constrained problem (9),

$$q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H) = \begin{cases} (\mu_H a_{Hj}(\varphi)/\lambda_{Hj})^{-\sigma_H}, & \text{if } \varphi \in \Phi_{Hj}, \\ 0, & \text{otherwise}, \end{cases} \tag{10}$$

with $\Phi_{Hj} \equiv \{\varphi : (\mu_H - 1)a_{Hj}(\varphi)(\mu_H a_{Hj}(\varphi)/\lambda_{Hj})^{-\sigma_H} \geq f_{Hj}(\varphi)\}$ the set of domestically produced varieties sold in country j.[13]

Let us now turn to the outer problem that minimizes over N_H. At an interior solution, the derivative of the value function associated with the inner problem should be equal to zero. By the Envelope Theorem, this condition simplifies into

$$f_H^e = \sum_{j=H,F} \int_\Phi (\lambda_{Hj}(q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H))^{1/\mu_H} - l_{Hj}(q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H), \varphi))dG_H(\varphi). \tag{11}$$

This determines the optimal measure of domestic entrants, $N_H(Q_{HH}, Q_{HF})$. The optimal micro quantities are then given by $q_{Hj}(\varphi|Q_{HH}, Q_{HF}) \equiv q_{Hj}(\varphi|Q_{HH}, Q_{HF}, N_H(Q_{HH}, Q_{HF}))$.

By comparing equations (1), (2), (4), and (5), on the one hand, and equations (9b), (10), and (11), on the other hand, one can check that conditional on Q_{HH} and Q_{HF}, the output levels and number of entrants in the decentralized equilibrium with zero taxes and the solution to the relaxed planning problem coincide. This reflects the efficiency of firm's level decision under monopolistic competition with Constant Elasticity of Substitution (CES) utility conditional on industry size; see Dixit and Stiglitz (1977) and Dhingra and Morrow (2012) for closed economy versions of this result. As shown in Section 4, this feature implies that the home government may want to impose a uniform import tariff or

[13]Given the previous definition, equation (10) implies that when indifferent between producing or not, the planner chooses producing. Since indifference is measure zero, this is without loss of generality.

an export tax—in order to manipulate the fraction of labor allocated to domestic production rather than export—but that it never wants to impose taxes that vary across domestic firms, regardless of whether they sell on the domestic or foreign market.

3.2 Second Micro Problem: Foreign's Offer Curve

Next consider the problem of maximizing Home's imports, $Q_{FH}(Q_{HF})$, conditional on its aggregate exports, Q_{HF}, subject to Foreign's equilibrium conditions, namely conditions (1) and (4) for $i = F$ and $j = F$, (2), (5), (6) for $i = F$, and (3) for $j = F$. This second micro problem can be reduced to

$$Q_{FH}^{1/\mu_F}(Q_{HF}) \equiv \max_{\mathbf{q_{FH}}, Q_{FF}, N_F} \int_{\Phi} N_F q_{FH}^{1/\mu_F}(\varphi) dG_F(\varphi)) \tag{12a}$$

$$L_F = P_{FF}(Q_{FF}, N_F)(Q_{FF} + MRS_F(Q_{HF}, Q_{FF})Q_{HF}) \tag{12b}$$

$$N_F f_F^e = \Pi_{FF}(Q_{FF}, N_F)$$
$$+ N_F \int [\mu_F a_{FH}(\varphi) q_{FH}(\varphi) - l_{FH}(q_{FH}(\varphi), \varphi)] dG_F(\varphi), \tag{12c}$$

$$L_F = N_F f_F^e + L_{FF}(Q_{FF}, N_F) + N_F \int_{\Phi} l_{FH}(q_{FH}(\varphi), \varphi) dG_F(\varphi), \tag{12d}$$

$$\mu_F a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}(q_{FH}(\varphi), \varphi), \tag{12e}$$

where $MRS_F(Q_{HF}, Q_{FF}) \equiv \frac{\partial U_F(Q_{HF}, Q_{FF})/\partial Q_{HF}}{\partial U_F(Q_{HF}, Q_{FF})/\partial Q_{FF}}$ denotes the marginal rate of substitution in Foreign and $\Pi_{FF}(Q_{FF}, N_F)$ and $L_{FF}(Q_{FF}, N_F)$ denote the total profits and total employment associated with the local sales of foreign firms. As established in Appendix A.2, constraint (12b) summarizes Foreign's utility maximization problem, whereas constraints (12c) and (12d) summarize its free entry and labor market clearing conditions, respectively, after taking into account the equilibrium values of local prices and quantities in Foreign, $\mathbf{p_{FF}}(Q_{FF}, N_F)$, $P_{FF}(Q_{FF}, N_F)$, and $\mathbf{q_{FF}}(Q_{FF}, N_F)$. Together with the new utility constraint (8) for $i = F$ and $j = H$, the value function in (12) will characterize Foreign's offer curve.

Like in the case of Home's production possibility frontier, it is convenient to focus first on the subproblem that takes Q_{FF} and N_F as given and maximizes over $\mathbf{q_{FH}}$. To deal with the non-smoothness and non-convexities of this minimization problem and derive necessary properties that any of its solution must satisfy, we can follow a similar strategy as in Section 3.1. Technical details can be found in Appendix A.2.

Consider the one-dimensional subproblem of finding the amount of foreign imports of variety φ that solves

$$\max_q q^{1/\mu_F} - \lambda_E \mu_F a_{FH}(\varphi) q + (\lambda_E - \lambda_L) l_{FH}(q,\varphi) \tag{13a}$$

$$\mu_F a_{FH}(\varphi) q \geq l_{FH}(q,\varphi), \tag{13b}$$

where λ_E and λ_L are the Lagrange multipliers associated with (12c) and (12d). The solution to the unconstrained problem, ignoring inequality (13b), is given by

$$q^u_{FH}(\varphi) = \begin{cases} (\mu_F \chi_{FH} a_{FH}(\varphi))^{-\sigma_F}, & \text{if } \theta_{FH}(\varphi) \geq (\max\{0, (\lambda_L - \lambda_E)/\chi_{FH}\})^{1/\sigma_F}, \\ 0, & \text{otherwise,} \end{cases}$$

with $\theta_{FH}(\varphi) \equiv (1/\mu_F \chi_{FH})[(\mu_F - 1)(a_{FH}(\varphi))^{1-\sigma_F}/f_{FH}(\varphi)]^{1/\sigma_F}$ and $\chi_{FH} \equiv \lambda_L + (\mu_F - 1)\lambda_E > 0$.[14] In what follows, we refer to $\theta_{FH}(\varphi)$ as the "profitability" of foreign varieties in Home's market. If $q^u_{FH}(\varphi)$ satisfies constraint (13b), then it is also a solution to (13). If it does not, then the solution to (13) is given either by zero or by $q^c_{FH}(\varphi) > q^u_{FH}(\varphi)$ such that (13b) exactly binds, that is

$$q^c_{FH}(\varphi) = f_{FH}(\varphi)/((\mu_F - 1)a_{FH}(\varphi)).$$

The former case occurs if $(q^c_{FH}(\varphi))^{1/\mu_F} + \chi_{FH} a_{FH}(\varphi) q^c_{FH}(\varphi) + (\lambda_E - \lambda_L) f_{FH}(\varphi) > 0$, while the latter case occurs otherwise. Accordingly, we can express the solution to our second micro problem in a compact way as

$$q_{FH}(\varphi|Q_{HF}) = \begin{cases} (\mu_F \chi_{FH} a_{FH}(\varphi))^{-\sigma_F} & \text{, if } \varphi \in \Phi^u_{FH}, \\ f_{FH}(\varphi)/((\mu_F - 1)a_{FH}(\varphi)) & \text{, if } \varphi \in \Phi^c_{FH}, \\ 0 & \text{, otherwise,} \end{cases} \tag{14}$$

with the two sets of imported varieties defined by

$$\Phi^u_{FH} \equiv \{\varphi : \theta_{FH}(\varphi) \in [(\max\{1, (\lambda_L - \lambda_E)/(\lambda_L + (\mu_F - 1)\lambda_E)\})^{1/\sigma_F}, \infty)\},$$
$$\Phi^c_{FH} \equiv \{\varphi : \theta_{FH}(\varphi) \in [\lambda_L/(\lambda_L + (\mu_F - 1)\lambda_E), 1)\}.$$

Given a solution to the inner problem, $\{q_{FH}(\varphi|Q_{HF}, N_F)\}$, the optimal measure of entrants and local output in Foreign, $N_F(Q_{HF})$ and $Q_{FF}(Q_{HF})$, can be solved for in a stan-

[14] $\chi_{FH} > 0$ is necessary for the solution of the Lagrangian problem to satisfy constraints (12c) and (12d). Since λ_L and λ_E are associated with equality constraints, however, we cannot rule out at this point that one of these two multipliers is negative. We come back to this issue in detail below.

dard manner, as described in Appendix A.2. The optimal micro quantities are then given by $q_{FH}(\varphi|Q_{HF}) \equiv q_{HF}(\varphi|Q_{FF}(Q_{HF}), N_F(Q_{HF}))$.

The set Φ^c_{FH} will play a key role in our subsequent analysis. For varieties $\varphi \in \Phi^c_{FH}$, Home finds it optimal to raise its imports in order to make sure that the least profitable firms in Foreign are willing to produce and export strictly positive amounts, a situation that we will refer to as positive discrimination. As can be seen from the above expression, whether or not Φ^c_{FH} is empty boils down to whether the Lagrange multiplier on the free entry condition, λ_E, is strictly positive or not. At the optimal level of exports, which we characterize next, we will demonstrate that the former case necessarily arises.

3.3 Macro Problem: Manipulating Terms-of-Trade

Finally, consider the choice of macro quantities, (Q_{HH}, Q_{FH}, Q_{HF}), that maximize U_H subject to the last two constraints of Home's relaxed planning problem: Home's resource constraint, i.e., condition (6) for $i = H$, and the new utility constraint (8) for $i = F$ and $j = H$. Given the analysis of Sections 3.1 and 3.2, these two constraints (6) and (8) can be expressed as $L_H(Q_{HH}, Q_{HF}) = L_H$ and $Q_{FH}(Q_{FF}) \geq Q_{FH}$. Thus, optimal aggregate quantities must solve the following macro problem,

$$\max_{Q_{HH}, Q_{FH}, Q_{HF}} U_H(Q_{HH}, Q_{FH}) \tag{15a}$$

$$Q_{FH} \leq Q_{FH}(Q_{HF}), \tag{15b}$$

$$L_H(Q_{HH}, Q_{HF}) = L_H. \tag{15c}$$

At this point, it should be clear that we are back to a standard terms-of-trade manipulation problem with constraints (15b) and (15c) describing Foreign's offer curve and Home's production possibility frontier, as in Baldwin (1948). Like in a perfectly competitive model of international trade, foreign technology, endowments, and preferences only matter through their combined effect on Foreign's offer curve, the elasticity of which will determine the optimal level of trade trade protection.

To characterize the solution of the macro problem (15), let us define Home's terms-of-trade as,

$$P(Q_{FH}, Q_{HF}) \equiv P_{HF}(Q_{HF}) / \tilde{P}_{FH}(Q_{HF}, Q_{FH}),$$

where $P_{HF}(Q_{HF})$ and $\tilde{P}_{FH}(Q_{HF}, Q_{FH})$ are the price of Home's exports and the average cost of Home's imports, respectively,

$$P_{HF}(Q_{HF}) = P_{FF}(Q_{FF}(Q_{HF}), N_F(Q_{HF}))MRS_F(Q_{HF}, Q_{FF}(Q_{HF})),$$
$$\tilde{P}_{FH}(Q_{HF}, Q_{FH}) = N_F(Q_{HF}) \int_{\Phi} \mu_F a_{FH}(\varphi) q_{FH}(\varphi|Q_{HF}) dG_F(\varphi)/Q_{FH}.$$

The tilde symbol emphasizes the fact that the import price index, P_{FH}, faced by Home's consumer in the decentralized equilibrium will differ from $\tilde{P}_{FH}(Q_{HF}, Q_{FH})$: the former is inclusive of trade taxes, whereas the latter is not. As shown in Appendix A.3, at an interior solution to (15), which we focus on throughout our analysis, the necessary first-order conditions imply

$$MRT_H^* P^* / MRS_H^* = 1/\eta^*, \tag{16}$$

where $MRS_H^* \equiv (\partial U_H / \partial Q_{HH}) / (\partial U_H / \partial Q_{FH})$ and $MRT_H^* \equiv (\partial L_H / \partial Q_{HH}) / (\partial L_H / \partial Q_{HF})$ are the marginal rate of substitution and marginal rate of transformation in Home, respectively, and $\eta^* \equiv d \ln Q_{FH} / d \ln Q_{HF}$ is the elasticity of Foreign's offer curve, all evaluated at the solution to the macro problem.

The left-hand side of equation (16) describes the optimal wedge between the relative price faced by Home's consumer, which must be equal to MRS_H^* in a decentralized equilibrium, and the relative price abroad adjusted by the trade costs, $MRT_H^* P^*$. If there are no trade frictions, including no fixed exporting costs, $MRT_H^* = 1$ and equation (16) reduces to the well-known optimal tariff formula, $P^*/MRS_H^* - 1 = 1/\eta^* - 1$, as in Dixit (1985). Finally, note that since trade balance requires $Q_{FH}(Q_{HF}) = P(Q_{FH}, Q_{HF})Q_{HF}$, the elasticity of Foreign's offer curve must satisfy

$$\eta = \frac{1 + \rho_{HF}}{1 - \rho_{FH}},$$

where $\rho_{ij} \equiv \partial \ln P(Q_{FH}, Q_{HF}) / \partial \ln Q_{ij}$ denotes Home's terms-of-trade elasticities. Thus, like in a Walrasian economy, the more Home's terms-of-trade deteriorate with increases in exports or imports —i.e., the lower ρ_{HF}, ρ_{FH}, and hence, η are—the higher the optimal level of trade protection should be.

3.4 The Case for Positive Discrimination

We are ready to come back to the issue of whether the solution to Home's relaxed planning problem exhibits positive discrimination. As discussed above, establishing positive discrimination is formally equivalent to establishing that, at the optimum, $\lambda_E > 0$. In Section 4, we will show that this particular feature of the solution to Home's relaxed planning problem implies lower import taxes on the least profitable firms exporting from Foreign. Here, we present a heuristic proof; the complete proof is given in Appendix A.4.

Let us first argue that λ_E must be non-negative. This is equivalent to showing that one can relax constraint (12c) into

$$N_F f_F^e \geq \Pi_{FF}(Q_{FF}, N_F) + N_F \int [\mu_F a_{FH}(\varphi) q_{FH}(\varphi) - l_{FH}(q_{FH}(\varphi), \varphi)] dG_F(\varphi). \tag{17}$$

To do so, it is convenient to again separate (12) into an inner problem that takes Q_{FF} and N_F as given and maximizes over $\mathbf{q_{FH}}$ and an outer problem that maximizes over Q_{FF} and N_F. The first key observation is that if inequality (17) is slack, then the inner problem reduces to

$$Q_{FH}^{1/\mu_F}(Q_{FF}, N_F) \equiv \max_{\mathbf{q_{FH}}} \int_\Phi N_F q_{FH}^{1/\mu_F}(\varphi) dG_F(\varphi))$$
$$L_F = N_F f_F^e + L_{FF}(Q_{FF}, N_F) + N_F \int_\Phi l_{FH}(q_{FH}(\varphi), \varphi) dG_F(\varphi),$$

which is just the dual of minimizing the foreign labor cost of aggregate imports. Hence, the optimal quantities $q_{FH}(\varphi|Q_{FF}, N_{FF})$ must satisfy the same conditions as in Section 3.1. Differentiating the previous expression and invoking the Envelope Theorem, the same algebra now implies

$$\begin{aligned}\frac{\partial Q_{FH}^{1/\mu_F}(Q_{FF}, N_F)}{\partial N_F} = & -\lambda_L [f_F^e - \Pi_{FF}(Q_{FF}, N_F)/N_F \\ & - \int_\Phi (\mu_F a_{FH}(\varphi) q_{FH}(\varphi|Q_{FF}, N_F) - l(q_{FH}(\varphi|Q_{FF}, N_F), \varphi)) dG_F(\varphi)] < 0,\end{aligned} \tag{18}$$

where the sign of the inequality derives from (17).

Now consider the outer problem of maximizing the previous value function with respect to Q_{FF} and N_F subject to constraint (12b). Since the upper-level elasticity of substitution between Q_{HF} and Q_{FF} is always strictly greater than one, $MRS_F(Q_{HF}, Q_{FF})Q_{HF}$ must be increasing in Q_{HF}. Hence, at a solution to Home's relaxed planning problem, constraint (12b) can also be relaxed into

$$L_F \leq P_{FF}(Q_{FF}, N_F)(Q_{FF} + MRS_F(Q_{HF}, Q_{FF})Q_{HF}).$$

If this constraint was slack, Home could reduce Q_{HF} and increase Q_{HH}, while still satisfying (15c), and hence increase the utility function in (15a). Noting that price of foreign varieties, $P_{FF}(Q_{FF}, N_F)$, must be decreasing in N_F , we can then rearrange this constraint as an upper-bound on the measure of foreign entrants,

$$N_F \leq N_F(Q_{HF}, Q_{FF}). \tag{19}$$

This our second key observation. Together conditions (18) and (19) imply that (17) cannot be slack at an optimum. If it were, the derivative of the value function of the inner problem would be strictly negative, which would require N_F to be bounded from below, not above.

The previous argument establishes that $\lambda_E \geq 0$. The final issue is whether the relaxed free entry condition (17) could be binding, but with $\lambda_E = 0$. This case is not a theoretical curiosity; this is precisely what happens when entry is efficient, as established in Section 3.1. In other words, if Foreign was also operating on its production possibility frontier, as will be the case in Section 5, then the previous condition would hold. Here, however, the measure of foreign entrants must be inefficient. To see this, note that if condition (17) was satisfied with equality but $\lambda_E = 0$, then the same relationship between the sign of expected profits and the derivative of the value function of the inner problem would lead to

$$\frac{\partial Q_{FH}^{1/\mu_F}(Q_{FF}, N_F)}{\partial N_F} = 0. \tag{20}$$

But starting from such a situation, Home could always increase utility by: lowering the measure of foreign entrants, N_F; lowering its exports, Q_{HF}, so that constraint (12b) is still satisfied; and raising its consumption of the local good, Q_{HH}, so that the resource constraint (15c) also holds. By equation (20), the welfare loss caused by the change in aggregate imports must be at most second order, whereas the welfare gain caused by the increase in local consumption is first order.

Intuitively, Home internalizes the fact that by varying the measure of entrants in Foreign, it can manipulate its terms of trade. At an optimum, such considerations lead Home to select a lower measure of foreign entrants than under *laissez faire*. This tends to reduce foreign labor demand and to raise the expected profits of foreign firms. Yet, in equilibrium, foreign labor market must clear and foreign firms must still make zero profits. Hence, the lower measure of entrants must be compensated by an expansion of production by the least profitable firms in Foreign, which positive discrimination delivers.

4 Optimal Taxes

We have derived three necessary conditions—equations (10), (14), and (16)—that micro quantities, $\{q^*_{HH}(\varphi) \equiv q_{HH}(\varphi|Q^*_{HH}, Q^*_{HF})\}$, $\{q^*_{HF}(\varphi) \equiv q_{HF}(\varphi|Q^*_{HH}, Q^*_{HF})\}$, $\{q^*_{FH}(\varphi) \equiv q_{FH}(\varphi|Q^*_{HF})\}$, and macro quantities, Q^*_{HH}, Q^*_{HF}, and Q^*_{FH}, solving Home's relaxed planning problem must satisfy. We now use these conditions to derive necessary properties that ad-valorem taxes implementing such a solution must satisfy (Sections 4.1-4.3). We will then use these properties to establish the existence of such taxes (Section 4.4). Since they replicate the solution to Home's relaxed planning problem, they a fortiori solve the home government's problem described in Definition 1.

4.1 Micro-level Taxes on Domestic Varieties

Consider first a schedule of domestic taxes, $\{s^*_{HH}(\varphi)\}$ and $\{t^*_{HH}(\varphi)\}$, that implements the optimal micro quantities, $\{q^*_{HH}(\varphi)\}$. Fix a benchmark variety φ_{HH} that is sold domestically, $q^*_{HH}(\varphi_{HH}) > 0$. Denote by $s^*_{HH} \equiv s^*_{HH}(\varphi_{HH})$ and $t^*_{HH} \equiv t^*_{HH}(\varphi_{HH})$ the domestic taxes imposed on that variety. Now take any other variety $\varphi \in \Phi_{HH}$ that is sold domestically. By equations (1) and (2), we must have

$$\frac{q^*_{HH}(\varphi_{HH})}{q^*_{HH}(\varphi)} = \left(\frac{(1+t^*_{HH})a_{HH}(\varphi_{HH})}{(1+s^*_{HH})} \frac{(1+t^*_{HH}(\varphi))}{(1+s^*_{HH}(\varphi))a_{HH}(\varphi)} \right)^{-\sigma_H}.$$

Combining this expression with equation (10), we obtain our first result.

Lemma 1. *In order to implement an allocation solving the relaxed planning problem, domestic taxes should be such that*

$$(1+s^*_{HH}(\varphi))/(1+t^*_{HH}(\varphi)) = (1+s^*_{HH})/(1+t^*_{HH}) \text{ if } \varphi \in \Phi_{HH}. \tag{21}$$

While we have focused on domestic taxes, there is nothing in the previous proposition that hinges on domestic varieties being sold in the domestic market rather than abroad. Thus, we can use the exact same argument to characterize the structure of export taxes, $\{s^*_{HF}(\varphi)\}$, that implements $\{q^*_{HF}(\varphi)\}$. In line with the previous analysis, let φ_{HF} denote a benchmark variety that is exported, with $s^*_{HF} \equiv s^*_{HF}(\varphi_{HF})$. The following result must hold.

Lemma 2. *In order to implement an allocation solving the relaxed planning problem, export taxes should be such that*

$$s^*_{HF}(\varphi) = s^*_{HF} \text{ if } \varphi \in \Phi_{HF}. \tag{22}$$

4.2 Micro-level Taxes on Foreign Varieties

Now consider a schedule of import taxes, $\{t^*_{FH}(\varphi)\}$, that implements the desired allocation, $\{q^*_{FH}(\varphi)\}$. Fix a benchmark variety $\varphi_{FH} \in \Phi^u_{FH}$ that is imported. In line with our previous analysis, let $t^*_{FH} \equiv t^*_{FH}(\varphi_{FH})$ denote the import tax imposed on that benchmark variety. For any other variety $\varphi \in \Phi_{FH} \equiv \Phi^u_{FH} \cup \Phi^c_{FH}$ that is imported, equations (1) and (2) now imply

$$\frac{q^*_{FH}(\varphi_{FH})}{q^*_{FH}(\varphi)} = \left(\frac{(1+t^*_{FH})a_{FH}(\varphi_{FH})}{(1+t^*_{FH}(\varphi))a_{FH}(\varphi)}\right)^{-\sigma_F}. \tag{23}$$

There are two possible cases to consider. If $\varphi \in \Phi^u_{FH}$, then equations (14) and (23) imply

$$t^*_{FH}(\varphi) = t^*_{FH}.$$

If $\varphi \in \Phi^c_{FH}$, then equations (14) and (23) imply

$$t^*_{FH}(\varphi) = (1+t^*_{FH})\theta_{FH}(\varphi) - 1.$$

This leads to our third result.

Lemma 3. *In order to implement an allocation solving the relaxed planning problem, import taxes should be such that*

$$t^*_{FH}(\varphi) = (1+t^*_{FH})\min\{1, \theta_{FH}(\varphi)\} - 1 \text{ if } \varphi \in \Phi_{FH}, \tag{24}$$

with the profitability index $\theta_{FH}(\varphi) \equiv (\lambda_{FH}/\chi_{FH}\mu_F)[(\mu_F - 1)(a_{FH}(\varphi))^{1-\sigma_F}/f_{FH}(\varphi)]^{1/\sigma_F}$.

In the context of a canonical model of intra-industry trade where heterogeneous firms select into exporting, optimal import taxes are higher for more profitable exporters. However, such heterogeneous taxes do not reflect the home government's desire to prevent imports from more profitable exporters. Instead, they reflect the desire to import from less profitable exporters as well. This motive leads to import taxes that are constant among the most profitable exporters, but vary among the least profitable ones.

4.3 Overall Level of Taxes

Our next goal is to characterize the overall level of taxes that is necessary for a decentralized equilibrium to implement the desired allocation. In Sections 4.1 and 4.2, we have already expressed all other taxes as a function of t^*_{HH}, t^*_{FH}, s^*_{HH}, and s^*_{HF}. So, this boils down to characterizing these four taxes. To do so, we compare the ratio between the marginal rates of substitution at home and abroad, evaluated at the solution to Home's relaxed planning problem, and their ratio in the decentralized equilibrium with taxes. As expected, and as established formally in Appendix B.1, the inverse of the elasticity of Foreign's offer curve, η^*, that appears in the first-order conditions of Home's macro planning problem anchors the overall level of taxes in the decentralized equilibrium.

Lemma 4. *In order to implement an allocation solving the relaxed planning problem, the overall level of optimal taxes, t^*_{HH}, t^*_{FH}, s^*_{HH}, and s^*_{HF}, should be such that*

$$\frac{(1+t^*_{FH})/(1+t^*_{HH})}{(1+s^*_{HF})/(1+s^*_{HH})} = \frac{\int_{\Phi_{FH}} \left((\min\{1,\theta_{FH}(\varphi)\})^{\mu_F} a_{FH}(\varphi)\right)^{1-\sigma_F} dG_F(\varphi)}{\eta^* \int_{\Phi_{FH}} \left((\min\{1,\theta_{FH}(\varphi)\}) a_{FH}(\varphi)\right)^{1-\sigma_F} dG_F(\varphi)}. \tag{25}$$

Two remarks are in order. First, if Φ^c_{FH} is measure zero, then $\min\{1,\theta_{FH}(\varphi)\} = 1$ for all $\varphi \in \Phi_{FH}$ so optimal import taxes are uniform and equation (25) reduces to

$$\frac{(1+t^*_{FH})/(1+t^*_{HH})}{(1+s^*_{HF})/(1+s^*_{HH})} = 1/\eta^*.$$

This is what would happen in the absence of fixed exporting costs, as in Krugman (1980).[15] We come back to this situation more generally in Section 5 when we study optimal uniform taxes. Second, if Φ^c_{FH} is not measure zero, then $\mu_F > 1$ implies

$$\frac{(1+t^*_{FH})/(1+t^*_{HH})}{(1+s^*_{HF})/(1+s^*_{HH})} > 1/\eta^*.$$

This merely reflects our choice of benchmark variety for imports. t^*_{FH} is the tax on varieties $\varphi \in \Phi^u_{FH}$, and we know from Lemma 3 that import taxes should be lower on varieties $\varphi \in \Phi^c_{FH}$. So in order to implement the same wedge, the domestic government must now impose import taxes on varieties $\varphi \in \Phi^u_{FH}$ that, relative to other taxes, are strictly greater than $1/\eta^*$.

4.4 Implementation

Lemmas 1-4 provide necessary conditions that linear taxes have to satisfy so that the decentralized equilibrium replicates a solution to the relaxed planning problem. In the next lemma, which is proven in Appendix B.2, we show that that if the previous taxes

[15]In Section 3.4, we have established that Φ^c_{FH} is not empty. For arbitrary distributions of foreign blueprints, G_F, however, our analytical results do not rule out the possibility that the measure of blueprints in Φ^c_{FH} is zero. In all our simulations, we have found that if G_F was non degenerate, then Φ^c_{FH} had strictly positive measure.

are augmented with high enough taxes on the goods that are not consumed, $\varphi \notin \Phi_{HH}$, $\varphi \notin \Phi_{HF}$, and $\varphi \notin \Phi_{FH}$, then they are also sufficient to implement any allocation that solves the relaxed planning problem.

Lemma 5. *There exists a decentralized equilibrium with taxes that implements any allocation that solves the relaxed planning problem.*

Since Home's relaxed planning problem is, as its name indicates, a relaxed version of Home's government problem introduced in Definition 1, the taxes associated with a decentralized equilibrium that implements a solution to the relaxed planning problem must a fortiori solve Home's government problem. Lemmas 2-5 therefore imply that any taxes that solve Home's government problem must satisfy conditions (21), (22), (24), and (25). To summarize, we can characterize unilaterally optimal taxes as follows.

Proposition 1. *At the micro-level, unilaterally optimal taxes should be such that: (i) domestic taxes are uniform across all domestic producers (condition 21); (ii) export taxes are uniform across all exporters (condition 22); (iii) import taxes are uniform across Foreign's most profitable exporters and strictly increasing with profitability across its least profitable ones (condition 24). At the macro-level, unilaterally optimal taxes should reflect standard terms-of-trade considerations (condition 25).*

Note that condition (25) only pins down the relative levels of optimal taxes. In the proof of Lemma 5, we show how to implement the desired allocation using only import taxes, $t^*_{HH} = s^*_{HH} = s^*_{HF} = 0$. There is, however, a continuum of optimal taxes that would achieve the same allocation. For instance, we could have used a uniform export tax,

$$s^*_{HF} = \frac{\eta^* \int_{\Phi_{FH}} ((\min\{1, \theta_{FH}(\varphi)\}) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)}{\int_{\Phi_{FH}} ((\min\{1, \theta_{FH}(\varphi)\})^{\mu_F} a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)},$$

while setting the overall level other taxes such that $t^*_{HH} = s^*_{HH} = t^*_{FH} = 0$. This is an expression of Lerner symmetry, which must still hold under monopolistic competition. In this case, all varieties $\varphi \in \Phi^c_{FH}$ would receive an import subsidy equal to $\theta_{FH}(\varphi) - 1 < 0$. As alluded to in Section 3.1, the fact that domestic taxes can be dispensed with derives from the efficiency of the decentralized equilibrium with monopolistic competition and CES utility. Here, as in Bhagwati (1971), trade taxes are the preferred instruments to exploit monopoly and monopsony power in world markets.[16]

[16] In the present environment, however, the introduction of non-linear taxes would raise Home's welfare. By imposing two-part tariffs, Home could incentivize foreign firms to sell at marginal costs and compensate them (exactly) for the fixed exporting costs that they incur. Qualitatively, optimal taxes would remain

4.5 How Does Firm Heterogeneity Affect Optimal Trade Policy?

Using Proposition 1, we can take a first stab at describing how firm heterogeneity affects optimal trade policy. There are two broad insights that emerge from our analysis.

The first one is that a unique macro-elasticity, η^*, determines the wedge between Home and Foreign's marginal rates of substitution at the desired allocation and, in turn, the overall level of trade protection, as established by condition (25). In line with the equivalence result in Arkolakis, Costinot and Rodríguez-Clare (2012), this is true regardless of whether or not firms are heterogeneous and only the most profitable ones select into exporting. This first observation derives from the fact that at the macro-level, Home's relaxed planning problem can still be expressed as a standard terms-of-trade manipulation problem where Home chooses aggregate exports and imports taking into account the elasticity of Foreign's offer curve; see problem (15).

The second insight that emerges from Proposition 1 is that even conditioning on the previous macro-elasticity, firm heterogeneity does affect optimal trade policy, as it leads to optimal trade taxes that are heterogeneous across foreign exporters. In order to lower the aggregate price of its imports, the home government has incentives to impose tariffs that are increasing with the profitability of foreign exporters. Since the overall level of trade protection is fixed by the inverse of the elasticity of Foreign's offer curve, η^*, this implies that the import tariffs imposed on the most profitable firms from abroad are higher, relative to other taxes, than they would be in the absence of selection, as also established by condition (25).

These findings echo the results derived by Costinot, Donaldson, Vogel and Werning (2015) in the context of a Ricardian model. As they note, the equivalence emphasized by Arkolakis, Costinot and Rodríguez-Clare (2012) builds on the observation that standard gravity models, like Anderson and Van Wincoop (2003) and Eaton and Kortum (2002), are equivalent to endowment models in which countries directly exchange labor services. Hence, conditional on the elasticity of their labor demand curves, the aggregate implications of uniform changes in trade costs, i.e. exogenous labor demand shifters, must be the same in all gravity models. The previous observation, however, does not imply that optimal policy should be the same in all these models. To the extent that optimal trade taxes are heterogeneous across goods, they will not act as simple labor demand shifters, thereby breaking the equivalence in Arkolakis, Costinot and Rodríguez-Clare (2012). This is what Proposition 1 establishes in the context of a canonical model of trade with monopolistic

uniform across domestic firms and optimal import tariffs would remain biased against Foreign's most profitable exporters. The main difference is that discrimination would take the form of higher fixed fees for entering Home's market rather than higher linear taxes.

competition and firm-level heterogeneity à la Melitz (2003).

This general conclusion notwithstanding—micro-structure matters for optimal policy, even conditioning on macro-elasticities—it is worth noting that the specific policy prescriptions derived under perfect and monopolistic competition differ sharply. In Costinot, Donaldson, Vogel and Werning (2015), optimal export taxes should be heterogeneous, whereas optimal import tariffs should be uniform. This is the exact opposite of what conditions (22) and (24) prescribe under monopolistic competition. In a Ricardian economy, goods exported by domestic firms could also be produced by foreign firms. This threat of entry limits the ability of the home government to manipulate prices and leads to lower export taxes on "marginal" goods. Since this threat is absent under monopolistic competition, optimal export taxes are uniform instead. On the import side, lower tariffs on "marginal" goods under monopolistic competition derive from the existence of fixed exporting costs, which are necessarily absent under perfect competition.

5 Optimal Uniform Taxes

In the last two sections, we have characterized optimal trade policy under the assumption that the home government is not only free to discriminate between firms from different countries by using trade taxes, but also unlimited in its ability to discriminate between firms from the same country. While this provides a useful benchmark to study the normative implications of firm heterogeneity for trade policy, informational or legal constraints may make this type of taxation infeasible in practice. Here, we turn to the other polar case in which the home government is constrained to set uniform taxes: $t_{HF}(\varphi) = \bar{t}_{HF}$, $t_{HH}(\varphi) = \bar{t}_{HH}$, $s_{HF}(\varphi) = \bar{s}_{HF}$, and $s_{HH}(\varphi) = \bar{s}_{HH}$ for all φ.

5.1 Micro to Macro Once Again

To solve for optimal uniform taxes, we can follow the same approach as in Sections 3 and 4. The only difference is that the micro problems of Sections 3.1 and 3.2 should now include an additional constraint:

$$q_{ij}(\varphi')/q_{ij}(\varphi) = (a_{ij}(\varphi')/a_{ij}(\varphi))^{-\sigma_F} \text{ for any } \varphi, \varphi' \text{ such that } q_{ij}(\varphi'), q_{ij}(\varphi) > 0. \tag{26}$$

By construction, whenever the solution to Home's relaxed planning problem satisfies (26), it can be implemented with uniform taxes over the goods that are being produced. Furthermore, since Home always prefers to produce or import the most profitable goods, any solution that satisfies (26) can also be implemented with the same uniform taxes over the goods that are not produced or imported. Like in Section 4.4, strictly higher taxes on those goods can be dispensed with.

For varieties from Home that are sold in any market, $i = H$ and $j = H, F$, constraint (26) is satisfied by the solution to the relaxed problem (9). In this case, optimal taxes were already uniform, as established in Lemmas 1 and 2. So the value of $L_H(Q_{HH}, Q_{HF})$ remains unchanged. In contrast, for foreign varieties that are imported by Home, $i = F$ and $j = H$, constraint (26) will bind at the solution to (12). This leads to a new offer curve in Foreign, which we describe in the next subsection.

The other equations that characterize the solution to Home's relaxed planning problem are unchanged. In particular, one can still reduce Home's macro planning problem to (15). Following the same reasoning as in Section 4, one can therefore show that optimal uniform taxes must satisfy

$$\frac{(1+\bar{t}^*_{FH})/(1+\bar{t}^*_{HH})}{(1+\bar{s}^*_{HF})/(1+\bar{s}^*_{HH})} = 1/\eta^*. \tag{27}$$

Like in Section 4, the optimal wedge still depends exclusively on the elasticity of Foreign's offer curve.

In order to help compare our results to those in the existing literature, we set domestic and export taxes to zero in the rest of this section: $\bar{t}^*_{HH} = \bar{s}^*_{HH} = \bar{s}^*_{HF} = 0$. For the same reasons as in Section 4.4, this is without loss of generality. Under this normalization, we can talk equivalently about optimal uniform taxes and optimal uniform tariffs, $\bar{t}^*_{FH} = 1/\eta^* - 1$.

5.2 Terms-of-Trade Elasticities

In Section 3, terms-of-trade elasticities are complex objects that depend both on supply and demand conditions in Foreign as well as the optimal micro-level choices of Home's government. With uniform trade taxes, the constraints imposed on the latter makes the determinants of terms-of-trade elasticities simpler. We now take advantage of this simplicity to explore the deeper determinants of terms-of-trade elasticities.[17] In the next subsection, this information will allow us to address whether going from an economy without firm heterogeneity to an economy with firm heterogeneity affects the overall level of trade protection by changing the terms-of-trade elasticities.

[17] In that respect, our analysis bears some connection to Melitz and Redding (2015) who investigate how the introduction of firm heterogeneity affects the elasticity of trade flows with respect to trade costs and, in turn, the welfare gains from trade.

In the absence of taxes that vary at the micro-level, it is convenient to summarize technology in Foreign by the function

$$L_F(Q_{FH}, Q_{FF}) \equiv \min_{\mathbf{q_{FH}}, \mathbf{q_{FF}}, N_F} N_F[\sum_{j=H,F} \int_\Phi l_{Fj}(q_{Fj}(\varphi), \varphi) dG_F(\varphi) + f_F^e] \tag{28a}$$

$$N_F \int_\Phi (q_{Fj}(\varphi))^{1/\mu_F} dG_F(\varphi) \geq Q_{Fj}^{1/\mu_F}, \text{ for } j = H, F. \tag{28b}$$

By construction, Foreign's production possibility frontier corresponds to the set of aggregate output levels (Q_{FH}, Q_{FF}) such that $L_F(Q_{FH}, Q_{FF}) = L_F$. This is just the counterpart of equation (15c) for Home in Section 3.3. Building on the efficiency of the decentralized equilibrium under monopolistic competition with CES utility, one can then show that Foreign necessarily operates on its production possibility frontier with the marginal rate of transformation being equal to the price of foreign exports relative to foreign domestic output. On the demand side, we already know that the marginal rate of substitution must be equal to the price of foreign imports relative to foreign domestic consumption and that Foreign's total spending must be equal to its revenue. Thus foreign equilibrium conditions can be described compactly as follows; see Appendix C.1 for a formal proof.

Lemma 6. *Conditional on Q_{HF}, the decentralized equilibrium abroad is such that*

$$MRS_F(Q_{HF}, Q_{FF}(Q_{FH})) = P_{HF}/P_{FF}, \tag{29}$$

$$MRT_F(Q_{FH}, Q_{FF}(Q_{FH})) = \tilde{P}_{FH}/P_{FF}, \tag{30}$$

$$P_{HF}Q_{HF} = \tilde{P}_{FH}Q_{FH} \tag{31}$$

with $\tilde{P}_{FH}$ the untaxed price of Home's imports, and $Q_{FF}(Q_{FH})$ given by the implicit solution of

$$L_F(Q_{FH}, Q_{FF}) = L_F. \tag{32}$$

The key insight of Lemma 6 is that the decentralized equilibrium abroad under monopolistic competition with CES utility is isomorphic, in terms of aggregate quantities and prices, to a perfectly competitive equilibrium with three goods, one that is produced and consumed domestically (in quantity Q_{FF}), one that is produced but not consumed (exported in quantity Q_{FH}), and one that is consumed but not produced (imported in quantity Q_{HF}). The only distinction between the two equilibria is that under monopolistic competition, Foreign's production set may not be convex, as depicted in Figure 1. We come back to this point below.

Using Lemma 6, we can relate the elasticity of Foreign's offer curve to its aggregate elasticities of substitution and transformation. Let $\epsilon \equiv -d\ln(Q_{HF}/Q_{FF})/d\ln(P_{HF}/P_{FF})$

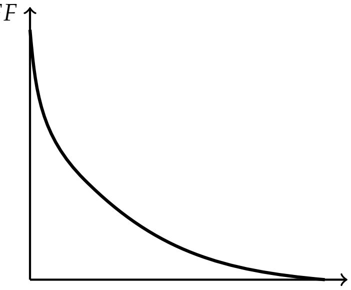

Figure 1: Aggregate Nonconvexities with Firm Heterogeneity

denote the elasticity of substitution between imports and domestic goods and let $\kappa \equiv d\ln(Q_{FH}/Q_{FF})/d\ln(\tilde{P}_{FH}/P_{FF})$ denote the elasticity of transformation between between exports and domestic goods (both in Foreign). Since the marginal rate of substitution and the marginal rate of transformation abroad are both homogeneous of degree zero,[18] equations (29) and (30) imply

$$\epsilon = -1/(d\ln MRS_F(Q_{HF}/Q_{FF},1)/d\ln(Q_{HF}/Q_{FF})), \tag{33}$$

$$\kappa = 1/(d\ln MRT_F(Q_{FH}/Q_{FF},1)/d\ln(Q_{FH}/Q_{FF})). \tag{34}$$

By equations (29) and (30), Home's terms of trade can be expressed as

$$P(Q_{FH},Q_{HF}) = MRS_F(Q_{HF},Q_{FF}(Q_{FH}))/MRT_F(Q_{FH},Q_{FF}(Q_{FH})). \tag{35}$$

Combining equation (35) with the trade balance condition (31), we can describe Foreign's offer curve implicitly as

$$P(Q_{FH},Q_{HF})Q_{HF} = Q_{FH}. \tag{36}$$

As in Section 3.3, totally differentiating the previous expression with respect to Home's aggregate exports and imports, Q_{HF} and Q_{FH}, we obtain

$$\eta = (1+\rho_{HF})/(1-\rho_{FH}), \tag{37}$$

where Home's terms-of-trade elasticities, $\rho_{ij} \equiv \partial\ln P(Q_{FH},Q_{HF})/\partial\ln Q_{ij}$, can be com-

[18]The homogeneity of degree zero of the marginal rate of substitution derives directly from our assumption that the foreign utility function is homothetic. Establishing the homogeneity of degree zero of the marginal rate of transformation is more subtle since the transformation function, $L_F(Q_{FH},Q_{FF})$, is not homogeneous of degree one. We do so formally in Appendix C.2.

puted using equations (33)-(35),

$$\rho_{HF} = -1/\epsilon, \tag{38}$$

$$\rho_{FH} = -(1/x_{FF} - 1)/\epsilon - 1/(x_{FF}\kappa), \tag{39}$$

with $x_{FF} \equiv P_{FF}Q_{FF}/L_F$ the share of expenditure on domestic goods in Foreign.[19]

When $\kappa \geq 0$, Foreign's production set is convex and, everything else being equal, an increase in Home's imports tends to worsen its terms of trade by raising the opportunity cost of foreign exports in terms of foreign domestic output. This is the mechanism at play in a neoclassical environment. When $\kappa < 0$ instead, aggregate nonconvexities imply that an increase in Home's imports tends to *lower* the opportunity cost of foreign exports, and in turn, *improve* its terms of trade.

5.3 A Generalized Optimal Tariff Formula

Combining equation (27)—under the restriction that $\bar{t}^*_{HH} = \bar{s}^*_{HH} = \bar{s}^*_{HF} = 0$—with equations (37)-(39), we obtain the following characterization of optimal uniform tariffs under monopolistic competition with firm heterogeneity.

Proposition 2. *Optimal uniform tariffs are such that*

$$\bar{t}^*_{FH} = \frac{1 + (\epsilon^*/\kappa^*)}{(\epsilon^* - 1)x^*_{FF}}, \tag{40}$$

where ϵ^*, κ^*, and x^*_{FF} are the values of ϵ, κ, and x_{FF} evaluated at those taxes.

Equation (40) is a strict generalization of the optimal tariff formula derived under monopolistic competition by Gros (1987), Demidova and Rodríguez-Clare (2009), and Felbermayr, Jung and Larch (2013). It applies to any economy in which: (i) Home's optimal choices of exports and imports correspond to the solution to a planning problem that can be reduced to (15); and (ii) the decentralized equilibrium in the rest of the world can be reduced to equations (29)-(32). Within that class of models, alternative assumptions

[19] To derive equation (39), we have also used the fact that

$$d\ln Q_{FF}(Q_{FH})/d\ln Q_{FH} = Q'_{FF}(Q_{FH})Q_{FH}/Q_{FF}(Q_{FH}) = -Q_{FH}MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))/Q_{FF}(Q_{FH}).$$

Together with equation (30), this implies

$$d\ln Q_{FF}(Q_{FH})/d\ln Q_{FH} = -(\tilde{P}_{FH}Q_{FH})/(P_{FF}Q_{FF}) = -(1/x_{FF} - 1).$$

about technology, preferences, and market structure only matter for the overall level of trade protection if they affect the three sufficient statistics: ϵ^*, κ^*, and x^*_{FF}.

Gros (1987) focuses on an economy à la Krugman (1980). There is no firm heterogeneity, no market-specific fixed costs, and the elasticity of substitution between domestic and foreign goods is constant, $\epsilon^* = \sigma_H = \sigma_F \equiv \sigma$. In this case, all firms export to all markets. Thus, equation (30) implies that the marginal rate of transformation abroad is constant and given by

$$MRT_F = \frac{\left(\int_\Phi (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)\right)^{1/(1-\sigma_F)}}{\left(\int_\Phi (a_{FF}(\varphi))^{1-\sigma_F} dG_F(\varphi)\right)^{1/(1-\sigma_F)}}.$$

In turn, the elasticity of transformation κ^* goes to infinity and equation (40) becomes

$$\bar{t}^*_{FH} = \frac{1}{(\sigma - 1) x^*_{FF}} > 0.$$

Proposition 2 demonstrates that Gros's (1987) formula remains valid for arbitrary distributions of firm-level productivity and arbitrary upper-level utility functions provided that Foreign's production possibility frontier is linear. A sufficient condition for this to be the case is that foreign firms face no fixed costs of selling in both markets, $f_{Fj}(\varphi) = 0$ for $j = H, F$.

Beside greater generality, a benefit of our analysis is that it helps identify the economic forces that determine optimal trade policy under monopolistic competition. Under the restriction that $\epsilon^* = \sigma_H = \sigma_F \equiv \sigma$, the optimal tariff formula derived by Gros (1987) can be interpreted in two ways, as discussed by Helpman and Krugman (1989). One can think of Home as manipulating its terms-of-trade, as we have emphasized in this paper, or of Home imposing a tariff equal to the markup charged on domestic goods so that the relative price of foreign to domestic goods equals the country's true opportunity cost of domestic goods. Indeed, the difference between Home firms' price and marginal cost for their domestic sales is equal to $\mu_H - 1 = 1/(\sigma_H - 1)$, which is the optimal tariff that a small open economy would choose when $\epsilon^* = \sigma_H$. By allowing the upper-level elasticity of substitution, ϵ^*, to differ from the lower-level elasticities of substitution, σ_H and σ_F, our analysis suggests that the first of these two interpretations is the most robust. When $\epsilon^* \neq \sigma_H$, Home firms still charge a markup $\mu_H = \sigma_H/(\sigma_H - 1)$ on the goods that they sell domestically. Yet, the only relevant elasticity in this case is ϵ^* because it is the one that shapes Home's terms-of-trade elasticities, as shown in equations (38) and (39). We come back to this issue in Section 6.3.

As noted above, Proposition 2 also generalizes the results of Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung and Larch (2013) who focus on an economy à la Melitz

(2003). Compared to the present paper, they assume a constant elasticity of substitution between domestic and foreign goods, $\epsilon^* = \sigma_H = \sigma_F \equiv \sigma$. They also assume that taxes are uniform across firms, that firms only differ in terms of their productivity, and that the distribution of firm-level productivity is Pareto. Under these assumptions, the decentralized equilibrium with taxes can be solved in closed-form. As discussed in Feenstra (2010), models of monopolistic competition with Pareto distributions lead to an aggregate production possibility frontier with constant elasticity of transformation,

$$\kappa^* = -\frac{\sigma\nu - (\sigma - 1)}{\nu - (\sigma - 1)} < 0, \tag{41}$$

where $\nu > \sigma - 1$ is the shape parameter of the Pareto distribution; see Appendix C.3.[20] Combining equations (40) and (41) and imposing $\epsilon^* = \sigma$, we obtain

$$\bar{t}^*_{FH} = \frac{1}{(\nu\mu - 1)x^*_{FF}} > 0,$$

as in Felbermayr, Jung and Larch (2013). In the case of a small open economy, the previous expression simplifies further into $1/(\nu\mu - 1)$, as in Demidova and Rodríguez-Clare (2009).

5.4 Firm Heterogeneity, Aggregate Nonconvexities, and Trade Policy

Since $\nu > \sigma - 1$, an intriguing implication of the results in Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung and Larch (2013) is that conditional on $\epsilon^* = \sigma$ and x^*_{FF}, the optimal level of trade protection is lower when only a subset of firms select into exports than when they all do, $1/((\nu\mu - 1)x^*_{FF}) < 1/((\sigma - 1)x^*_{FF})$. This specific parametric example, however, is silent about the nature and robustness of the economic forces leading up to this result.

Our general analysis isolates aggregate nonconvexities as the key economic channel through which firm heterogeneity tends to lower the overall level of trade protection. Mathematically, the previous observation is trivial. We know that the upper-level elasticity of substitution between Q_{HF} and Q_{FF} is strictly greater than one, $\epsilon^* - 1 > 0$. Since

[20] In his analysis of models of monopolistic competition with Pareto distributions, Feenstra (2010) concludes that firm heterogeneity leads to strictly convex production sets. In contrast, equation (41) implies that Foreign's production set is non-convex: $\kappa^* < 0$. Both results are mathematically correct. The apparently opposite conclusions merely reflect the fact that we have defined the aggregate production possibility frontier abroad as a function of the CES quantity aggregates, Q_{FH} and Q_{FF}, whereas Feenstra (2010) defines them, using our notation, in terms of Q_{FH}^{1/μ_F} and Q_{FF}^{1/μ_F}.

$\kappa^* \to \infty$ when firms are homogeneous, we arrive at the following corollary of Proposition 2.

Corollary 1. *Conditional on (ϵ^*, x^*_{FF}), optimal uniform tariffs are strictly lower with than without firm heterogeneity if and only if firm heterogeneity creates aggregate nonconvexities, $\kappa^* < 0$.*

Economically speaking, Home's trade restrictions derive from the negative effects of exports and imports on its terms of trade. By reducing the elasticity of Home's terms of trade with respect to its imports, in absolute value, aggregate nonconvexities dampen this effect, and in turn, reduce the optimal level of trade protection.

The final question that remains to be addressed is how likely it is that the selection of heterogeneous firms into exporting will lead to aggregate nonconvexities. It is instructive to consider first a hypothetical situation in which the measure of foreign firms, N_F, is exogenously given. In that situation, the selection of heterogeneous firms would necessarily lead to aggregate nonconvexities. To see this, note that equation (30) implies

$$MRT_F = \frac{(\int_{\Phi_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}{(\int_{\Phi_{FF}} (a_{FF}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}},$$

with the set of foreign varieties sold in market $j = H, F$ such that

$$\Phi_{Fj} = \{\varphi : (\mu_F - 1) a_{Fj}^{1-\sigma_F}(\varphi)(N_F \int_{\Phi_{Fj}} a_{Fj}^{1-\sigma_F}(\varphi) dG_F(\varphi))^{-\mu_F} Q_{Fj} \geq f_{Fj}(\varphi)\}.$$

If selection is active in market j, in the sense that some foreign firms are indifferent between selling and non-selling in market j, then Φ_{Fj} must expand as Q_{Fj} increases. Since consumers love variety, this must lead to a decrease in $(\int_{\Phi_{Fj}} (a_{Fj}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}$.[21] And since labor market clearing requires Q_{FF} to be decreasing in Q_{FH}, this implies that $MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))$ is decreasing in Q_{FH}, i.e. that there are aggregate nonconvexities.

Intuitively, an increase in foreign exports, Q_{FH}, has two effects. First, it expands the set of foreign firms that export, which lowers the unit cost of Foreign's exports. Second, it lowers Q_{FF}, which reduces the set of foreign firms that sell domestically and raises the unit cost of Foreign's domestic consumption. Both effects tend to lower Foreign's opportunity cost of exports in terms of domestic consumption.

Our next result provides sufficient conditions such that the previous selection forces dominate any additional effect that changes in aggregate exports, Q_{FH}, may have on the

[21] Formally, this requires that G_F has strictly positive density around blueprints φ with profitability such that foreign firms are indifferent between selling and not selling in market j. Whenever we say that selection is active in market j, we assume that this is the case.

number of foreign entrants, N_F, and in turn, the monotonicity of MRT_F. Let $N_F(Q_{FH}, Q_{FF})$ denote the measure of foreign firms associated with the solution to (28).

Lemma 7. *If the measure of foreign entrants increases with aggregate output to any market, $\partial N_F(Q_{FH}, Q_{FF})/\partial Q_{Fj} \geqslant 0$ for $j = H, F$, then firm heterogeneity creates aggregate nonconvexities, $\kappa^* \leq 0$, with strict inequality whenever selection is active in at least one market.*

We view the monotonicity condition in Lemma 7 as very mild. The measure of foreign entrants, $N_F(Q_{FH}, Q_{FF})$, is determined by free entry.[22] When a change in aggregate output in any of the two markets changes firms' expected profits, the measure of foreign entrants adjusts to bring them back to the fixed entry costs, f_F^e. In the absence selection effects, an increase in aggregate output in any market raises profits and, in turn, the measure of foreign entrants. In this case, the monotonicity condition in Lemma 7 would necessarily be satisfied. In the presence of selection effects, an increase in aggregate output in market j may actually *decrease* expected profits if the decrease in the price index associated with an expansion of Φ_{Fj} is large enough to offset the direct positive effect of aggregate output, Q_{Fj}, on firms' profits. For the monotonicity condition in Lemma 7 to be violated, there must be large selection effects in one, but only one of the two markets so that expected profits shift in opposite directions in response to changes in Q_{FH} and Q_{FF}. Under these circumstances, N_F^* cannot be increasing in both Q_{FH} and Q_{FF}. For the interested reader, Appendix C.5 constructs one such example in which, in spite of the selection of heterogeneous firms, Foreign's aggregate production set remains locally convex.

Combining Corollary 1 and Lemma 7, we arrive at the following proposition.

Proposition 3. *If the measure of foreign entrants increases with aggregate output to any market, then conditional on (ϵ^*, x_{FF}^*), optimal uniform tariffs are lower with than without firm heterogeneity, with strict inequality whenever selection is active in at least one market.*

The active selection of heterogeneous firms may actually lower the overall level of trade protection so much that the optimal uniform tariff may become an *import subsidy*, an instance of the Lerner paradox. To see this, note that as ϵ^* goes to infinity, the optimal uniform tariff in equation (40) converges towards

$$\bar{t}_{FH}^* = 1/(\kappa^* x_{FF}^*),$$

[22] Since the decentralized equilibrium is efficient, one can always interpret $N_F(Q_{FH}, Q_{FF})$ as the measure of foreign entrants in the decentralized equilibrium, conditional on the equilibrium values of Q_{FH} and Q_{FF}. This is the interpretation we adopt here. Formally, $N_F(Q_{FH}, Q_{FF})$ is given by equation (C.3) in the proof of Lemma 6.

which is strictly negative if there are aggregate nonconvexities abroad, $\kappa^* < 0$. In this limit case, foreign preferences are linear, which eliminates the last neoclassical force calling for an import tariff: diminishing marginal rates of substitution in Foreign. More generally, equation (40) implies that an import subsidy is optimal, $\bar{t}^*_{FH} < 0$, if and only if nonconvexities on the supply-side dominate convexities on the demand-side, $\kappa^* > -\epsilon^*$.[23,24]

The "new" trade theory synthesized by Helpman and Krugman (1985) and Helpman and Krugman (1989) is rich in paradoxical results. For instance, a country with higher demand for a particular good may be a net exporter of that good, the so-called home-market effect. Such paradoxes derive from the presence of increasing returns at the sector-level: when employment in a sector expands, more firms enter, and since consumers love varieties, the associated price index goes down. In a one-sector economy, however, these considerations are mute, which explains why the optimal tariff formula derived by Gros (1987) under monopolistic competition à la Krugman (1980) is the same as in a perfectly competitive Armington model, or why the formula for gains from trade derived by Arkolakis, Costinot and Rodríguez-Clare (2012) is the same for the two models.

Interestingly, the possibility of the Lerner paradox presented above derives from a very different type of nonconvexities, one that is unique to monopolistically competitive models with firm heterogeneity and selection, and one that matters for trade policy, even with only one sector. In a neoclassical environment with diminishing marginal returns, consumers and firms do not internalize the fact that, at the margin, an increase in imports must raise their opportunity costs and, in turn, the price of all infra-marginal units, which calls for a positive import tax. Here, in contrast, a government may lower the price of its imports by raising their volume and inducing more foreign firms to become exporters, which explains why an import subsidy may be optimal.

[23] Allowing for non-CES utility functions such that $\epsilon^* \neq \sigma$ is important for an import subsidy to be optimal. With CES utility functions, the same parameter σ would affect the curvature of both the production possibility frontier and the indifference curve abroad. In the Pareto case, for instance, one can check that the restriction $\nu > \sigma - 1$ further implies $\kappa^* < -\sigma$. So, in the environments considered by Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung and Larch (2013), an import subsidy would never be optimal.

[24] A careful reader may wonder whether the inequality $\kappa^* > -\epsilon^*$ conflicts with our previous decision to restrict our analysis to interior solutions of Home's relaxed planning problem that satisfy equation (16). The answer is no. Provided that the upper-level elasticity of substitution between domestic and foreign goods *at home* is low enough, Home's planner must prefer an interior allocation, regardless of whether $\kappa^* > -\epsilon^*$ or $\kappa^* \leq -\epsilon^*$ abroad.

6 Optimal Taxes with Intra- and Inter-Industry Trade

The monopolistically competitive model of Section 2 is commonly interpreted as a model of intra-industry trade where domestic and foreign firms specialize in differentiated varieties of the same product. We now consider a more general environment with both intra- and inter-industry trade across multiple sectors indexed by $k = 1, ..., K$. Formally, the utility function of the representative agent in each country is given by

$$U_i = U_i(U_i^1, .., U_i^K),$$
$$U_i^k = U_i^k(Q_{Hi}^k, Q_{Fi}^k),$$
$$Q_{ji}^k = [\int_\Phi N_j^k (q_{ji}^k(\varphi))^{1/\mu_j^k} dG_j^k(\varphi)]^{\mu_j^k},$$

with U_i^k the utility from consuming all varieties from sector k in country i, Q_{ji}^k the subutility associated with varieties from country j in that sector, and $\mu_j^k \equiv \sigma_j^k/(\sigma_j^k - 1)$, with $\sigma_j^k > 1$ the elasticity of substitution between varieties from country j in sector k. The model of Section 2 corresponds to the special case in which $K = 1$ and $U_i = U_i^1$. In line with our previous analysis, we assume that $U_i^k(\cdot, \cdot)$ is homogeneous of degree one for all i and k. Without loss of generality, Assumptions on technology and market structure are unchanged.

6.1 More Micro Problems and a More Complex Macro Problem

The first goal of this section is to show that the micro-to-macro approach that we have followed in previous sections readily extends to an economy with multiple industries. Let us start with the micro problems of Sections 3.1 and 3.2. Within each sector $k = 1, ..., K$, one can still define the minimum labor cost at home of producing domestic output, Q_{HH}^k, and exports, Q_{HF}^k,

$$L_H^k(Q_{HH}^k, Q_{HF}^k) \equiv \min_{\mathbf{q_{HH}^k}, \mathbf{q_{HF}^k}, N_H^k} N_H^k [\sum_{j=H,F} \int_\Phi l_{Hj}(q_{Hj}^k(\varphi), \varphi) dG_H^k(\varphi) + f_H^{e,k}]$$
$$N_H^k \int_\Phi (q_{Hj}^k(\varphi))^{1/\mu_H^k} dG_H^k(\varphi) \geq (Q_{Hj}^k)^{1/\mu_H^k}, \text{ for } j = H, F,$$

as well as the maximum amount of imports, conditional on aggregate exports, Q_{HF}^k, as well as Foreign's sectoral expenditure, E_F^k, and employment, L_F^k,

$$
\begin{aligned}
(Q_{FH}^k)^{1/\mu_F^k}(Q_{HF}^k, E_F^k, L_F^k) \equiv \max_{\mathbf{q_{FH}^k}, Q_{FF}^k, N_F^k} & \int_\Phi N_F^k q_{FH}^k{}^{1/\mu_F^k}(\varphi) dG_F^k(\varphi)) \\
& E_F^k = P_{FF}^k(Q_{FF}^k, N_F^k)(Q_{FF}^k + MRS_F^k(Q_{HF}^k, Q_{FF}^k)Q_{HF}^k) \\
& N_F^k f_F^{e,k} = \Pi_{FF}^k(Q_{FF}^k, N_F^k) \\
& + N_F^k \int [\mu_F^k a_{FH}(\varphi) q_{FH}^k(\varphi) - l_{FH}(q_{FH}^k(\varphi), \varphi)] dG_F^k(\varphi), \\
& L_F^k = N_F^k f_F^{e,k} + L_{FF}^k(Q_{FF}^k, N_F^k) + N_F^k \int_\Phi l_{FH}(q_{FH}^k(\varphi), \varphi) dG_F^k(\varphi), \\
& \mu_F^k a_{FH}(\varphi) q_{FH}^k(\varphi) \geq l_{FH}(q_{FH}^k(\varphi), \varphi).
\end{aligned}
$$

Using the same arguments as in Sections 4.1 and 4.2, one can then show that our qualitative results about the optimal structure of micro-level taxes are unchanged: domestic taxes should be uniform across firms within the same sector, whereas import taxes should be lower on the least profitable exporters from Foreign.

At the macro level, Home's production possibility frontier is now given by

$$\sum_k L_H^k(Q_{HH}^k, Q_{HF}^k) = L_H.$$

Compared to the one-sector case, the key difference is that Foreign's offer curve now also reflects the fact that Home's planner can choose the level of foreign expenditure and employment, E_F^k and L_F^k, in each sector k subject to the foreign resource constraint, as well as the optimality of foreign consumption across sectors. We describe the associated constraints in Appendix D.1.[25]

Not surprisingly, like in a neoclassical environment with arbitrarily many sectors, see e.g. Bond (1990), there is little that can be said, in general, about the optimal structure of macro-level taxes. To provide further insights into the forces that shape terms-of-trade manipulation under monopolistic competition, both within and between sectors, we turn to a simple example that has received particular attention in the previous literature.

[25]Throughout our analysis, we have restricted ourselves to a world economy with only two countries. We can deal with multi-country environments in the same way as we have dealt with multi-sector environments. Indeed, one can always reinterpret varieties from different countries as varieties from different sectors. At the micro-level, our qualitative results would still hold country-by-country. At the macro-level, the key difference, relative to the multi-sector case, is that optimal taxes would now also reflect the incentives to manipulate relative wages (since labor is immobile across countries).

6.2 A Simple Example with Homogeneous and Differentiated Goods

Suppose that there are two sectors, a homogeneous outside sector ($k = O$) and a differentiated sector ($k = D$), and that Foreign consumers have Cobb-Douglas preferences, as in the model with homogeneous firms of Venables (1987), Ossa (2011), and Campolmi, Fadinger and Forlati (2014), and the model with heterogeneous firms of Haaland and Venables (2014). To facilitate the connection between previous results in the literature and ours, we also restrict all taxes to be uniform within the same sector, as in Section 5. This is equivalent to adding the sector-level counterpart of constraint (26) to the sector-level micro problems in Section 6.1. We let $\bar{t}^D_{HH}$, $\bar{t}^D_{FH}$, $\bar{s}^D_{HH}$, and $\bar{s}^D_{HF}$ denote the uniform ad-valorem taxes in the differentiated sector and $\bar{t}^O_H$ denote the ad-valorem trade tax-cum-subsidy in the homogeneous sector.[26]

In the outside sector, we assume that $\sigma^O_H, \sigma^O_F \to \infty$, that there are no fixed costs of production and no trade costs, and that all firms at home and abroad have the same productivity, which we normalize to one. So, one can think of the homogeneous good as being produced by perfectly competitive firms in both countries. In the rest of this section, we use the outside good as our numeraire. As in the previous sections, we impose no restriction on the distributions of firm-level productivity and fixed costs in the differentiated sector, G^D_H and G^D_F, nor on the sector-level aggregator, U^D_H and U^D_F, which determines the substitutability between domestic and foreign varieties in both countries. Finally, we let β_F denote the share of expenditure on differentiated goods in Foreign. Given our Cobb-Douglas assumption, this share is constant.

In this environment, Foreign's offer curve, $Q^D_{FH}(Q^D_{HF}, X^O_H)$, depends both on the exports of the differentiated good, Q^D_{HF}, and of the outside good, $X^O_H \equiv Q^O_{HF} - Q^O_{FH}$. It is defined implicitly by

$$\tilde{P}^D_{FH}(X^O_H, Q^D_{FH})Q^D_{FH} = P^D_{HF}(X^O_H, Q^D_{FH}, Q^D_{HF})Q^D_{HF} + X^O_H,$$

where Home's import and export prices in the differentiated sector, $\tilde{P}^D_{FH}(X^O_H, Q^D_{FH})$ and $P^D_{HF}(X^O_H, Q^D_{HF}, Q^D_{FH})$, are such that

$$\tilde{P}^D_{FH}(X^O_H, Q^D_{FH}) = \mu^D_F L^D_{FH}(Q^D_{FH}, Q^D_{FF}(Q^D_{FH}, L^D_F(X^O_H))), \quad (42)$$

$$P^D_{HF}(X^O_H, Q^D_{HF}, Q^D_{FH}) = \mu^D_F L^D_{FF}(Q^D_{FH}, Q^D_{FF}(Q^D_{FH}, L^D_F(X^O_H)))MRS^D_F(Q^D_{HF}, Q^D_{FF}(Q^D_{FH}, L^D_F(X^O_H))), \quad (43)$$

[26] For notational convenience, we focus throughout this section on the structure of optimal trade taxes under the normalization that domestic taxes are zero in the homogeneous sector, though they may be positive or negative in the differentiated sector. As we will see, the difference in markups between the differentiated and homogeneous sectors implies that $\bar{t}^D_{HH}$ and $\bar{s}^D_{HH}$ will no longer be zero at an optimum.

with $L^D_{Fi} \equiv \partial L^D_F / \partial Q_{Fi}$ the marginal cost in Foreign of aggregate output for market $i = H, F$ and $MRS^D_F \equiv (\partial U^D_F / \partial Q^D_{HF}) / (\partial U^D_F / \partial Q^D_{FF})$ Foreign's marginal rate of substitution in the differentiated sector. Given Foreign's new offer curve, Home's macro planning problem then generalizes to

$$\max_{Q^O_H, X^O_H Q^D_{HH}, Q^D_{FH}, Q^D_{HF}} U_H(Q^O_H - X^O_H, U^D_H(Q^D_{HH}, Q^D_{FH}))$$
$$Q^D_{FH} \leq Q^D_{FH}(Q^D_{HF}, X^O_H),$$
$$Q^O_H + L^D_H(Q^D_{HH}, Q^D_{HF}) = L_H.$$

Note that exports of the outside good, X^O_H, affects Foreign's offer curve both directly through the trade balance condition and indirectly through the impact on the amount of labor left over for the differentiated sector. Foreign production of the differentiated good for its local market, $Q^D_{FF}(Q^D_{FH}, L^D_F(X^O_H)))$, not only depends on its exports of the differentiated good, Q^D_{FH}, but also on the total amount of labor allocated to the differentiated sector, $L^D_F(X^O_H)$, which now appears as a second argument. Given Cobb-Douglas preferences, this only depends on Home's net imports of the outside good. Since Foreign always spends $(1 - \beta_F)L_F$ on the outside good, the amount of labor allocated to that sector must be equal to $(1 - \beta_F)L_F - X^O_H$ and the amount allocated to the differentiated sector must be equal to L_F minus this number, $L^D_F(X^O_H) = \beta_F L_F + X^O_H$.

In spite of the introduction of an outside sector, the relative price of Home's exports in the differentiated sector, $P^D \equiv P^D_{HF} / \tilde{P}^D_{FH}$, still satisfies $P^D = MRS^D_F / MRT^D_F$, where $MRT^D_F \equiv L^D_{FH} / L^D_{FF}$ is the marginal trade of transformation in Foreign's differentiated sector. However, since there are now three aggregate goods that are traded internationally—Home's and Foreign's differentiated goods as well as the homogeneous good—there are two relative prices, P^D and $\tilde{P}^D_{FH}$, that Home can manipulate to improve its terms-of-trade both within and between sectors. Mathematically, these considerations are captured by the first-order conditions of Home's new macro problem, which imply

$$MRT^D_H P^D / MRS^D_H = (1 - \Delta) / \eta^D,$$
$$MRS^{FO}_H / \tilde{P}^D_{FH} = \Delta / \eta^O,$$

where MRT^D_H and MRS^D_H are Home's marginal rate of transformation and substitution in the differentiated sector, $MRS^{FO}_H \equiv (\partial U_H / \partial Q^D_{FH}) / (\partial U_H / \partial U^O_H)$ is Home's marginal rate of substitution between Foreign's differentiated good and the homogeneous good, $\eta^D \equiv d \ln Q^D_{FH}(Q^D_{HF}, X^O_H) / d \ln Q^D_{HF}$ and $\eta^O \equiv d \ln Q^D_{FH}(Q^D_{HF}, X^O_H) / d \ln X^O_H$ are Foreign's offer curve elasticities, and $\Delta \equiv \left(\tilde{P}^D_{FH} Q^D_{FH} - P^D_{HF} Q^D_{HF}\right) / \tilde{P}^D_{FH} Q^D_{FH}$ is a normalized measure of inter-industry trade, all evaluated at the optimum. Using the same argument as in Section 4, one can then show that

$$\frac{(1+\bar{t}^D_{FH})/(1+\bar{t}^D_{HH})}{(1+\bar{s}^D_{HF})/(1+\bar{s}^D_{HH})} = (1-\Delta)/\eta^D, \tag{44}$$

$$(1+\bar{t}^D_{FH})/(1+\bar{t}^O_{H}) = \Delta/\eta^O. \tag{45}$$

Finally, as shown in Appendix D.2, given the difference in markups between the differentiated and homogeneous sectors, the domestic government would like to use domestic taxes in order to undo the markup distortion,

$$(1+\bar{t}^D_{HH})/(1+\bar{s}^D_{HH}) = 1/\mu^D_H.$$

As in Section 5, we can link Foreign's offer curve elasticities to the terms-of-trade elasticities,

$$\eta^D = \frac{\left(1+\rho^D_{HF}\right)(\Delta-1)}{\rho^D_{HF}+(1-\Delta)\,\rho^D_{FH}-\Delta\zeta_{FH}}, \tag{46}$$

$$\eta^O = \frac{\Delta+(1-\Delta)\,\rho^D_X-\Delta\zeta_X}{1+(\Delta-1)\,\rho^D_{FH}+\Delta\zeta_{FH}}, \tag{47}$$

with $\rho^D_{HF} \equiv \partial\ln P^D/\partial\ln Q^D_{HF}$, $\rho^D_{FH} \equiv \partial\ln P^D/\partial\ln Q^D_{FH}$, $\rho^D_X \equiv \partial\ln P^D/\partial\ln X^O_H$, $\zeta_{FH} \equiv \partial\ln\tilde{P}^D_{FH}/\partial\ln Q^D_{FH}$, $\zeta_X \equiv \partial\ln\tilde{P}^D_{FH}/\partial\ln X^O_H$. The first group of price elasticities, ρ^D_{HF}, ρ^D_{FH}, and ρ^D_X, determine Home's incentives to manipulate terms of trade within the differentiated sector, whereas the second group of elasticities, ζ_{FH} and ζ_X, determine its incentives to manipulate terms of trade between the differentiated sector and the homogeneous sector.[27] In the absence of inter-industry trade, $\Delta = 0$, only the first group of elasticities affects the elasticity of Foreign's offer-curve and $\eta^D = \frac{1+\rho^D_{HF}}{1-\rho^D_{FH}}$, as in Section 5.

When there is no active selection of firms in the differentiated sector, as in the model with homogeneous firms of Venables (1987), Ossa (2011), and Campolmi, Fadinger and Forlati (2014), we can use equations (42) and (43) to express Home's terms-of-trade elasticities as a function of the elasticity of substitution within the differentiated sector in Foreign, ϵ^D, and the domestic expenditure and revenue shares, $x^D_{FF} \equiv P^D_{FF}Q^D_{FF}/(P^D_{FF}Q^D_{FF} + P^D_{HF}Q^D_{HF})$ and $r^D_{FF} \equiv P^D_{FF}Q^D_{FF}/(P^D_{FF}Q^D_{FF} + \tilde{P}^D_{FH}Q^D_{FH})$, as described in Appendix D.3. Com-

[27]The definitions of ρ^D_X and ζ_X implicitly assume that Home is an exporter of the homogeneous good, $X^O_H > 0$. If Home is an importer of the homogeneous good, one can simply rewrite all our formulas in terms of $\partial\ln P^D/\partial\ln(-X^O_H)$ and $\partial\ln P^D_{FH}/\partial\ln(-X^O_H)$. None of our results depends on this convention.

bining these expressions with equations (44)-(47), we get that

$$\frac{(1+\bar{t}^D_{FH})/(1+\bar{t}^D_{HH})}{(1+\bar{s}^D_{HF})/(1+\bar{s}^D_{HH})} = 1 + \frac{1}{(\epsilon^D - 1)x^D_{FF}}, \tag{48}$$

$$(1+\bar{t}^D_{FH})/(1+\bar{t}^O_H) = 1 - \frac{(1-r^D_{FF})((1-\Delta)/r^D_{FF} + \Delta\epsilon^D)}{\epsilon^D(\sigma^D_F - 1) + (1-r^D_{FF})(\sigma^D_F (1-\Delta)/r^D_{FF} + \Delta\epsilon^D)}. \tag{49}$$

From equation (48), we see that Gros's (1987) formula, which determines the optimal level of trade protection within the differentiated sector remains unchanged. Although the domestic government now wants to manipulate its terms-of-trade both within and between sectors, the latter consideration only affects the choice of $(1+\bar{t}^D_{FH})/(1+\bar{t}^O_H)$.

According to equation (49), if Home is a net exporter of the homogeneous good—and so a net importer of the differentiated good, $\Delta > 0$)—then optimal taxes must be such that $(1+\bar{t}^D_{FH})/(1+\bar{t}^O_H) < 1$. This can be achieved, for example, by subsidizing imports of the differentiated good, $\bar{t}^D_{FH} < 0$ with $\bar{t}^O_H = 0$, or by subsidizing exports of the homogeneous good, $\bar{t}^D_{FH} = 0$ with $\bar{t}^O_H > 0$. Intuitively, an increase in Home's exports of the homogeneous good creates a home-market effect: it increases Foreign's employment in the differentiated sector, $\beta_F L_F + X^O_H$, which leads to more entry of foreign firms in this sector and, because of love of variety, a lower price of Foreign's differentiated good relative to the homogeneous good. When Home is an exporter of the homogeneous good, this creates a first improvement in its terms of trade. In addition, an increase in either imports of the differentiated good or exports of the homogeneous good raises foreign production of the differentiated good for its local market. Since $P^D \propto P^D_{HF}/P^D_{FF} = MRS^D_F$ in the absence of selection, this must be accompanied by a decrease in the relative price of Foreign's differentiated goods relative to Home's differentiated goods, a second improvement in Home's terms of trade.[28] When Home is a small open economy in the sense that $r^D_{FF} = 1$, it cannot manipulate entry or output abroad, which leads to zero subsidies: $(1+\bar{t}^D_{FH})/(1+\bar{t}^O_H) = 1$. The same is true when σ^D_F goes to infinity. In this case, the relative price of Foreign's differentiated goods relative to the homogeneous good is fixed. Hence, Home can only manipulate P^D, which it will do optimally by setting an import tariff or an export tax in the differentiated sector according to equation (48).

When there is active selection, equations (46)-(45) offer a strict generalization of the results of Haaland and Venables (2014). In line with the papers cited in Section 5.3,

[28] If Home is an importer of the homogeneous good, $z > 1$, then Home's terms of trade unambiguously improve if both P^D and P^D_{HF} increase. Although a decrease in Home's imports of the homogeneous good imports of differentiated goods necessarily increases P^D and lowers P^D_{FH}, it only increases P^D_{HF} if Foreign's elasticity of substitution between domestic and foreign goods, ϵ^D, is low enough. Accordingly, Home only taxes imports of the homogeneous good in this case if $\epsilon^D < z/(r^D_{FF}(z-1))$.

they assume a constant elasticity of substitution between domestic and foreign goods, $\epsilon^D = \sigma_H^D = \sigma_F^D \equiv \sigma^D$, that firms only differ in terms of their productivity, and that the distribution of firm-level productivity is Pareto. Crucially, they also assume that Home is small relative to Foreign in the sense that it cannot affect the number of foreign entrants, N_F^D, nor local output, Q_{FF}^D, in the differentiated sector. Under this restriction, and regardless of whether firm-level productivity is distributed Pareto, Appendix D.4 establishes that

$$\frac{(1+\bar{t}_{FH}^D)/(1+\bar{t}_{HH}^D)}{(1+\bar{s}_{HF}^D)/(1+\bar{s}_{HH}^D)} = 1 + \frac{1+\epsilon^D/\kappa^D}{\epsilon^D - 1}, \tag{50}$$

$$(1+\bar{t}_{FH}^D)/(1+\bar{t}_H^O) = 1 + 1/\kappa^D, \tag{51}$$

where κ^D is the elasticity of transformation between between exports and domestic goods in Foreign's differentiated sector. By equation (50), the structure of optimal trade protection within the differentiated sector is again exactly the same as in the one-sector case, with firm heterogeneity lowering trade protection if and only if there is active selection of foreign firms into exporting.[29] Furthermore, by equation (51), the same aggregate nonconvexities, $\kappa^D < 0$, should lead to less trade protection in the differentiated sector relative to the homogeneous sector: $(1+\bar{t}_{FH}^D)/(1+\bar{t}_H^O) < 1$. This reflects the fact that given aggregate nonconvexities, the import price in the differentiated sector, $\tilde{P}_{FH}^D$, is a *decreasing* function of import volumes, Q_{FH}^D. This can again be achieved by subsidizing imports of the differentiated good, $\bar{t}_{FH}^D < 0$ with $\bar{t}_H^O = 0$, or by subsidizing exports of the homogeneous good, $\bar{t}_{FH}^D = 0$ with $\bar{t}_H^O > 0$, an expression of Lerner symmetry.

6.3 Terms-of-Trade Manipulation and Optimal Trade Policy Redux

The existing literature on optimal trade policy under monopolistic competition draws a sharp distinction between models with only intra-industry trade, like the one studied by Gros (1987), and models with both intra- and inter-industry, like the one studied by Venables (1987). In the former class of models, the standard view, as put forward by Helpman and Krugman (1989), is that terms-of-trade manipulation can be thought of as the rationale behind optimal trade policy since a strategic country can affect its relative wage. In the latter class of models, however, the standard view would be that such terms-

[29] All formulas in this section are implicitly derived under the assumption that Home and Foreign produce in both sectors. A small open economy, however, is likely to be completely specialized in only one of them. When Home is completely specialized in the differentiated sector, one can show that both equations (50) and (51) must still hold. When Home is completely specialized in the outside sector, equation (51) must again hold, but equation (50), while consistent with an optimum, is no longer necessary. Details are available upon request.

of-trade motives are absent whenever the existence of an outside good pins down relative wages between countries, and accordingly, that the rationale behind trade policy must lie somewhere else, like the existence of so-called home-market effects.

Our analysis offers a different perspective, one suggesting that the terms-of-trade motive has greater scope than previously recognized. According to this view, imperfect competition and firm heterogeneity matter for the design of macro-level trade taxes, but only to the extent that they affect terms-of-trade elasticities. In the simple example of Section 6.2, Home's relative wage is fixed, whereas the number of foreign entrants in the differentiated sector is free to vary. Yet, if all elasticities of world prices are zero, that is if Home has no market power, then optimal wedges and optimal trade taxes are zero, as can be seen from equations (46) and (47). Our analysis echoes the results of Bagwell and Staiger (2012b,a, 2015) who argue that terms-of-trade externalities remain the sole motive for international trade agreements under various market structures.

The importance of the terms-of-trade motive in our analysis clearly depends on the availability of a full set of domestic instruments. In the presence of domestic distortions, trade policy can also be used as a second-best instrument, which means that if one were to restrict the set of domestic taxes, such considerations would also affect the level optimal trade taxes, as in Flam and Helpman (1987). This is true regardless of whether markets are perfectly or monopolistically competitive and we have little to add to this observation.

The core of the difference between the standard view and ours has a simpler origin. We define terms-of-trade manipulation at the macro-level as the manipulation of the relative price of sector-level aggregate prices, not the manipulation of relative wages. In the one-sector case studied by Gros (1987), the two definitions coincide, but not otherwise. While one may view the previous distinction as semantic, this does not mean that it is either irrelevant or trivial. Part of the reason why one builds theory is to develop a common language that can be applied under seemingly different circumstances. The perspective pushed forward in this paper is that within the class of models that we consider, international trade remains another transformation activity that turns aggregate exports into aggregate imports, as summarized by Foreign's offer curve, the shape of which determines the structure of optimal trade policy at the macro-level.

7 Concluding Remarks

Few economic mechanisms have received as much empirical support as the selection of heterogeneous firms into exporting; see e.g. Bernard and Jensen (1999), Bernard, Eaton, Jensen and Kortum (2003), Bernard, Jensen, Redding and Schott (2007), and Eaton, Kortum and Kramarz (2011). Policy makers have paid attention. As documented in the World Trade Report 2016, there were only two regional trade agreements (RTA) with provisions related to small- and medium-sized enterprises (SME) prior to 1990. As of March 2016, 133 RTAs, representing 49% of all the notified RTAs, include at least one provision mentioning explicitly SMEs. The Trans-Pacific Partnership (TPP) is now the first U.S. free trade agreement to include a separate chapter on SMEs.[30]

Ironically, there has been very little work to date about the policy implications of the endogenous selection of firms into exporting. In this paper, we have tackled this issue in the context of a generalized version of the trade model with monopolistic competition and firm-level heterogeneity developed by Melitz (2003). We have organized our analysis around two polar assumptions about the set of available policy instruments. In our baseline environment, ad-valorem taxes are unrestricted so that governments are free to impose different taxes on different firms. In our extensions, ad-valorem taxes are uniform so that governments cannot discriminate between firms from the same country.

When ad-valorem taxes are unrestricted, we have shown that optimal trade policy requires micro-level policies. Specifically, a welfare-maximizing government should impose firm-level import taxes that discriminate against the most profitable foreign exporters. In contrast, export taxes that discriminate against or in favor of the most profitable domestic exporters can be dispensed with. When taxes are uniform, we have shown that the selection of heterogeneous firms into exporting tends to create aggregate nonconvexities that lowers the overall level of trade protection. Under both assumptions, we have highlighted the central role that terms-of-trade manipulation plays in determining the structure of optimal trade taxes at the macro-level, thereby offering a unifying perspective on previous results about trade policy under monopolistic competition.

We conclude by pointing out a number of limitations of the present analysis that could be relaxed in future research. The first one is the assumption that all firms charge a constant markup. In general, a government that manipulates its terms-of-trade may do so by imposing different taxes on different firms in order to incentivize them to charge different markups. In practice, we know that firms of different sizes tend to have different markups and different pass-through rates; see e.g. Berman, Martin and Mayer (2012), Goldberg, Loecker, Khandelwal and Pavcnik (2015), and Amiti, Itskhoki and Konings (2015). While this channel is not directly related to the selection of heterogeneous firms into exporting, this is another potentially important mechanism through which firm heterogeneity may affect the design of optimal trade policy.

[30]Details of the chapter on SMEs can be found at https://medium.com/the-trans-pacific-partnership/small-and-medium-sized-businesses-8de15a02d843#.c68wvlrwc

The second limitation is that fixed exporting costs are assumed to be paid in the exporting country. This implies that all trade is trade in goods. If fixed costs were paid in the importing country, trade would also include trade in services, and manipulating the prices of such services would also be part of the objective of a welfare-maximizing government. More generally, our analysis abstracts from intermediate goods and global supply chains, which is another exciting area for future research on optimal trade policy; see Blanchard, Bown and Johnson (2015) for a first step in this direction.

The third limitation is that governments have access to a full set of tax instruments. As discussed in the previous section, when domestic instruments are restricted, trade policy would be called for not only to improve a country's terms of trade, but also to help in mitigating domestic distortions. We know little about the implications of trade models with firm heterogeneity for the design of optimal industrial policy. They may be particularly relevant in economies where credit markets are imperfect.

The final limitation is that we have only characterized the optimal policy of a country when the rest of the world consists of a single country that imposes no taxes of any sort. To further understand the implications of firm heterogeneity and selection for trade policy, future research should strive to characterize the Nash equilibrium in which all countries attempt to manipulate their terms of trade, and then study how trade agreements would be structured to avoid the associated negative welfare implications. Much remains to be done on the normative side of the literature to close the gap between theory and practice.

References

Amiti, Mary, Oleg Itskhoki, and Jozef Konings, "International Shocks and Domestic Prices: How Large Are Strategic Complementarities?," *mimeo Princeton University*, 2015.

Anderson, James E. and Eric Van Wincoop, "Gravity with Gravitas: A Solution to the Border Puzzle," *The American Economic Review*, 2003, *93* (1), 170–192.

Arkolakis, Costas, Arnaud Costinot, and Andres Rodríguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, 2012, *102* (1), 94–130.

Bagwell, Kyle and Robert W. Staiger, "The economics of trade agreements in the linear Cournot delocation model," *Journal of International Economics*, 2012, *88*, 32–46.

_ **and** _ , "Profit shifting and trade agreements in imperfectly competitive markets," *International Economic Review*, 2012, *53* (4), 1067–1104.

_ **and** _ , "Delocation and trade agreements in imperfectly competitive markets," *Research in Economics*, 2015, *69*, 132–156.

_ **and Seung Lee**, "Trade policy under monopolistic competition with firm selection," *mimeo Stanford*, 2015.

Baldwin, Robert E., "Equilibrium in International Trade: A Diagrammatic Approach," *Quarterly Journal of Economics*, 1948, *62* (5), 748–762.

Berman, Nicolas, Philippe Martin, and Thierry Mayer, "How do different exporters react to exchange rate changes?," *Quarterly Journal of Economics*, 2012, *127* (1), 437–492.

Bernard, Andew B., Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum, "Plants and Productivity in International Trade," *American Economic Review*, 2003, *93* (4), 1268–1290.

Bernard, Andrew B. and J. Bradford Jensen, "Exceptional Exporter Performance: Cause, Effect, or Both," *Journal of International Economics*, 1999, *47*, 1–25.

Bernard, Andrew, Bradford Jensen, Steve Redding, and Peter J. Schott, "FIrms in International Trade," *Journal of Economic Perspectives*, 2007, *21* (3), 105–130.

Bhagwati, Jagdish N., *The Generalized Theory of Distortions and Welfare*, Vol. Trade, Balance of Payments, and Growth: Papers in International Economics in Honor of Charles P. Kindleberger, Amsterdam: North-Holland, 1971.

Blanchard, Emily J., Chad P. Bown, and Robert C. Johnson, "Global Supply Chains and Trade Policy," *mimeo Dartmouth*, 2015.

Bond, Eric W, "The Optimal Tariff Structure in Higher Dimensions," *International Economic Review*, February 1990, *31* (1), 103–16.

Campolmi, Alessia, Harald Fadinger, and Chiara Forlati, "Trade Policy: Home Market Effect versus Terms-of-Trade Externality," *Journal of International Economics*, 2014, *93* (1), 92–107.

Costinot, Arnaud, Dave Donaldson, Jonathan Vogel, and Ivan Werning, "Comparative Advantage and Optimal Trade Policy," *Quarterly Journal of Economics*, 2015, *130* (2), 659–702.

_ , Guido Lorenzoni, and Ivan Werning, "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *Journal of Political Economy*, 2014, *122* (1), 77–128.

Demidova, Svetlana, "Trade Policies, Firm Heterogeneity, and Variable Markups," *mimeo McMaster University*, 2015.

_ and Andres Rodríguez-Clare, "Trade policy under firm-level heterogeneity in a small economy," *Journal of International Economics*, 2009, *78* (1), 100–112.

Dhingra, Swati and John Morrow, "The Impact of Integration on Productivity and Welfare Distortions Under Monopolistic Competition," *mimeo, LSE*, 2012.

Dixit, Avinash, "Tax policy in open economies," in A. J. Auerbach and M. Feldstein, eds., *Handbook of Public Economics*, Vol. 1 of *Handbook of Public Economics*, Elsevier, 1985, chapter 6, pp. 313–374.

Dixit, Avinash K. and Joseph E. Stiglitz, "Monopolistic Competition and Optimum Product Diversity," *The American Economic Review*, 1977, *67* (3), 297–308.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

_ , _ , and Francis Kramarz, "An Anatomy of International Trade: Evidence from French Firms," *Econometrica*, 2011, *79* (5), 1453–1498.

Everett, Hugh, "Generalized Lagrange Multiplier Methods for Solving Problems of Optimal Allocation of Resources," *Operations Research*, 1963, *11* (3), 399–417.

Feenstra, Robert C., "Measuring the Gains from Trade under Monopolistic Competition," *Canadian Journal of Economics*, 2010, pp. 1–28.

Felbermayr, Gabriel J., Benjamin Jung, and Mario Larch, "Optimal Tariffs, Retaliation and the Welfare Loss from Tariff Wars in the Melitz Model," *Journal of International Economics*, 2013, *89* (1), 13–25.

Flam, Harry and Elhanan Helpman, "Industrial Policy under Monopolistic Competition," *Journal of International Economics*, 1987, *22* (79-102).

Goldberg, Pinelopi, Jan De Loecker, Amit Khandelwal, and Nina Pavcnik, "Prices, Markups and Trade Reform," *forthcoming, Econometrica*, 2015.

Gopinath, Gita, Elhanan Helpman, and Kenneth Rogoff, eds, *Handbook of International Economics*, Vol. 4, Elsevier, 2014.

Gros, Daniel, "A Note on the Optimal Tariff, Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade," *Journal of Intermational Economics*, 1987, *23*, 357–367.

Haaland, Jan I. and Anthony J. Venables, "Optimal Trade Policy with Monopolistic Competition and Heterogeneous Firms," *mimeo University of Oxford*, 2014.

Helpman, Elhanan and Paul Krugman, *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy*, Cambridge, Massachusetts: MIT Press, 1985.

_ **and Paul R. Krugman**, *Trade Policy and Market Structure*, Cambridge, Massachussetts: MIT Press, 1989.

Kim, In Song, "Political Cleavages within Industry: Firm-level Lobbying for Trade Liberalization," *mimeo MIT*, 2016.

Krugman, Paul, "Scale Economies, Product Differentiation, and the Pattern of Trade," *The American Economic Review*, 1980, *70* (5), 950–959.

Luenberger, David G., *Optimization by Vector Space Methods*, John Wiley and Sons, Inc, 1969.

Maggi, Giovanni, "International Trade Agreements," 2014, *4.*

Matsuyama, Kiminori, "Making Monopolistic Competition More Useful," *The Hoover Institution Working Papers*, 1992.

Melitz, Marc and Stephen Redding, *Heterogeneous Firms and Trade*, Vol. 4 of *Handbook of International Economics*, Elsevier, 2014.

_ **and** _ , "New Trade models, New Welfare Implications," *American Economic Review*, 2015, *105* (3), 1105–1146.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

_ **and Gianmarco I. P. Ottaviano**, "Market Size, Trade, and Productivity," *The Review of Economic Studies*, 2008, *75* (1), 295–316.

Ossa, Ralph, "A "New Trade" Theory of GATT/WTO Negotiations," *Journal of Political Economy*, 2011, *119* (1), 112–152.

Ruhl, Kim, "The aggregate impact of antidumping policies," *mimeo NYU*, 2014.

Venables, Anthony J., "Trade and trade policy with differentiated products: A Chamberlinian-Ricardian model," *Economic Journal*, 1987, *97* (700-717).

A Proofs of Section 3

In Sections 3.1-3.3, we have described the solution to Home's relaxed planning problem:

$$\max_{\{\mathbf{q_{ij}},Q_{ij}\}_{i,j=H,F},\mathbf{p_{FF}},\mathbf{p_{FH}},P_{FF},P_{HF},\{N_i\}_{i=H,F}} U_H(Q_{HH},Q_{FH}) \tag{A.1a}$$

subject to:

$$Q_{ij}^{1/\mu_i} \leq \int_\Phi N_i(q_{ij}(\varphi))^{1/\mu_i} dG_i(\varphi), \text{ for } i = H \text{ or } j = H, \tag{A.1b}$$

$$q_{FF}(\varphi) = \begin{cases} \bar{q}_{FF}(\varphi) & \text{if } \mu_F a_{FF}(\varphi)\bar{q}_{FF}(\varphi) \geq l_{FF}(\bar{q}_{FF}(\varphi),\varphi), \\ 0 & \text{otherwise,} \end{cases} \tag{A.1c}$$

$$P_{FF}^{1-\sigma_j} = \int_\Phi N_F[p_{FF}(\varphi)]^{1-\sigma_i} dG_F(\varphi), \tag{A.1d}$$

$$p_{Fj}(\varphi) = \begin{cases} \bar{p}_{Fj}(\varphi) & \text{if } \mu_F a_{Fj}(\varphi)q_{Fj}(\varphi) \geq l_{Fj}(q_{Fj}(\varphi),\varphi), \\ \infty & \text{otherwise,} \end{cases} \text{ for } j = H, F, \tag{A.1e}$$

$$f_F^e = \sum\nolimits_{j=H,F} \int_\Phi [\mu_F a_{Fj}(\varphi) q_{Fj}(\varphi) - l_{Fj}(q_{Fj}(\varphi),\varphi)] dG_F(\varphi), \tag{A.1f}$$

$$Q_{HF}, Q_{FF} \in \arg\max_{\tilde{Q}_{HF},\tilde{Q}_{FF}} \{U_F(\tilde{Q}_{HF},\tilde{Q}_{FF}) | \sum\nolimits_{i=H,F} P_{iF}\tilde{Q}_{iF} = w_F L_F\}, \tag{A.1g}$$

$$L_i = N_i[\sum\nolimits_{j=H,F} \int_\Phi l_{ij}(q_{ij}(\varphi),\varphi) dG_i(\varphi) + f_i^e], \text{ for } i = H, F. \tag{A.1h}$$

We now provide the formal arguments used to characterize this solution.

A.1 Home's Production Possibility Frontier (Section 3.1)

This appendix discusses some technical details behind our analysis and characterization of optimal policy. Our approach in Section 3.1 was to derive necessary conditions for optimality, appealing to global Lagrangian necessity theorems. This appendix clarifies how we can invoke such results, despite the apparent non convexity of the problems.

It is convenient to separate (9) into an inner problem that takes N_H as given and minimizes over $\mathbf{q_{HH}}$ and $\mathbf{q_{HF}}$ and an outer problem that minimizes over N_H. As stated, the inner problem in 3.1 is not a convex optimization problem. We first convexify this problem by allowing randomization: instead of specifying a single quantity $q(\varphi)$ for each blueprint φ, we let the planner choose a distribution over q conditional on φ. Formally, for each φ there is a CDF over q_{Hj} given by $M_{Hj}(q;\varphi)$. Letting $\mathbf{M_{Hj}} \equiv \{M_{Hj}(q;\varphi)\}$, the inner planning problem becomes

$$L_H(Q_{HH},Q_{HF},N_H) \equiv \min_{\mathbf{M_{HH}},\mathbf{M_{HF}}\in\mathcal{M}} N_H \left(\sum_{j=H,F} \int_\Phi \int_{[0,\infty)} l_{Hj}(q,\varphi) dM_{Hj}(q;\varphi) dG_H(\varphi) + f_H^e \right)$$

$$N_H \int_\Phi \int_{[0,\infty)} q^{1/\mu_H} dM_{Hj}(q;\varphi) dG_H(\varphi) \geq Q_{Hj}^{1/\mu_H}, \text{ for } j = H, F,$$

where $\mathcal{M}$ is the set of all families of CDFs.

Note that $\mathcal{M}$ is a convex subset of a vector space. As stated, the above planning problem is linear and, thus, convex in $\mathbf{M_{HH}}$ and $\mathbf{M_{HF}}$.We have relaxed the equality to an inequality constraint to ensure that there exists an interior point, i.e. an $\mathbf{M_{Hj}}$ such that the constraint holds with strict inequality. Thus, we can apply a Lagrangian necessity theorem such as Theorem 1, p. 217 from Luenberger (1969). This guarantees that there exists $\lambda_{Hj} \geq 0$ such that any solution to the above problem must also minimize

$$N_H \left(\sum_{j=H,F} \int_{\Phi} \int_{[0,\infty)} l_{Hj}(q,\varphi) dM_{Hj}(q;\varphi) dG_H(\varphi) + f_H^e \right) + \sum_{j=H,F} \lambda_{Hj} \left(Q_{Hj}^{1/\mu_H} - N_H \int_{\Phi} \int_{[0,\infty)} q^{1/\mu_H} dM_{Hj}(q;\varphi) dG_H(\varphi) \right)$$

over $\mathbf{M_{HH}}, \mathbf{M_{HF}} \in \mathcal{M}$.

Next, we argue that this minimization must be attained without randomization. In other words, it can be described by two functions $q_{Hj}(\varphi)$ for $j = H, F$. This follows from the following two observations: (i) any $\mathbf{M_{Hj}} \in \mathcal{M}$ is dominated by $\hat{\mathbf{M}}_{\mathbf{Hj}} \in \mathcal{M}$ that assigns probability one to the set of points where $l_{Hj}(q;\varphi) - q^{1/\mu_H}$ is minimized; and (ii) the set of minimizers of $l_{Hj}(q;\varphi) - q^{1/\mu_H}$ is almost everywhere unique. To verify (ii), note that from the characterization in Section 3.1, $l_{Hj}(q;\varphi) - q^{1/\mu_H}$ has multiple minimizers only when

$$(\mu_H - 1)(\mu_H/\lambda_{Hj})^{-\sigma_H}(a_{Hj}(\varphi))^{1-\sigma_H} = f_{Hj}(\varphi).$$

Since we have assumed that for any $f_{Hj} > 0$ the distribution over a_{Hj} is smooth, this condition can only hold on a set with probability zero.

At this point, we have established that the solution to the inner problem in 3.1, with randomization, needs to minimize the associated Lagrangian and that the solution to the Lagrangian problem does not involve randomization. This implies that any solution to the inner problem in 3.1, without randomization, must also minimize

$$N_H[\sum_{j=H,F} \int_{\Phi} \left(l_{Hj}(q_{Hj}(\varphi),\varphi) - \lambda_{Hj}(q_{Hj}(\varphi))^{1/\mu_H} \right) dG_H(\varphi) + f_H^e].$$

Finally, note that for the solution of the Lagrangian problem to satisfy (9b), λ_{Hj} must also be non-zero, as stated in the main text.

A.2 Foreign's Offer Curve (Section 3.2)

The full problem of maximizing Home's imports, Q_{FH}, conditional on its aggregate exports, Q_{HF}, subject to Foreign's equilibrium conditions, i.e, conditions (1) and (4) for $i = F$ and $j = F$ and (2), (5), (6) for $i = F$, and (3) for $j = F$, is given by

$$Q_{FH}^{1/\mu_F}(Q_{HF}) \equiv \max_{\mathbf{q_{FF}},\mathbf{q_{FH}},\mathbf{p_{FF}},\mathbf{p_{FH}},P_{HF},P_{FF},Q_{FF},N_F} \int_\Phi N_F q_{FH}^{1/\mu_F}(\varphi) dG_F(\varphi)) \tag{A.2a}$$

$$q_{FF}(\varphi) = \begin{cases} \bar{q}_{FF}(\varphi) & \text{if } \mu_F a_{FF}(\varphi)\bar{q}_{FF}(\varphi) \geq l_{FF}(\bar{q}_{FF}(\varphi),\varphi), \\ 0 & \text{otherwise,} \end{cases} \tag{A.2b}$$

$$P_{FF}^{1-\sigma_j} = \int_\Phi N_F[p_{FF}(\varphi)]^{1-\sigma_i} dG_F(\varphi), \tag{A.2c}$$

$$p_{FF}(\varphi) = \begin{cases} \bar{p}_{FF}(\varphi) & \text{if } \mu_F a_{FF}(\varphi) q_{FF}(\varphi) \geq l_{FF}(q_{FF}(\varphi),\varphi), \\ \infty & \text{otherwise,} \end{cases} \tag{A.2d}$$

$$Q_{HF}, Q_{FF} \in \arg\max_{\tilde{Q}_{HF},\tilde{Q}_{FF}} \{U_F(\tilde{Q}_{HF},\tilde{Q}_{FF}) | \textstyle\sum_{i=H,F} P_{iF}\tilde{Q}_{iF} = L_F\}, \tag{A.2e}$$

$$f_F^e = \textstyle\sum_{j=H,F} \int_\Phi [\mu_F a_{Fj}(\varphi) q_{Fj}(\varphi) - l_{Fj}(q_{Fj}(\varphi),\varphi)] dG_F(\varphi), \tag{A.2f}$$

$$L_F = N_F[\textstyle\sum_{j=H,F} \int_\Phi l_{Fj}(q_{Fj}(\varphi),\varphi) dG_F(\varphi) + f_F^e]. \tag{A.2g}$$

Constraints (A.2b)-(A.2d) can be used to solve for the local micro quantities and prices in Foreign, as a function of Q_{FF} and N_F,

$$q_{FF}(\varphi|Q_{FF},N_F) = \begin{cases} \bar{q}_{FF}(\varphi|Q_{FF},N_F) & \text{, if } \mu_F a_{FF}(\varphi)\bar{q}_{FF}(\varphi|Q_{FF},N_F) \geq l_{FF}(\bar{q}_{FF}(\varphi|Q_{FF},N_F),\varphi), \\ 0 & \text{, otherwise;} \end{cases} \tag{A.3}$$

$$p_{FF}(\varphi|Q_{FF},N_F) = \begin{cases} \mu_F a_{FF}(\varphi) & \text{, if } \mu_F a_{FF}(\varphi) q_{FF}(\varphi|Q_{FF},N_F) \geq l_{Fj}(q_{FF}(\varphi|Q_{FF},N_F),\varphi), \\ \infty & \text{, otherwise;} \end{cases} \tag{A.4}$$

$$P_{FF}(Q_{FF},N_F) = \left(\int_\Phi N_F(p_{FF}(\varphi|Q_{FF},N_F))^{1-\sigma_F} dG_F(\varphi)\right)^{1/(1-\sigma_F)}, \tag{A.5}$$

with $\bar{q}_{FF}(\varphi|Q_{FF},N_F) \equiv [\mu_F a_{FF}(\varphi)/P_{FF}(Q_{FF},N_F)]^{-\sigma_F} Q_{FF}$. Total profits and total employment associated with the local sales of foreign firms, $\Pi_{FF}(Q_{FF},N_F)$ and $L_{FF}(Q_{FF},N_F)$, are then given by

$$\Pi_{FF}(Q_{FF},N_F) \equiv N_F[\int_\Phi \mu_F a_{FF}(\varphi) q_{FF}(\varphi|Q_{FF},N_F) dG_F(\varphi) - \int_\Phi l_{FF}(q_{FF}(\varphi|Q_{FF},N_F),\varphi) dG_F(\varphi)], \tag{A.6}$$

$$L_{FF}(Q_{FF},N_F) \equiv N_F[\int_\Phi l_{FF}(q_{FF}(\varphi|Q_{FF},N_F),\varphi) dG_F(\varphi)]. \tag{A.7}$$

In turn, constraint (A.2e) can be used to solve for Home's export price as a function of Q_{HF}, Q_{FF}, and N_F. The necessary first order conditions for utility maximization in Foreign imply

$$P_{HF}(Q_{HF},Q_{FF},N_F) = P_{FF}(Q_{FF},N_F) MRS_F(Q_{HF},Q_{FF}), \tag{A.8}$$

where $MRS_F(Q_{HF}, Q_{FF}) \equiv (\partial U_F/\partial Q_{HF})/(\partial U_F/\partial Q_{FF})$ is the marginal rate of substitution in Foreign. Combining the previous equation with Foreign's budget constraint, we can rearrange constraint (A.2e) more compactly as

$$L_F = P_{FF}(Q_{FF}, N_F)(Q_{FF} + MRS_F(Q_{HF}, Q_{FF})Q_{HF}).$$

Substituting the previous expressions into problem A.2, we get that Home's optimal import quantities, $\mathbf{q_{FH}}$, as well as the measure of foreign entrants, N_F, and local output, Q_{FF}, must solve (12).

We can derive necessary conditions for optimality of $\mathbf{q_{FH}}$, appealing to global Lagrangian necessity theorems, as we did in Section 3.1. Consider the subproblem that takes Q_{FF} and N_F as given and maximizes over $\mathbf{q_{FH}}$. Specifically, allowing for randomization we can rewrite the problem as a choice over CDF $M_{FH}(q;\varphi)$. The problem then becomes

$$\begin{aligned}
&\max_{\mathbf{M_{FH}} \in \mathcal{M}_{FH}} \int_\Phi \int_{[0,\infty)} N_F q^{1/\mu_F} dM_{FH}(q;\varphi)\, dG_F(\varphi) \\
&N_F f_F^e = \Pi_{FF}(Q_{FF}, N_F) \\
&+ N_F \int_\Phi \int_{[0,\infty)} [\mu_F a_{FH}(\varphi) q - l_{FH}(q,\varphi)] dM_{FH}(q;\varphi)\, dG_F(\varphi), \\
&L_F = N_F f_F^e + L_{FF}(Q_{FF}, N_F) + N_F \int_\Phi \int_{[0,\infty)} l_{FH}(q,\varphi)\, dM_{FH}(q;\varphi)\, dG_F(\varphi),
\end{aligned}$$

where

$$\mathcal{M}_{FH} = \{\mathbf{M_{FH}} \in \mathcal{M} : \int_\Phi \int_{\{q:\mu_F a_{FH}(\varphi) q - l_{FH}(q,\varphi) < 0\}} dM_{FH}(q;\varphi)\, dG_F(\varphi) = 0\}$$

is the set of probability distributions that ensures positive profits almost everywhere. As stated, this problem is linear and, thus, convex. It features an inequality and two equality constraints. Invoking the Lagrangian necessity theorem given by Theorem 1, p. 217 from Luenberger (1969) extended in Exercise 8.8.7 (p. 236), there exist multipliers λ_E and λ_L for the equality constraints so that any solution must also maximize the Lagrangian

$$\int_\Phi \int_{[0,\infty)} [q^{1/\mu_F} - \lambda_E \mu_F a_{FH}(\varphi) q + (\lambda_E - \lambda_L) l_{FH}(q,\varphi)] dM_{FH}(q;\varphi)\, dG_F(\varphi)$$

over $\mathbf{M_{FH}} \in \mathcal{M}_{FH}$. As before, since the objective is linear in $\mathbf{M_{FH}}$ it follows that we can focus on a "bang bang" solution that puts full weight on any point q for each φ that minimizes

$$q^{1/\mu_F} - \lambda_E \mu_F a_{FH}(\varphi) q + (\lambda_E - \lambda_L) l_{FH}(q,\varphi)$$

over the set of q satisfying $\mu_F a_{FH}(\varphi) q \geq l_{FH}(q,\varphi)$. By virtue of the analysis carried out in Section 3.2, the solution to this problem is unique almost everywhere. This follows since indifference obtains only if $f_{FH}(\varphi) > 0$ and for at most two values of $\theta_{FH}(\varphi)$, namely $\theta_{FH}(\varphi) = ((\lambda_L - \lambda_E)/(\lambda_L + (\mu_F - 1)\lambda_E))^{1/\sigma_F}$, if $\lambda_E < 0$, or $\theta_{FH}(\varphi) = (\lambda_L/(\lambda_L + (\mu_F - 1)\lambda_E)$, otherwise.

But under our assumption that for any $f_{FH} > 0$ the distribution over $a_{FH}(\varphi)$ is smooth, it follows that indifference happens with probability zero.

Once optimal quantities, $q_{FH}(\varphi|Q_{FF}, N_F)$, have been solved for, optimal import prices are given by

$$p_{FH}(\varphi|Q_{FF}, N_F) = \begin{cases} \mu_F a_{FH}(\varphi) & \text{, if } \mu_F a_{FH}(\varphi) q_{FH}(\varphi|Q_{FF}, N_F) \geq l_{FH}(q_{FH}(\varphi|Q_{FF}, N_F), \varphi), \\ \infty & \text{, otherwise.} \end{cases} \tag{A.9}$$

Finally, the optimal local output and measure of entrants in Foreign, $Q_{FF}(Q_{HF})$ and $N_F(Q_{HF})$, are then given by the solution to the outer problem

$$\begin{aligned} Q_{FF}(Q_{HF}), N_F(Q_{HF}) \in & \text{argmax}_{(Q_{FF}, N_F) \in \Omega_F} \int_\Phi N_F q_{FH}^{1/\mu_F}(\varphi|Q_{FF}, N_F) dG_F(\varphi)) \\ & L_F = P_{FF}(Q_{FF}, N_F)(Q_{FF} + MRS_F(Q_{HF}, Q_{FF}) Q_{HF}), \end{aligned} \tag{A.10}$$

with Ω_F the set of (Q_{FF}, N_F) for which a solution to the inner maximization problem exists.

A.3 First-Order Conditions of the Macro Problem (Section 3.3)

At an interior solution to the macro problem (15), the necessary first-order conditions are given by

$$\begin{aligned} U^*_{HH} &= \Lambda_H L^*_{HH}, \\ U^*_{FH} &= \Lambda_T, \\ \Lambda_T Q'_{FH}(Q_{HF}) &= \Lambda_H L^*_{HF}, \end{aligned}$$

where $U^*_{iH} \equiv \partial U_H / \partial Q_{iH}$ denotes the marginal utility at home of the aggregate good from country $i = H, F$; $L^*_{Hj} \equiv \partial L_H / \partial Q_{Hj}$ denotes the marginal cost of producing and delivering one unit of the home good in country $j = H, F$; and Λ_T and Λ_H are the Lagrange multipliers associated with constraints (15b) and (15c). After eliminating the Lagrange multipliers, we obtain

$$\frac{U^*_{HH}}{U^*_{FH}} = \frac{L^*_{HH} Q'_{FH}(Q^*_{HF})}{L^*_{HF}}. \tag{A.11}$$

To conclude, note that at a solution to (12), constraints (12c) and (12d) imply

$$\begin{aligned} L_F = & \Pi_{FF}(Q_{FF}(Q^*_{HF}), N_F(Q^*_{HF})) + L_{FF}(Q_{FF}(Q^*_{HF}), N_F(Q^*_{HF})) \\ & + N_F(Q^*_{HF}) \int \mu_F a_{FH}(\varphi) q_{FH}(\varphi|Q^*_{HF}) dG_F(\varphi), \end{aligned}$$

which can be rearranged as

$$L_F = P_{FF}(Q_{FF}(Q^*_{HF}), N_F(Q^*_{HF})) Q_{FF}(Q^*_{HF}) + \tilde{P}_{FH}(Q^*_{HF}, Q^*_{FH}) Q^*_{FH}.$$

Together with constraint (12b), this leads to the trade balance condition,

$$P_{HF}(Q^*_{HF})Q^*_{HF} = \tilde{P}_{FH}(Q^*_{HF}, Q^*_{FH})Q^*_{FH}.$$

Combining this expression with equation (A.11), we finally get

$$\frac{U^*_{HH}}{U^*_{FH}} = \frac{L^*_{HH}}{L^*_{HF}} \frac{P_{HF}(Q^*_{HF})}{\tilde{P}_{FH}(Q^*_{HF}, Q^*_{FH})} \frac{Q^*_{HF}Q'_{FH}(Q^*_{HF})}{Q_{FH}(Q^*_{HF})}.$$

Equation (16) follows from this equation and the definitions of $MRS^*_H \equiv U^*_{HH}/U^*_{FH}$, $MRT^*_H \equiv L^*_{HH}/L^*_{HF}$, $P^* \equiv P_{HF}(Q^*_{HF})/\tilde{P}_{FH}(Q^*_{HF}, Q^*_{FH})$, and $\eta^* \equiv d\ln Q_{FH}/d\ln Q_{HF}$.

A.4 Positive Discrimination (Section 3.4)

Consider the relaxed version of (12),

$$\tilde{Q}^{1/\mu_F}_{FH}(Q_{HF}) \equiv \max_{\mathbf{q_{FH}}, Q_{FF}, N_F} \int_\Phi N_F q^{1/\mu_F}_{FH}(\varphi) dG_F(\varphi))$$

$$L_F \geq P_{FF}(Q_{FF}, N_F)(Q_{FF} + MRS_F(Q_{HF}, Q_{FF})Q_{HF}) \tag{A.12a}$$

$$N_F f^e_F \geq \Pi_{FF}(Q_{FF}, N_F)$$
$$+ N_F \int [\mu_F a_{FH}(\varphi) q_{FH}(\varphi) - l_{FH}(q_{FH}(\varphi), \varphi)] dG_F(\varphi), \tag{A.12b}$$

$$L_F = N_F f^e_F + L_{FF}(Q_{FF}, N_F) + N_F \int_\Phi l_{FH}(q_{FH}(\varphi), \varphi) dG_F(\varphi), \tag{A.12c}$$

$$\mu_F a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}(q_{FH}(\varphi), \varphi).$$

The first goal of this appendix is to show that if Q^*_{HF} is part of an interior solution to the macro problem,

$$\max_{Q_{HH}, Q_{FH}, Q_{HF}} U_H(Q_{HH}, Q_{FH}) \tag{A.13a}$$

$$\tilde{Q}_{FH}(Q_{HF}) \geq Q_{FH}, \tag{A.13b}$$

$$L_H(Q_{HH}, Q_{HF}) = L_H, \tag{A.13c}$$

then constraints (A.12a) and (A.12b) must be satisfied with equality at a solution to (A.12). It follows that any $\mathbf{q^*_{FH}}$ that is part of an interior solution to Home's relaxed planning problem must also be part of a solution to (A.12). The second part of this appendix will then show that if $\mathbf{q^*_{FH}}$ is part of an interior solution to Home's relaxed planning problem, then positive discrimination must arise at this solution.

Consider first inequality (A.12a). We proceed by contradiction. Suppose that inequality (A.12a) is slack for Q^*_{HF} that is part of an interior solution to (A.13). Starting from this allocation, Home could strictly decrease aggregate exports, Q_{HF}, and strictly increase domestic output, Q_{HH}, while still satisfying constraints (A.13b) and (A.13c). Such a deviation would strictly increase Home's utility, thereby contradicting the optimality of Q^*_{HF}.

Next consider inequality (A.12b). By the same randomization arguments as in Sections A.1 and A.2, we know that there exist $\lambda_E \geq 0$ and λ_L such that any solution to the inner problem associated with (A.12),

$$\tilde{Q}^{1/\mu_F}_{FH}(Q_{FF}, N_F) \equiv \max_{\mathbf{q_{FH}}} \int_\Phi N_F q^{1/\mu_F}_{FH}(\varphi) dG_F(\varphi))$$

$$N_F f^e_F \geq \Pi_{FF}(Q_{FF}, N_F)$$

$$+ N_F \int [\mu_F a_{FH}(\varphi) q_{FH}(\varphi) - l_{FH}(q_{FH}(\varphi), \varphi)] dG_F(\varphi),$$

$$L_F = N_F f^e_F + L_{FF}(Q_{FF}, N_F) + N_F \int_\Phi l_{FH}(q_{FH}(\varphi), \varphi) dG_F(\varphi),$$

$$\mu_F a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}(q_{FH}(\varphi), \varphi),$$

also maximizes the associated Lagrangian,

$$\begin{aligned}\mathcal{L}(Q_{FF}, N_F) \equiv & \max_{\mathbf{q_{FH}} \in \mathcal{Q}_{FH}} \int_\Phi N_F (q_{FH}(\varphi))^{1/\mu_F} dG_F(\varphi) \\ & + \lambda_E (N_F f^e_F - \Pi_{FF}(Q_{FF}, N_F) - N_F \int [\mu_F a_{FH}(\varphi) q_{FH}(\varphi) - l_{FH}(q_{FH}(\varphi), \varphi)] dG_F(\varphi)) \\ & + \lambda_L (L_F - N_F f^e_F - L_{FF}(Q_{FF}, N_F) - \int_\Phi N_F l(q_{FH}(\varphi), \varphi) dG_F(\varphi)),\end{aligned}$$

where

$$\mathcal{Q}_{FH} = \{\mathbf{q_{FH}} : \mu_F a_{FH}(\varphi) q_{FH}(\varphi) \geq l_{FH}(q_{FH}(\varphi), \varphi) \text{ for all } \varphi \in \Phi\}.$$

We again proceed by contradiction. Suppose that inequality (A.12b) is slack. Then by complementary slackness, the Lagrange multiplier associated with this constraint must be zero, $\lambda_E = 0$. For the same reasons as in Section 3.2, $\chi_{FH} \equiv \lambda_L + (\mu_F - 1)\lambda_E$ must be strictly positive, hence the Lagrange multiplier associated with (A.12c) must be strictly positive as well, $\lambda_L > 0$. By the Envelope Theorem, we must therefore have

$$\frac{\partial \mathcal{L}(Q_{FF}, N_F)}{\partial N_F} = \int_\Phi (q_{FH}(\varphi | Q_{FF}, N_F))^{1/\mu_F} dG_F(\varphi) + \lambda_L (-f^e_F - \frac{dL_{FF}(Q_{FF}, N_F)}{dN_F} - \int_\Phi l(q_{FH}(\varphi), \varphi | Q_{FF}, N_F)) dG_F(\varphi)),$$

where $q_{FH}(\varphi | Q_{FF}, N_F)$ is the solution to the Lagrangian problem with $\lambda_E = 0$. Using the characterization of this solution derived in Section 3.2 (equation 14), we can rearrange the previous derivative as

$$\frac{\partial \mathcal{L}(Q_{FF}, N_F)}{\partial N_F} = \lambda_L \left[-\frac{\partial L_{FF}(Q_{FF}, N_F)}{\partial N_F} + \int_\Phi (\mu_F a_{FH}(\varphi) q_{FH}(\varphi | Q_{FF}, N_F) - l(q_{FH}(\varphi | Q_{FF}, N_F), \varphi)) dG_F(\varphi) - f^e_F \right]. \tag{A.14}$$

Since the decentralized equilibrium under CES is efficient, $L_{FF}(Q_{FF}, N_F)$ must be such that

$$L_{FF}(Q_{FF}, N_F) = \min_{\mathbf{q_{FF}}} N_F \int_\Phi l_{FF}(q_{FF}(\varphi), \varphi | Q_{FF}, N_F) dG_F(\varphi)$$
$$\int_\Phi N_F q_{FH}^{1/\mu_F}(\varphi) dG_F(\varphi) = Q_{FF}.$$

Thus invoking again the Envelope Theorem and using the characterization of the solution to the previous problem, as described in Section 3.1, one can also establish that

$$\frac{\partial L_{FF}(Q_{FF}, N_F)}{\partial N_F} = -(\int_\Phi (\mu_F a_{FF}(\varphi) q_{FF}(\varphi | Q_{FF}, N_F) - l(q_{FF}(\varphi | Q_{FF}, N_F), \varphi)) dG_F(\varphi)). \tag{A.15}$$

Combining equations (A.14) and (A.15), we obtain

$$\frac{\partial \mathcal{L}(Q_{FF}, N_F)}{\partial N_F} = \lambda_L (\sum_{j=H,F} \int_\Phi (\mu_F a_{Fj}(\varphi) q_{Fj}(\varphi | Q_{FF}, N_F) - l(q_{Fj}(\varphi | Q_{FF}, N_F), \varphi)) dG_F(\varphi) - f_F^e) < 0, \tag{A.16}$$

where the sign of the inequality follows from the fact that inequality (A.12b) is slack and $\lambda_L > 0$. To conclude, note that at a solution to (A.12), (Q_{FF}^*, N_F^*) must solve the outer problem

$$\max_{Q_{FF}, N_F \in \Omega_F} \tilde{Q}_{FH}(Q_{FF}, N_F)$$
$$P_{FF}(Q_{FF}, N_F) Q_{FF} + P_{HF}(Q_{HF}, Q_{FF}, N_F) Q_{HF} \geq L_F,$$

where Ω_F denotes the set of (Q_{FF}, N_F) for which a solution to the inner problem exists. Since $P_{FF}(Q_{FF}, N_F) Q_{FF} + P_{HF}(Q_{HF}, Q_{FF}, N_F) Q_{HF}$ is strictly decreasing in N_F, the previous constraint can be rearranged as

$$N_F \leq \bar{N}(Q_{HF}, Q_{FF}).$$

Note also that any (Q_{FF}, N_F) on the boundary of Ω_F must satisfy $\tilde{Q}_{FH}(Q_{FF}, N_F) = 0$, whereas any interior point must lead to $\tilde{Q}_{FH}(Q_{FF}, N_F) > 0$. Together the two previous observations imply that a solution to (A.12) must satisfy

$$\frac{\partial \tilde{Q}_{FH}(Q_{FF}^*, N_F^*)}{\partial N_F} \geq 0.$$

Since $\mathcal{L}(Q_{FF}, N_F) = \tilde{Q}_{FH}^{1/\mu_F}(Q_{FF}, N_F)$, this contradicts inequality (A.16).

At this point, we have established that if Q_{HF}^* is part of an interior solution to the macro problem (A.13), then constraints (A.12a) and (A.12b) must be satisfied with equality at a solution to (A.12). This implies that if $\mathbf{q}_{\mathbf{FH}}^*$ is part of an interior solution to Home's relaxed planning problem, then it is also part of a solution to (A.12). We now establish that positive discrimination must arise at this solution.

From our analysis in Section 3.2, we know that positive discrimination is equivalent to λ_E being strictly positive. And since (A.12b) is an inequality constraint, we already know that λ_E

is nonnegative. So we only need to establish that λ_E cannot be zero. We can again proceed by contradiction. Suppose that $\lambda_E = 0$. Then the same argument as above implies

$$\frac{\partial \mathcal{L}(Q_{FF}, N_F)}{\partial N_F} = \lambda_L(\sum_{j=H,F} \int_\Phi (\mu_F a_{Fj}(\varphi) q_{Fj}(\varphi | Q_{FF}, N_F) - l(q_{Fj}(\varphi | Q_{FF}, N_F), \varphi)) dG_F(\varphi) - f_F^e) = 0, \tag{A.17}$$

where the second equality uses the fact that (A.12b) is satisfied with equality. Starting from such an allocation, Home could strictly decrease both N_F and Q_{HF} such that (A.12a) still holds, and then increase Q_{HH} such that (A.13c) holds as well. By equation (A.17), the change in N_F would have at most second-order effects on aggregate imports and hence Home's utility, whereas the change in Q_{HH} would lead to a first-order increase in Home's utility, thereby contradicting the optimality of the initial allocation.

B Proofs of Section 4

Let $(\{\mathbf{q}_{\mathbf{ij}}^*, Q_{ij}^*\}_{i,j=H,F}, \mathbf{p}_{\mathbf{FF}}^*, \mathbf{p}_{\mathbf{FH}}^*, P_{FF}^*, P_{HF}^*, \{N_i^*\}_{i=H,F})$ denote a solution to Home's relaxed planning problem. In the main text, we have already described some of these variables. Before establishing Lemmas 4 and 5, we provide a complete characterization of this solution. The three macro quantities, $(Q_{HH}^*, Q_{HF}^*, Q_{FH}^*)$, are given by the solution to (15). Conditional on Q_{HH}^* and Q_{HF}^*, the domestic micro quantities, $\mathbf{q}_{\mathbf{HH}}^* = \{q_{HH}(\varphi | Q_{HH}^*, Q_{HF}^*)\}$ and $\mathbf{q}_{\mathbf{HF}}^* = \{q_{HF}(\varphi | Q_{HH}^*, Q_{HF}^*)\}$, as well as the measure of domestic entrants, N_H^*, are given by equations (10) and (11). Conditional on Q_{HF}^*, the local output and the measure of entrants in Foreign, $Q_{FF}^* = Q_{FF}(Q_{HF}^*)$ and $N_F^* = N_F(Q_{HF}^*)$ are given by condition (A.10), whereas the foreign micro quantities, $\mathbf{q}_{\mathbf{FH}}^* = \{q_{FH}(\varphi | Q_{HF}^*)\}$ and $\mathbf{q}_{\mathbf{FF}}^* = \{q_{FF}(\varphi | Q_{FF}(Q_{HF}^*), N_F(Q_{HF}^*))\}$ are given by conditions (14) and (A.3). Finally, the prices of foreign varieties, $\mathbf{p}_{\mathbf{FF}}^* = \{p_{FF}(\varphi | Q_{FF}(Q_{HF}^*), N_F(Q_{HF}^*))\}$ and $\mathbf{p}_{\mathbf{FH}}^* = \{p_{FH}(\varphi | Q_{FF}(Q_{HF}^*), N_F(Q_{HF}^*))\}$, are given by equations (A.4) and (A.9), whereas the aggregate price indices, $P_{HF}^* = P_{HF}(Q_{HF}^*)$ and $P_{FF}^* = P_{FF}(Q_{FF}(Q_{HF}^*), N_F(Q_{HF}^*))$, are given by equations (A.8), and (A.5). We also let $\tilde{P}_{FH}^* = \tilde{P}_{FH}(Q_{HF}^*, Q_{FH}^*)$ denote the average cost of imports at home. Note that $\tilde{P}_{FH}^*$ differs from the import price index faced by Home consumers in the decentralized equilibrium, P_{FH}, which is inclusive of taxes.

B.1 Lemma 4

Proof of Lemma 4. First, consider the marginal rate of substitution, $MRS_j^* \equiv U_{Hj}^* / U_{Fj}^*$, in country $j = H, F$ at a solution to Home's relaxed planning problem. In Foreign, the necessary first order conditions for utility maximization imply $MRS_F^* = P_{HF}^* / P_{FF}^*$. Combining this expression with equation (A.5), we obtain

$$MRS_F^* = \frac{P_{HF}^*}{(\int_\Phi N_F^* (p_{FF}^*(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}. \tag{B.1}$$

At home, we already know from equation (16) that

$$MRS_H^* = \eta^* MRT_H^* (P_{HF}^* / \tilde{P}_{FH}^*).$$

By the Envelope Theorem, we also know that

$$MRT_H^* = (\lambda_{HH}/\lambda_{HF})(Q_{HH}^*/Q_{HF}^*)^{-1/\sigma_H}.$$

From equations (9b) and (10), we also know that the Lagrange multipliers satisfy

$$\lambda_{Hj} = [N_H^* \int_{\Phi_{Hj}} (\mu_H a_{Hj}(\varphi))^{1-\sigma_H} dG_H(\varphi)]^{1/(1-\sigma_H)} (Q_{Hj}^*)^{1/\sigma_H}.$$

Combining the two previous expressions, we get

$$MRT_H^* = \frac{(\int_{\Phi_{HH}} (a_{HH}(\varphi))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)}}{(\int_{\Phi_{HF}} (a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)}}, \tag{B.2}$$

and in turn,

$$MRS_H^* = \frac{\eta^* (\int_{\Phi_{HH}} (a_{HH}(\varphi))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)} P_{HF}^*}{(\int_{\Phi_{HF}} (a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)} \tilde{P}_{FH}^*}. \tag{B.3}$$

Next, consider a decentralized equilibrium with taxes that implements a solution to the relaxed planning problem. The marginal rate of substitution for each of the two countries is determined by conditions (2)-(4). Using the fact that the set of varieties available for consumption in the decentralized equilibrium must be the same as in the solution to the relaxed planning problem, we obtain

$$MRS_F^* = \frac{(\int_{\Phi_{HF}} N_H^* (\mu_H w_H a_{HF}(\varphi)/(1+s_{HF}^*(\varphi)))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)}}{(\int_{\Phi} N_F^* (p_{FF}^*(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}, \tag{B.4}$$

$$MRS_H^* = \frac{(\int_{\Phi_{HH}} N_H^* ((1+t_{HH}^*(\varphi))\mu_H w_H a_{HH}(\varphi)/(1+s_{HH}^*(\varphi)))^{1-\sigma_H} dG_H(\varphi))^{1/(1-\sigma_H)}}{(\int_{\Phi_{FH}} N_F^* ((1+t_{FH}^*(\varphi))\mu_F a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}. \tag{B.5}$$

Combining equations (B.1), (B.3), (B.4), and (B.5) with the micro-level taxes in Lemmas 1-3, we get

$$\frac{(1+t_{FH}^*)/(1+t_{HH}^*)}{(1+s_{HF}^*)/(1+s_{HH}^*)} = \frac{\tilde{P}_{FH}^*}{\eta^* (\int_{\Phi_{FH}} N_F^* (\min\{1, \theta_{FH}(\varphi)\} \mu_F a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}. \tag{B.6}$$

By definition of $\tilde{P}_{FH}^*$, we know that

$$\tilde{P}_{FH}^* Q_{FH}^* = \int_{\Phi} N_F^* \mu_F a_{FH}(\varphi) q_{FH}^*(\varphi) dG_F(\varphi).$$

Together with equation (14), this implies

$$\frac{\tilde{P}^*_{FH} Q^*_{FH}}{(N^*_F)^{1/(1-\sigma_F)}\mu_F} = \frac{\int_{\Phi^u_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi^c_{FH}} \left((\theta_{FH}(\varphi))^{\mu_F} a_{FH}(\varphi)\right)^{1-\sigma_F} dG_F(\varphi)}{(\mu_F \chi_{FH})^{\sigma_F} (N^*_F)^{\sigma_F/(1-\sigma_F)}}.$$

At a solution to Home's relaxed planning problem, constraint (15b) must be satisfied with equality. Otherwise, Home could raise its imports, Q_{FH}, and hence the utility of its representative agent. Using equations (14) and (15b), one can also check that

$$\frac{(\mu_F \chi_{FH})^{\sigma_F - 1} (Q^*_{FH})^{1/\mu_F}}{N^*_F} = \int_{\Phi^u_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi^c_{FH}} (\theta_{FH}(\varphi) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi).$$

Combining the two previous expressions, we then obtain

$$\frac{\tilde{P}^*_{FH}}{(N^*_F)^{1/(1-\sigma_F)}\mu_F} = \frac{\int_{\Phi^u_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi^c_{FH}} ((\theta_{FH}(\varphi))^{\mu_F} a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)}{\left(\int_{\Phi^u_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi^c_{FH}} (\theta_{FH}(\varphi) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)\right)^{\sigma_F/(\sigma_F - 1)}}. \quad \text{(B.7)}$$

Substituting into equation (B.6) and using the definition of Φ^u_{FH} and Φ^c_{FH} we get equation (25). □

B.2 Lemma 5

Proof of Lemma 5. In order to show the existence of a decentralized equilibrium that implements the desired allocation, we follow a guess and verify strategy. Consider: (i) quantities such that

$$q_{ij}(\varphi) = q^*_{ij}(\varphi), \quad \text{(B.8)}$$

$$Q_{ij} = Q^*_{ij}; \quad \text{(B.9)}$$

(ii) measures of entrants such that

$$N_i = N^*_i \text{ for all } i; \quad \text{(B.10)}$$

(iii) wages such that

$$w_H = P^*_{HF}/\mu_H L^*_{HF}, \quad \text{(B.11)}$$

$$w_F = 1; \quad \text{(B.12)}$$

(iv) goods prices such that

$$p_{Hj}(\varphi) = \begin{cases} \bar{p}_{Hj}(\varphi) & \text{, if } \mu_H a_{Hj}(\varphi) q_{Hj}(\varphi) \geq l_{Hj}(q_{Hj}(\varphi), \varphi), \\ \infty & \text{, otherwise,} \end{cases} \quad \text{(B.13)}$$

$$p_{Fj}(\varphi) = p^*_{Fj}(\varphi), \quad \text{(B.14)}$$

and

$$P_{HH}^{1-\sigma_H} = \int_\Phi N_H[(1+t_{HH}(\varphi))p_{HH}(\varphi)]^{1-\sigma_H} dG_H(\varphi), \tag{B.15}$$
$$P_{HF} = P_{HF}^*, \tag{B.16}$$
$$P_{FH} = \eta^* \tilde{P}_{FH}^*, \tag{B.17}$$
$$P_{FF} = P_{FF}^*; \tag{B.18}$$

(v) taxes such that

$$s_{Hj}(\varphi) = s_{Hj}^*, \text{ for all } \varphi \text{ and for } j = H, F, \tag{B.19}$$

$$t_{HH}(\varphi) = t_{HH}^*, \text{ for all } \varphi, \tag{B.20}$$

$$t_{FH}(\varphi) = t_{FH}^*(\varphi), \text{ if } \varphi \in \Phi_{FH}, \tag{B.21}$$
$$t_{FH}(\varphi) \geq t_{FH}^*, \text{ otherwise}, \tag{B.22}$$

with $s_{Hj}^* = 0$ for $j = H, F$, $t_{HH}^* = 0$, $t_{FH}^*(\varphi)$ given by equation (24), and t_{FH}^* given by equation (25); and (vi) a lump-sum transfer such that

$$T_H = \sum\nolimits_{j=H,F} [\int_\Phi N_j t_{jH}(\varphi) p_{jH}(\varphi) q_{jH}(\varphi) dG_j(\varphi) - \int_\Phi N_H s_{Hj}(\varphi) p_{Hj}(\varphi) q_{Hj}(\varphi) dG_H(\varphi)]. \tag{B.23}$$

We now check that the previous allocation and prices satisfy the equilibrium conditions (1)-(7).

First, consider condition (7). Since it is equivalent to equation (B.23), it is trivially satisfied by construction.

Second, consider condition (2). For goods that are produced by home firms, they are equivalent to equations (B.13). So, it is again trivially satisfied. For goods that are produced by foreign firms, condition (2) derives from equations (A.4), (A.9), (B.8), and (B.14).

Third, consider condition (4). For goods locally sold by home firms, it directly derives from condition (B.15). For goods exported by home firms, one can use the same argument as in the proof of Lemma 4 to show that

$$L_{HF}^* = \lambda_{HF} (Q_{HF}^*)^{-1/\sigma_H} / \mu_H, \tag{B.24}$$
$$\lambda_{HF} = [N_H^* \int_{\Phi_{HF}} (\mu_H a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi)]^{1/(1-\sigma_H)} (Q_{HF}^*)^{1/\sigma_H},$$

which imply

$$L_{HF}^* = [N_H^* \int_{\Phi_{HF}} (\mu_H a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi)]^{1/(1-\sigma_H)} / \mu_H. \tag{B.25}$$

Combining the previous expression with equation (B.11), we get

$$P_{HF}^* = [N_H^* \int_{\Phi_{HF}} (\mu_H w_H a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi)]^{1/(1-\sigma_H)}.$$

Condition (4) then derives from the previous equation and equations (B.10), (B.13), and (B.19). Next, consider goods locally sold by foreign firms. For those, condition (4) derives from equations (A.4), (A.5), (B.8), (B.10), (B.14), and (B.18). Finally, for goods exported by foreign firms, we already know from equation (B.6) that

$$\frac{(1+t^*_{FH})/(1+t^*_{HH})}{(1+s^*_{HF})/(1+s^*_{HH})} = \frac{\eta^* \tilde{P}^*_{FH}}{(\int_{\Phi_{FH}} N^*_F(\min\{1, \theta_{FH}(\varphi)\}\mu_F a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}}.$$

Combining the previous expression with equations (24), (B.21), (B.22), and using the fact that $s^*_{Hj} = 0$ for $j = H, F$ and $t^*_{HH} = 0$, we then get

$$(\int_{\Phi_{FH}} N^*_F((1+t_{FH}(\varphi))\mu_F a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)} = \eta^* \tilde{P}^*_{FH}.$$

Condition (4) derives from the previous expression and equations (A.9), (B.10), (B.14), and (B.17).

Fourth, consider condition (1). For goods locally sold by foreign firms, condition (1) directly derives from equations (A.3), (B.8), (B.9), and (B.18). For goods exported by home firms, note that by equations (B.8), (B.11), (B.13), (B.9), and (B.19) with $s^*_{HF} = 0$, condition (1) holds if

$$(\mu_H a_{HF}(\varphi)/\lambda_{HF})^{-\sigma_H} = [P^*_{HF} a_{HF}(\varphi)/(L^*_{HF} P_{HF})]^{-\sigma_H} Q^*_{HF}.$$

Since the previous equation follows from equations (B.24) and (B.16), condition (1) must hold for goods exported by home firms. We can use a similar logic to analyze micro-level quantities sold at Home. Given equations (B.8), (B.11), (B.13), (B.9), and (B.20) with $t^*_{HH} = 0$, condition (1) holds for goods locally sold by home firms if

$$(\mu_H a_{HH}(\varphi)/\lambda_{HH})^{-\sigma_H} = (P^*_{HF} a_{HH}(\varphi)/(L^*_{HF} P_{HH}))^{-\sigma_H} Q^*_{HH}. \quad \text{(B.26)}$$

Using the same argument as in the proof of Lemma 4, one can also show that

$$L^*_{HH} = \lambda_{HH}(Q^*_{HH})^{-1/\sigma_H}/\mu_H.$$

Hence, condition (B.26) is equivalent to

$$P^*_{HF}/P_{HH} = L^*_{HF}/L^*_{HH}, \quad \text{(B.27)}$$

which follows from equations (B.2), (B.13), (B.15), (B.16), (B.19), (B.20), as well as the fact that condition (4) holds for goods exported by home firms. Lastly, consider goods exported by foreign firms. Given equations (B.8), (B.9), (B.12), (B.14), (B.21), and (B.22), condition (1) holds if

$$(\mu_F \chi_{FH} a_{FH}(\varphi))^{-\sigma_F} = [(1+t^*_{FH})\mu_F a_{FH}(\varphi)/P_{FH}]^{-\sigma_F} Q^*_{FH}, \text{ if } \varphi \in \Phi^u_{FH},$$
$$f_{FH}(\varphi)/((\mu_F - 1)a_{FH}(\varphi)) = [(1+t^*_{FH})\theta_{FH}(\varphi)\mu_F a_{FH}(\varphi)/P_{FH}]^{-\sigma_F} Q^*_{FH}, \text{ if } \varphi \in \Phi^c_{FH},$$

Given the definition of $\theta_{FH}(\varphi)$, both conditions reduce to

$$1/\chi_{FH} = (Q^*_{FH})^{1/\sigma_F} P_{FH}/(1+t^*_{FH}). \tag{B.28}$$

Using equations (14) and (15b), one can again use the same strategy as in the proof of Lemma 4 to show that

$$1/\chi_{FH} = (Q^*_{FH})^{1/\sigma_F} (\int_{\Phi_{FH}} N^*_F (\mu_F(\min\{1, \theta_{FH}(\varphi)\}) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}.$$

Since condition (4) holds for goods exported by foreign firms, we also know from equations (B.10), (B.21), and (B.22) that

$$P_{FH} = (1+t^*_{FH})(\int_{\Phi_{FH}} N^*_F (\mu_F(\min\{1, \theta_{FH}(\varphi)\}) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi))^{1/(1-\sigma_F)}.$$

Equation (B.28) derives from the two previous observations. Hence, condition (1) must also hold for goods exported by foreign firms.

Fifth, consider the free entry condition (5). Abroad, this condition derives from equations (12c), (A.6), (B.8), (B.9), and (B.10). At home, it derives from equations (11) and (B.8).

Sixth, consider the labor market condition (6). Abroad, this condition derives from equations (12d), (A.7), (B.8), (B.9), and (B.10). At home, constraint (15c) implies

$$L_H(Q^*_{HH}, Q^*_{HF}) = L_H. \tag{B.29}$$

Condition (6) then derives from the definition of $L_H(Q_{HH}, Q_{HF})$ and equations (B.8), (B.9), (B.10), and (B.29).

Finally, consider condition (3). Abroad, it is trivially satisfied by construction. At Home, we know from equation (16) that at the desired allocation

$$U^*_{FH}/U^*_{HH} = \eta^*((L^*_{HF}\tilde{P}^*_{FH})/(L^*_{HH}P^*_{HF})), \tag{B.30}$$

Equations (B.27) and (B.30) imply

$$U^*_{FH}/U^*_{HH} = \eta^*(\tilde{P}^*_{FH}/P_{HH}).$$

By equation (B.17), we then get

$$U^*_{FH}/U^*_{HH} = P_{FH}/P_{HH}. \tag{B.31}$$

At the desired allocation, constraint (15b) also implies

$$\tilde{P}^*_{FH} Q^*_{FH} = P^*_{HF} Q^*_{HF},$$

and in turn, using equation (B.9),

$$P_{HH} Q_{HH} + \tilde{P}^*_{FH} Q_{FH} = P_{HH} Q_{HH} + P^*_{HF} Q_{HF}. \tag{B.32}$$

Since conditions (1) and (4) hold for goods sold by home firms at home and abroad, we know that

$$P_{Hj} Q_{Hj} = N_H \int_\Phi p_{Hj}(\varphi) q_{Hj}(\varphi) dG_H(\varphi).$$

Combining this observation with equations (B.13), (B.16), (B.19), and (B.20) we get

$$\begin{aligned} P_{HH} Q_{HH} + P^*_{HF} Q_{HF} = N_H w_H (&\int_\Phi \mu_H a_{HH}(\varphi) q_{HH}(\varphi) dG_H(\varphi) \\ &+ \int_\Phi \mu_H a_{HF}(\varphi) q_{HF}(\varphi) dG_H(\varphi)). \end{aligned}$$

Since condition (5) holds at home, this can be rearranged as

$$P_{HH} Q_{HH} + P^*_{HF} Q_{HF} = N_H w_H (\textstyle\sum_{j=H,F} \int_\Phi l_{Hj}(q_{Hj}(\varphi), \varphi) dG_H(\varphi) + f^e_H).$$

Since condition (6) also holds, we then get

$$P_{HH} Q_{HH} + P^*_{HF} Q_{HF} = w_H L_H.$$

Combining this expression with equation (B.32), we obtain

$$P_{HH} Q_{HH} + \tilde{P}^*_{FH} Q_{FH} = w_H L_H. \tag{B.33}$$

Since conditions (1) and (4) hold for goods sold by foreign firms at home, we must have

$$P_{FH} Q_{FH} = N_F \int_\Phi (1 + t_{FH}(\varphi)) p_{FH}(\varphi) q_{FH}(\varphi) dG_H(\varphi),$$

which, using equation (B.14), leads to

$$P_{FH} Q_{FH} = N_F \int_\Phi \mu_F (1 + t_{FH}(\varphi)) a_{FH}(\varphi) q_{FH}(\varphi) dG_H(\varphi). \tag{B.34}$$

From the definition of $\tilde{P}^*_{FH}$ as well as equations (B.9) and (B.10), we also know that

$$\tilde{P}^*_{FH} Q_{FH} = N_F \int_\Phi \mu_F a_{FH}(\varphi) q_{FH}(\varphi) dG_F(\varphi). \tag{B.35}$$

Combining equation (B.23) with equations (B.33), (B.34), and (B.35), we finally obtain

$$P_{HH}Q_{HH} + P_{FH}Q_{FH} = w_H L_H + T_H. \tag{B.36}$$

Condition (3) at home derives from equations (B.31) and (B.36). □

C Proofs of Section 5

C.1 Lemma 6

Proof of Lemma 6. Equation (29) derives directly from utility maximization abroad. To establish equation (32), we can follow similar steps as in Section 3.1. Any solution to (28) must be such that the optimal quantity of good φ produced for country $j = H, F$ satisfies

$$q^*_{Fj}(\varphi) = \begin{cases} (\mu_F a_{Fj}(\varphi)/\lambda_{Fj})^{-\sigma_F}, & \text{if } \varphi \in \Phi_{Fj}, \\ 0, & \text{otherwise,} \end{cases} \tag{C.1}$$

with the set of varieties with non-zero output such that

$$\Phi_{Fj} = \{\varphi : \mu_F a_{Fj}(\varphi)(\mu_F a_{Fj}(\varphi)/\lambda_{Fj})^{-\sigma_F} \geq l_{Fj}((\mu_F a_{Fj}(\varphi)/\lambda_{Fj})^{-\sigma_F}, \varphi)\},$$

and the Lagrange multiplier associated with (28b) such that

$$\lambda_{Fj} = [N^*_F \int_{\Phi_{Fj}} (\mu_F a_{Fj}(\varphi))^{1-\sigma_F} dG_F(\varphi)]^{1/(1-\sigma_F)} Q_{Fj}^{1/\sigma_F}. \tag{C.2}$$

Any solution to (28) must also be such that

$$\sum_{j=H,F} \int_{\Phi_{Fj}} [\mu_F a_{Fj}(\varphi) q^*_{Fj}(\varphi) - l_{Fj}(q^*_{Fj}(\varphi), \varphi)] dG_F(\varphi) = f^e_F. \tag{C.3}$$

The comparison of equations (1), (4), (2), and (5), on the one hand, and equations (C.1), (C.2), and (C.3), on the other hand, imply that the outputs of foreign varieties and the measure of foreign entrants in the decentralized equilibrium must coincide with the solution of (28), conditional on Q_{FH} and Q_{FF}. Since the outputs of foreign varieties and the measure of foreign entrants satisfy (6), we must therefore have

$$L_F(Q_{FH}, Q_{FF}) = L_F,$$

which establishes equation (32) and implicitly defines $Q_{FF}(Q_{FH})$. Let us now turn to equation (31). From Foreign's budget constraint, we know that

$$P_{FF}Q_{FF} + P_{HF}Q_{HF} = L_F$$

Combining this expression with (32), we obtain

$$P_{HF}Q_{HF} = N_F^*(f_F^e + \sum_{j=H,F} \int_{\Phi_{Fj}} l_{Fj}(q_{Fj}^*(\varphi), \varphi)]dG_F(\varphi) - \int_{\Phi_{FF}} [\mu_F a_{FF}(\varphi) q_{FF}^*(\varphi) dG_F(\varphi))).$$

Equation (31) derives from the previous equation and equation (C.3). To conclude, note that by the Envelope Theorem, we must have

$$\partial L_F(Q_{FH}, Q_{FF})/\partial Q_{Fj} = \lambda_{Fj} Q_{Fj}^{-1/\sigma_F}/\mu_F. \tag{C.4}$$

Conditional on Q_{FH} and Q_{FF}, equations (4) (with $w_F = 1$) and (C.2) further imply that

$$\lambda_{FF} = P_{FF} Q_{FF}{}^{1/\sigma_F} \tag{C.5}$$

and

$$\lambda_{FH} = \tilde{P}_{FH} Q_{FH}{}^{1/\sigma_F}. \tag{C.6}$$

Equation (30) follows from equations (C.4)-(C.6). □

C.2 Marginal Rate of Transformation is Homogeneous of Degree Zero (Section 5.2)

In Section 5.2, we have argued that $MRT_F(Q_{FH}, Q_{FF})$ is homogeneous of degree zero. We now establish this result formally. In the proof of Lemma 6, we have already shown that the solution of (28) satisfies equations (C.1), (C.2), and (C.3). Combining these three conditions, one can check that the measure of foreign firms is such that

$$N_F(Q_{FH}, Q_{FF}) = (M_F(Q_{FH}, Q_{FF}))^{1/\mu_F},$$

with $M_F(Q_{FH}, Q_{FF})$ implicitly given by the solution to

$$M_F = \frac{\sum_{j=H,F} Q_{Fj} \mathbb{A}_{Fj}(M_F/Q_{Fj})}{(\sigma_F - 1)\left[f_F^e + \sum_{j=H,F} \mathbb{F}_{Fj}(M_F/Q_{Fj})\right]}, \tag{C.7}$$

with

$$\mathbb{A}_{Fj}(M_F/Q_{Fj}) \equiv (\int_{\Phi_{Fj}(M_F/Q_{Fj})} a_{Fj}^{1-\sigma_F}(\varphi) dG_F(\varphi))^{1/(1-\sigma_F)}, \tag{C.8}$$

$$\mathbb{F}_{Fj}(M_F/Q_{Fj}) \equiv \int_{\Phi_{Fj}(M_F/Q_{Fj})} f_{Fj}(\varphi) dG_F(\varphi), \tag{C.9}$$

and

$$\Phi_{Fj}(M_F/Q_{Fj}) \equiv \{\varphi : a_{Fj}^{1-\sigma_F}(\varphi) \geq \frac{f_{Fj}(\varphi)}{(\mu_F - 1)} \frac{M_F}{Q_{Fj}} (\int_{\Phi_{Fj}(M_F/Q_{Fj})} a_{Fj}^{1-\sigma_F}(\varphi) dG_F(\varphi))^{\mu_F}\}. \tag{C.10}$$

From equation (30), we know that

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{[\int_{\Phi_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)]^{1/(1-\sigma_F)}}{[\int_{\Phi_{FF}} (a_{FF}(\varphi))^{1-\sigma_F} dG_F(\varphi)]^{1/(1-\sigma_F)}}.$$

Using the notation above, this can be rearranged as

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{\mathbb{A}_{FH}(M_F^*(Q_{FH}, Q_{FF})/Q_{FH})}{\mathbb{A}_{FF}(M_F^*(Q_{FH}, Q_{FF})/Q_{FF})}.$$

By equation (C.7), $M_F(Q_{FH}, Q_{FF})$ is homogeneous of degree one. Together with the previous expression, this implies that $MRT_F(Q_{FH}, Q_{FF})$ is homogeneous of degree zero.

C.3 Marginal Rate of Transformation in the Pareto Case (Section 5.3)

In Section 5.3, we have argued that under the assumptions that (i) firms only differ in terms of their productivity, $f_{ij}(\varphi) = f_{ij}$, (ii) the distribution of firm-level productivity is Pareto, $a_{ij}(\varphi) = \tau_{ij}/\varphi$ with $G_F(\varphi) = 1 - (b_F/\varphi)^{\nu_F}$ for all $\varphi \geq b_F$, and (iii) there is active selection of Foreign firms in both the Foreign and Home markets, then the elasticity of transformation, κ^*, satisfies equation (41). We now establish this result formally.

The same arguments as in the proof of Lemma 6 imply

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{(\lambda_{FH} Q_{FH}^{-1/\sigma_F}/\mu_F)}{(\lambda_{FF} Q_{FF}^{-1/\sigma_F}/\mu_F)} \tag{C.11}$$

with the Lagrange multipliers such that

$$\lambda_{Fj} = [N_F^* \int_{\Phi_{Fj}} (\mu_F a_{Fj}(\varphi))^{1-\sigma_F} dG_F(\varphi)]^{1/(1-\sigma_F)} Q_{Fj}^{1/\sigma_F}$$

and the set of imported varieties such that

$$\Phi_{Fj} = \{\varphi : \mu_F a_{Fj}(\varphi)(\mu_F a_{Fj}(\varphi)/\lambda_{Fj})^{-\sigma_F} \geq l_{Fj}((\mu_F a_{Fj}(\varphi)/\lambda_{Fj})^{-\sigma_F}, \varphi)\}.$$

Under assumption (i), the set of imported varieties must be such that $\Phi_{Fj} = \{\varphi \geq \varphi_{Fj}^*\}$, with the productivity cut-off such that

$$(\mu_F - 1)(\tau_{Fj}/\varphi_{Fj}^*)^{1-\sigma_F}(\mu_F/\lambda_{Fj})^{-\sigma_F} = f_{Fj}, \tag{C.12}$$

while assumptions (ii) and (iii) imply that $\varphi^*_{Fj} \geq b_F$ and that the Lagrange multiplier must be such that

$$\lambda_{Fj} = [N^*_F \nu_F (b_F)^{\nu_F} \int_{\varphi^*_{Fj}} (\mu_F \tau_{Fj}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F -1} d\varphi]^{1/(1-\sigma_F)} Q_{Fj}^{1/\sigma_F}. \tag{C.13}$$

Equations (C.11) and (C.13) imply

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{(\int_{\varphi^*_{FH}} (\tau_{FH}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F-1} d\varphi)^{1/(1-\sigma_F)}}{(\int_{\varphi^*_{FF}} (\tau_{FF}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F-1} d\varphi)^{1/(1-\sigma_F)}}, \tag{C.14}$$

whereas equations (C.12) and (C.13) imply

$$\varphi^*_{Fj} = \frac{\tau_{Fj}(f_{Fj}/(\mu_F - 1))^{1/(\sigma_F-1)} Q_{Fj}^{1/(1-\sigma_F)}}{[N^*_F \nu_F (b_F)^{\nu_F} \int_{\varphi^*_{Fj}} (\tau_{Fj}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F-1} d\varphi]^{\sigma_F/((\sigma_F-1)(1-\sigma_F))}}.$$

We can use the last expression to solve for φ^*_{Fj}. We obtain

$$\varphi^*_{Fj} = \frac{\tau_{Fj}^{(\sigma_F-1)/((\sigma_F-1)-\nu_F\sigma_F)} (f_{Fj}/(\mu_F-1))^{(1-\sigma_F)/((\sigma_F-1)-\nu_F\sigma_F)} Q_{Fj}^{(\sigma_F-1)/((\sigma_F-1)-\nu_F\sigma_F)}}{[N^*_F \nu_F (b_F)^{\nu_F} / (\nu_F - (\sigma_F - 1))]^{\sigma_F/((\sigma_F-1)-\nu_F\sigma_F)}},$$

and, in turn,

$$(\int_{\varphi^*_{Fj}} (\tau_{Fj}/\varphi)^{1-\sigma_F} \varphi^{-\nu_F-1} d\varphi)^{1/(1-\sigma_F)} = \frac{\tau_{Fj}^{\frac{\nu_F(\sigma_F-1)}{\nu_F\sigma_F-(\sigma_F-1)}} (f_{Fj}/(\mu_F-1))^{\frac{\nu_F-(\sigma_F-1)}{\nu_F\sigma_F-(\sigma_F-1)}} Q_{Fj}^{-\frac{\nu_F-(\sigma_F-1)}{\nu_F\sigma_F-(\sigma_F-1)}}}{[N^*_F \nu_F (b_F)^{\nu_F}]^{\frac{\sigma_F(\sigma_F-\nu_F-1)}{(1-\sigma_F)((\sigma_F-1)-\nu_F\sigma_F)}} (\nu_F - (\sigma_F-1))^{-\frac{\sigma_F-1}{\nu_F\sigma_F-(\sigma_F-1)}}}.$$

Substituting into equation (C.14) leads to

$$MRT_F(Q_{FH}, Q_{FF}) = (\tau_{FH}/\tau_{FF})^{\frac{\nu_F(\sigma_F-1)}{\nu_F\sigma_F-(\sigma_F-1)}} (f_{FH}/f_{FF})^{\frac{\nu_F-(\sigma_F-1)}{\sigma_F\nu_F-(\sigma_F-1)}} (Q_{FH}/Q_{FF})^{-\frac{\nu_F-(\sigma_F-1)}{\sigma_F\nu_F-(\sigma_F-1)}}.$$

For $\nu_F = \nu$ and $\sigma_F = \sigma$, the previous expression and equation (34) imply equation (41).

C.4 Lemma 7

Proof of Lemma 7. In Section C.2, we have established that

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{\mathbb{A}_{FH}(M_F(Q_{FH}, Q_{FF})/Q_{FH})}{\mathbb{A}_{FF}(M_F(Q_{FH}, Q_{FF})/Q_{FF})}.$$

with M_F, $\mathbb{A}_{FH}$, and $\mathbb{A}_{FF}$ implicitly determined by equations (C.7)-(C.10). Taking log and totally differentiating the previous expression with respect to Q_{FH}, we get

$$\frac{d \ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d \ln Q_{FH}} = \epsilon^{\mathbb{A}}_{FH}(-(1-\epsilon^M_{FH}) + \epsilon^Q_F \epsilon^M_{FF}) + \epsilon^{\mathbb{A}}_{FF}(-\epsilon^M_{FH} + \epsilon^Q_F(1-\epsilon^M_{FF})),$$

with

$$\epsilon_{Fj}^{\mathbb{A}} = \frac{d\ln \mathbb{A}_{Fj}(M_F/Q_{Fj})}{d\ln(M_F/Q_{Fj})} \geq 0,$$
$$\epsilon_{F}^{Q} = \frac{d\ln Q_{FF}(Q_{FH})}{d\ln Q_{FH}} < 0,$$
$$\epsilon_{Fj}^{M} = \frac{\partial \ln M_F^*(Q_{FH}, Q_{FF})}{\partial \ln Q_{Fj}},$$

where the non-negativity of $\epsilon_{Fj}^{\mathbb{A}}$ directly follows from equations (C.8) and (C.10). In Section C.2, we have already argued that $M_F(Q_{FF}, Q_{FH})$ is homogeneous of degree one. Thus, we must have $\epsilon_{FH}^{M} + \epsilon_{FF}^{M} = 1$, which leads to

$$\frac{d\ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d\ln Q_{FH}} = \left(\epsilon_{FF}^{\mathbb{A}}\epsilon_{FH}^{M} + \epsilon_{FH}^{\mathbb{A}}\epsilon_{FF}^{M}\right)(\epsilon_F^Q - 1). \tag{C.15}$$

Since $\epsilon_F^Q - 1 < 0$, $\epsilon_{Fj}^{\mathbb{A}} \geq 0$, and (by assumption) $\epsilon_{FH}^{M}, \epsilon_{FF}^{M} \geq 0$ with $\epsilon_{FH}^{M} + \epsilon_{FF}^{M} = 1$, we can conclude that if selection is active in at least one market, $\epsilon_{FF}^{\mathbb{A}} > 0$ or $\epsilon_{FH}^{\mathbb{A}} > 0$, then

$$\frac{d\ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d\ln Q_{FH}} < 0,$$

which is equivalent to $\kappa^* < 0$ by equation (34). □

C.5 Locally Convex Production Sets with Selection (Section 5.4)

The goal of this subsection is to construct an economy where: (i) the number of entrants in Foreign is strictly decreasing with aggregate output in one market and (ii) Foreign's production set is locally convex.

Suppose that firms in Foreign differ only in terms of their productivity, $a_{ij}(\varphi) = \tau_{ij}/\varphi$ and $f_{ij}(\varphi) = f_{ij}$ for all φ. We assume that there are no iceberg trade costs, $\tau_{FF} = \tau_{FH} = 1$, that fixed exporting costs are equal to zero, $f_{FH} = 0$, but that fixed costs of selling domestically are not, $f_{FF} > 0$. Starting from these assumptions and the characterization of the solution to (28)—equations (C.1)-(C.3) in the proof of Lemma 6—we can follow the same strategy as in Section C.2 and write $M_F(Q_{FH}, Q_{FF}) = (N_F(Q_{FH}, Q_{FF}))^{\mu_F}$ as the implicit solution of

$$M_F = \frac{Q_{FH}\mathbb{A}_{FH} + Q_{FF}\mathbb{A}_{FF}(M_F/Q_{FF})}{(\sigma_F - 1)\left[f_F^e + \mathbb{F}_{FF}(M_F/Q_{FF})\right]}, \tag{C.16}$$

with

$$\mathbb{A}_{FH} = (\int_{\Phi} \varphi^{\sigma_F - 1} dG_F(\varphi))^{1/(1-\sigma_F)}, \tag{C.17}$$

$$\mathbb{A}_{FF}(M_F/Q_{FF}) = (\int_{\varphi^*_{FF}} \varphi^{\sigma_F - 1} dG_F(\varphi))^{1/(1-\sigma_F)}, \tag{C.18}$$

$$\mathbb{F}_{FF}(M_F/Q_{FF}) = f_{FF}(1 - G_F(\varphi^*_{FF})), \tag{C.19}$$

and the productivity cut-off for foreign firms in their domestic market such that

$$(\varphi^*_{FF})^{\sigma_F - 1} = \frac{f_{FF} M_F}{(\mu_F - 1) Q_{FF}} (\int_{\varphi^*_{FF}} \varphi^{\sigma_F - 1} dG_F(\varphi))^{\mu_F}. \tag{C.20}$$

By equation (C.16), a sufficient condition for M_F to be decreasing in Q_{FH} is that

$$\epsilon^{\mathbb{A}}_{FF} \frac{\mathbb{A}_{FF}(M_F/Q_{FF})}{(Q_{FH}/Q_{FF})\mathbb{A}_{FH} + \mathbb{A}_{FF}(M_F/Q_{FF})} - \epsilon^{\mathbb{F}}_{FF} \frac{\mathbb{F}_{FF}(M_F/Q_{FF})}{f^e_F + \mathbb{F}_{FF}(M_F/Q_{FF})} > 1,$$

with $\epsilon^{\mathbb{A}}_{FF} = d \ln \mathbb{A}_{FF}(M_F/Q_{FF})/d \ln(M_F/Q_{FF}) \geq 0$ and $\epsilon^{\mathbb{F}}_{FF} \equiv d \ln \mathbb{F}_{FF}(M_F/Q_{FF})/d \ln(M_F/Q_{FF})$. In the limit where Home is small relative to Foreign, $Q_{FH}/Q_{FF} \to 0$, and fixed entry costs are small relative to the fixed cost of selling domestically, $f^e_F/\mathbb{F}_{FF}(M_F/Q_{FF}) \to 0$, the previous condition reduces to

$$\epsilon^{\mathbb{A}}_{FF} - \epsilon^{\mathbb{F}}_{FF} > 1. \tag{C.21}$$

We will now provide sufficient conditions on G_F such that the previous inequality holds. By equation (C.20), we know that

$$\epsilon_{FF} = \frac{1}{\sigma_F - 1 + \frac{\mu_F \varphi^*_{FF} g_F(\varphi^*_{FF})}{\int_{\varphi^*_{FF}} (\varphi/\varphi^*_{FF})^{\sigma_F - 1} dG_F(\varphi)}},$$

with $\epsilon_{FF} \equiv d \ln \varphi^*_{FF}(M_F/Q_{FF})/d \ln(M_F/Q_{FF})$. Combining this expression with equations (C.18) and (C.19), we get

$$\epsilon^{\mathbb{A}}_{FF} = \frac{1 - (\sigma_F - 1)\epsilon_{FF}}{\sigma_F},$$

$$\epsilon^{\mathbb{F}}_{FF} = -\frac{(1 - (\sigma_F - 1)\epsilon_{FF}) \int_{\varphi^*_{FF}} (\varphi/\varphi^*_{FF})^{\sigma_F - 1} dG_F(\varphi)}{\mu_F(1 - G_F(\varphi^*_{FF}))}.$$

and, in turn,

$$\epsilon^{\mathbb{A}}_{FF} - \epsilon^{\mathbb{F}}_{FF} = (\frac{1 - (\sigma_F - 1)\epsilon_{FF}}{\sigma_F})(1 + \frac{(\sigma_F - 1) \int_{\varphi^*_{FF}} (\varphi/\varphi^*_{FF})^{\sigma_F - 1} dG_F(\varphi)}{1 - G_F(\varphi^*_{FF})}).$$

Hence, the sufficient condition (C.21) can be rearranged as

$$\frac{\int_{\varphi^*_{FF}} (\varphi/\varphi^*_{FF})^{\sigma_F-1} dG_F(\varphi)}{1-G_F(\varphi^*_{FF})} - 1 > \frac{(\sigma_F-1)\int_{\varphi^*_{FF}} (\varphi/\varphi^*_{FF})^{\sigma_F-1} dG_F(\varphi)}{\varphi^*_{FF} g_F(\varphi^*_{FF})}.$$

Now pick G_F with finite support, $[\underline{\varphi}, \bar{\varphi}]$, and density bounded from below, $g_F(\varphi) > b > 0$, and set $\frac{f_{FF}}{(\mu_F-1)}$ such that given equation (C.20), φ^*_{FF} converges to $\bar{\varphi}$. For φ^*_{FF} close enough to $\bar{\varphi}$, the previous inequality must be satisfied. At this point, we have established that there exist sufficient conditions under which $M_F(Q_{FH}, Q_{FF})$ and hence $N_F(Q_{FH}, Q_{FF})$ is strictly decreasing in Q_{FH}. To conclude, recall that by equation (C.15), we must have

$$\frac{d \ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d \ln Q_{FH}} = \left(\epsilon^{\mathbb{A}}_{FF}\epsilon^M_{FH} + \epsilon^{\mathbb{A}}_{FH}\epsilon^M_{FF}\right)(\epsilon^Q_F - 1).$$

In the present economy, equations (C.17) and (C.18) imply $\epsilon^{\mathbb{A}}_{FF} > 0$ and $\epsilon^{\mathbb{A}}_{FH} = 0$. We have just provided sufficient conditions under which $\epsilon^M_{FH} < 0$. Since $\epsilon^Q_F - 1 < 0$, we therefore obtain

$$\frac{d \ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d \ln Q_{FH}} > 0,$$

which concludes our proof.

D Proofs of Section 6

D.1 Foreign's Offer Curve (Section 6.1)

The additional constraints for Home's macro problem arising in the multi-sector economy are Foreign's resource constraint,

$$\sum_k L^k_F = L_F, \tag{D.1}$$

and the optimality of foreign consumption across sectors,

$$(\partial U_F/\partial U^{k_1}_F)/(\partial U_F/\partial U^{k_2}_F) = P^{k_1}_F/P^{k_2}_F, \text{ for all } k_1, k_2, \tag{D.2}$$

$$\sum_k E^k_F = L_F, \tag{D.3}$$

where $P^k_F \equiv \min_{Q_{HF},Q_{FF}}\{P^k_{HF}Q_{HF} + P^k_{FF}Q_{FF} | U^k_F(Q_{HF}, Q_{FF}) \geq 1\}$ is the foreign price index in sector k, with Foreign's expenditure in that sector such that $E^k_F = P^k_F U^k_F$. These considerations lead to a new offer curve that corresponds to all the vectors of imports and exports, $\mathbf{Q_{FH}} \equiv (Q^1_{FH}, ..., Q^K_{FH})$ and $\mathbf{Q_{HF}} \equiv (Q^1_{HF}, ..., Q^K_{HF})$, such that

$$Q^k_{FH}(Q^k_{HF}, E^k_F, L^k_F) \geq Q^k_{FH} \text{ for all } k,$$

for some vectors of expenditure and employment, $\mathbf{E_F} \equiv (E_F^1, ..., E_F^K)$ and $\mathbf{L_F} \equiv (L_F^1, ..., L_F^K)$, satisfying conditions (D.1)-(D.3).

D.2 Domestic Taxes (Section 6.2)

The goal of this subsection is to show that if the assumptions of Section 6.2 hold then,

$$(1 + \bar{t}_{HH}^D)/(1 + \bar{s}_{HH}^D) = 1/\mu_H^D. \tag{D.4}$$

The first-order conditions associated with Home's relaxed planning problem imply

$$MRS_H^{HO} = L_{HH}^D, \tag{D.5}$$

with $MRS_H^{HO} \equiv (\partial U_H/\partial Q_{HH}^D)/(\partial U_H/\partial U_H^O)$ the marginal rate of substitution for Home between Home's differentiated good and the homogeneous good and $L_{HH}^D \equiv \partial L_H^D/\partial Q_{HH}^D$ the marginal cost of aggregate output for the local market at home. Like in Section 4.3, one can use the Envelope Theorem to show that

$$L_{HH}^D = (\int_{\Phi_{HH}^D} N_H^D (a_{HH}(\varphi))^{1-\sigma_H^D} dG_H^D(\varphi))^{1/(1-\sigma_H^D)}. \tag{D.6}$$

In the decentralized equilibrium with taxes, utility maximization at home implies

$$MRS_H^{HO} = P_{HH}^D, \tag{D.7}$$

with the aggregate price index such that

$$P_{HH}^D = (\int_{\Phi_{HH}^D} N_H^D ((1 + \bar{t}_{HH}^D)\mu_H^D a_{HH}(\varphi)/(1 + \bar{s}_{HH}^D))^{1-\sigma_H^D} dG_H^D(\varphi))^{1/(1-\sigma_H^D)}. \tag{D.8}$$

Equations (D.5)-(D.8) imply that in order to implement the solution of Home's relaxed planning problem, domestic taxes must be such that equation (D.4) holds.

D.3 Trade Taxes without Active Selection (Section 6.2)

The goal of this subsection is to establish equations (48) and (49) under the assumption that there is no active selection. We first compute Home's terms-of-trade elasticities within the differentiated sector, $\rho_{HF}^D \equiv \partial \ln P^D/\partial \ln Q_{HF}^D$ and $\rho_{FH}^D \equiv \partial \ln P^D/\partial \ln Q_{FH}^D$. Since $P^D \equiv P_{HF}^D/\tilde{P}_{FH}^D$, still satisfies $P^D = MRS_F^D/MRT_F^D$, ρ_{HF}^D and ρ_{FH}^D must satisfy the counterparts of equations (38) and (39),

$$\rho_{HF}^D = -1/\epsilon^D, \tag{D.9}$$

$$\rho_{FH}^D = -(1/r_{FF}^D - 1)/\epsilon^D - 1/(r_{FF}^D \kappa^D), \tag{D.10}$$

where ϵ^D and κ^D denote the elasticities of substitution and transformation, respectively, within the differentiated sector in Foreign and $r^D_{FF} \equiv P^D_{FF}Q^D_{FF}/(P^D_{FF}Q^D_{FF} + \tilde{P}^D_{FH}Q^D_{FH})$ denote Foreign's domestic share of revenue in the differentiated good.[31] In the absence of active selection, Foreign's production possibility frontier for the differentiated sector is linear, $\kappa^D \to \infty$, so equation (D.10) simplifies into

$$\rho^D_{FH} = -(1/r^D_{FF} - 1)/\epsilon^D. \tag{D.11}$$

Now, consider $\rho^D_X \equiv \partial \ln P^D / \partial \ln X^O_H$. The same steps used to compute η^D_{FH} implies

$$\rho^D_X = (d \ln Q^D_{FF} / d \ln X^O_H)/\epsilon^D. \tag{D.12}$$

In the decentralized equilibrium abroad, we know that

$$Q^D_{FF} = (\beta_F L_F + X^O_H)/P^D_{FF} - (\tilde{P}^D_{FH}/P^D_{FF})Q^D_{FH}$$

with price indices such that

$$\begin{aligned}
P^D_{FF} &= (\int_{\Phi_{FF}} N^D_F (\mu^D_F a_{FF}(\varphi))^{1-\sigma^D_F} dG^D_F(\varphi))^{1/(1-\sigma^D_F)}, \\
\tilde{P}^D_{FH} &= (\int_{\Phi_{FH}} N^D_F (\mu^D_F a_{FH}(\varphi))^{1-\sigma^D_F} dG^D_F(\varphi))^{1/(1-\sigma^D_F)}, \\
N^D_F &= \frac{\beta_F L_F + X^O_H}{\sigma^D_F [f^{e,D}_F + \sum_{j=H,F} \int_{\Phi_{Fj}} f_{Fj}(\varphi) dG^D_F(\varphi))]}.
\end{aligned}$$

In the absence of active selection, we can treat Φ_{FF} and Φ_{FH} as fixed. Thus, the previous equations imply

$$d \ln Q^D_{FF} / d \ln X^O_H = (\mu^D_F X^O_H)/(P^D_{FF}Q^D_{FF}).$$

Combining this expression with equation (D.12), we obtain

$$\rho^D_X = (\mu^D_F X^O_H)/(\epsilon^D P^D_{FF} Q^D_{FF}). \tag{D.13}$$

Finally, consider $\zeta_{FH} \equiv \partial \ln P^D_{FH} / \partial \ln Q^D_{FH}$ and $\zeta_X \equiv \partial \ln P^D_{FH} / \partial \ln X^O_H$. In the absence of active selection, we must have

$$\zeta_{FH} = 0, \tag{D.14}$$

$$\zeta_X = \frac{1}{1-\sigma^D_F} \frac{X^O_H}{(P^D_{FF}Q^D_{FF} + P^D_{FH}Q^D_{FH})}. \tag{D.15}$$

[31] In Section 5.2, we have expressed ρ_{FH} as a function of the expenditure share, $x_{FF} \equiv P_{FF}Q_{FF}/L_F$. It should be clear that with only one sector, shares of revenues and expenditures are equal by trade balance.

Combining equations (46) and (47) with equations (D.9), (D.11), (D.13), (D.14), and (D.15), we obtain

$$(1-\Delta)/\eta^D = 1 + \frac{1}{(\epsilon^D - 1)x^D_{FF}},$$
$$\Delta/\eta^O = 1 - \frac{(1-r^D_{FF})((1-\Delta)/r^D_{FF} + \Delta\epsilon^D)}{\epsilon^D(\sigma^D_F - 1) + (1-r^D_{FF})(\sigma^D_F(1-\Delta)/r^D_{FF} + \Delta\epsilon^D)},$$

where the first expression uses the fact foreign expenditure and revenue shares are related through $(1/x^D_{FF} - 1) = (1/r^D_{FF} - 1)(1-\Delta)$. Equations (48) and (49) derive from equations (44) and (45) and the two previous expressions.

D.4 Trade Taxes in a Small Open Economy (Section 6.2)

The goal of this subsection is to establish equations (50) and (51) under the assumption that Home is a small open economy. If Home is a small open economy, then $\zeta_X = \rho^D_X = 0$ and $\zeta_{FH} = 1/\kappa^D$. In addition, setting $r^D_{FF} = 1$ in equation (D.10), we obtain $\rho^D_{FH} = -1/\kappa^D$. The last elasticity, ρ^D_{HF}, is unaffected by the fact that Home is a small open economy: $\rho^D_{HF} = -1/\epsilon^D$ by equation (D.9). Combining the previous observations with equations (46) and (47), we get

$$(1-\Delta)/\eta^D = 1 + (1+\epsilon^D/\kappa^D)/(\epsilon^D - 1),$$
$$\Delta/\eta^O = 1 + 1/\kappa^D.$$

Equations (50) and (51) derive from equations (44) and (45) and the two previous expressions.