Advanced Game Theory

4. Games of incomplete information

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Games of incomplete information

Definition

A **game of incomplete information** is a game in which players do not know all the payoffs.

Classification of games of incomplete information: private vs common values

Simplest case: independent private value

Private values each player knows his/her own payoff, but does not know his/her opponent's payoff.

Independent private values each player's belief regarding the other players' payoff is independent on his/her own payoff.

- Suppose that, *before learning his/her payoff*, player A has a belief regarding player's B payoff.
- Now player A learns his/her payoff.
- *Independence of private value* implies that the A's beliefs regarding B's payoff is unchanged.

Simplest case: independent private value

- Example: auctioning off a dinner.

More complex case: common private values

- Example: auctioning off unproven oil fields.
- Each person bidding have the same valuation (but may be differentially informed about it).

How to model incomplete information

Modeling approach

1. Parametrize player *i*'s possible payoffs by a variable called *i*'s **type.**
2. Specify beliefs (knowledge) by a **probability distribution** over i's type.

Analyzing the game:

- What is a **strategy** for player i?
  - **A mapping of types into actions!**
- *Equilibrium: strategies that are mutual best responses.*

Example: war of attrition.

- Two firms, each with a valuation *V* randomly drawn between $100 million and $800 million,
- Choice: how long to stay in the game. Each year in the game "costs" 100 million,
- The last firm in the market earns its valuation,
- Questions:
  - what are the "types"?
  - what are "strategies"?
  - what is an equilibrium?

Third Predicament of Game theory:

you know too much!

you have to think how you would play *for each type you could be*

Entry game with unknown costs

Entry game

- Two firms, one managed by Anna, the other by Boris.
- each of them decides simultaneously whether to enter a new market.
- Anna's fixed cost of production C_a, known only to Anna.
- Boris' fixed cost of production C_b, known only to Boris.
  - both firms have zero marginal costs.

- Anna's **type** is C_a; Boris's **type** is C_b.
- A **strategy** is a decision to enter or not, **as a function of own cost**.

Each player knows his/her cost, but it has to derive its optimal action for every possible cost!

Each firm has beliefs over the other firm's cost

- Anna's beliefs about Boris' cost: C_b is distributed uniformly over $[0, 4]$.
- Boris' beliefs about Anna's cost: C_a is distributed uniformly over $[0, 4]$.

Entry game

		Boris	
		out	*in*
Anna	*out*	0,0	$0, 3 - C_b$
	in	$3 - C_a, 0$	$1 - C_a, 1 - C_b$

- When does Boris have a dominant action (for some C_b)?
- When does Anna have a dominant action (for some C_a)?

Cut-off strategies

- Suppose Boris enters for a cost equal to 2.4
- Will he enter for costs lower than 2.4?
- Equilibrium strategies can be expressed as **cutoff levels**
  - Enter if your own cost is below a threshold.
- To solve for the equilibrium, we need to find those cutoffs.

Boris' best response

- Suppose Boris **believes that Anna will enter if her cost is below $\bar{C}_a$.**
- The **probability that Anna enters** is

$$\text{pr}\{C_a < \bar{C}_a\} = \frac{\bar{C}_a}{4}$$

- Given this, should Boris enter?
  - Expected profits if he enters:

$$\text{pr}\{C_a < \bar{C}_a\}(1 - C_b) + (1 - \text{pr}\{C_a < \bar{C}_a\})(3 - C_b) = 3 - C_b - \frac{\bar{C}_a}{2}$$

  - Expected profits if he does not enter: 0
- **Boris' best response**: enter if $C_b < 3 - \frac{\bar{C}_a}{2}$
- i.e. Boris plays a cutoff strategy with cutoff $3 - \frac{\bar{C}_a}{2}$

Anna's best response

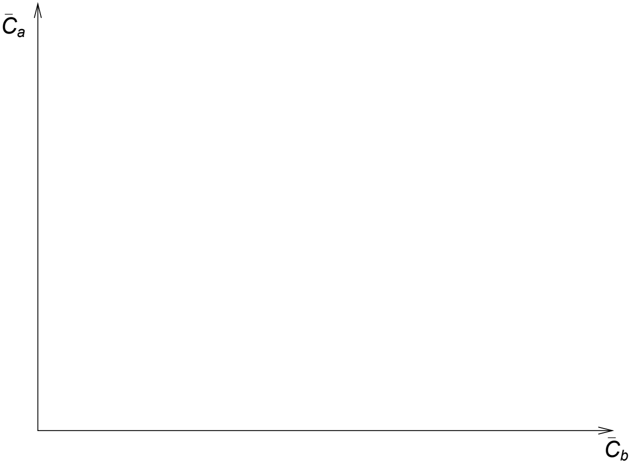
- Suppose Anna **believes that Boris will enter if his costs are below $\bar{C}_b$.**
- The **probability that Boris enters** is

$$\text{pr}\{C_b < \bar{C}_b\} = \frac{\bar{C}_b}{4}$$

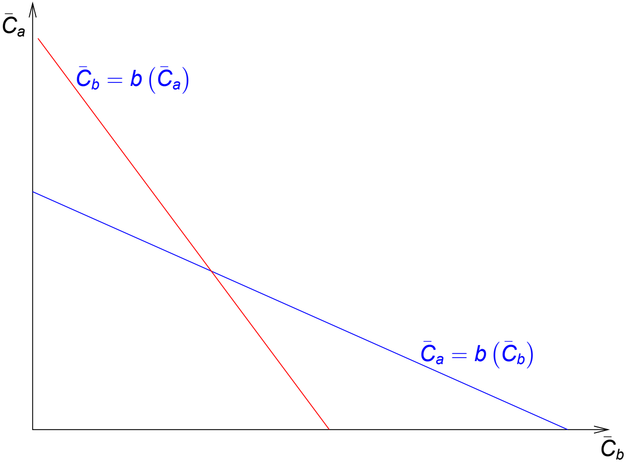
- **Anna's best response**: enter if $C_a < 3 - \frac{\bar{C}_b}{2}$

- i.e. Anna plays a cutoff strategy with cutoff $3 - \frac{\bar{C}_b}{2}$

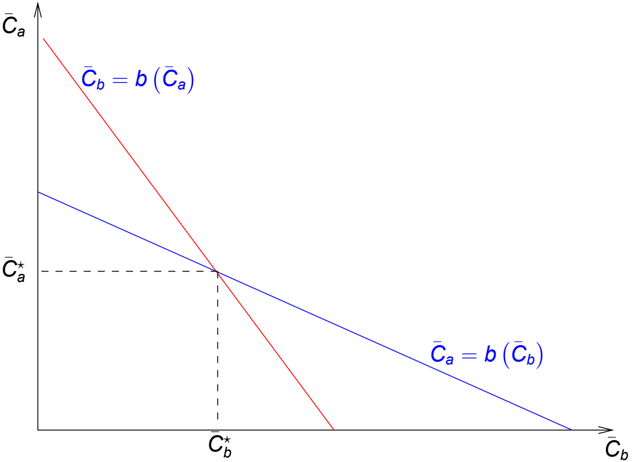
Reaction curves



Reaction curves



The equilibrium



The equilibrium

- We can find the equilibrium by solving for:

$$\bar{C}_b^\star = 3 - \frac{\bar{C}_a^\star}{2}$$

$$\bar{C}_a^\star = 3 - \frac{\bar{C}_b^\star}{2}$$

- Solution:

$$\bar{C}_a^\star = \bar{C}_b^\star = 2$$

In class game: bargaining with patient / impatient types

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Game of alternating offer between two players

- Total amount to be split is $100
- In every period, one of the two players proposes a given split to the other player.
- If an offer is accepted, the game ends.
- If an offer is rejected, the other player can propose a given split,
  - and so on
- The game lasts 10 rounds.
  - if the game ends with no agreement, each player earns zero.

In class game: bargaining with patient / impatient types

- Two types of players

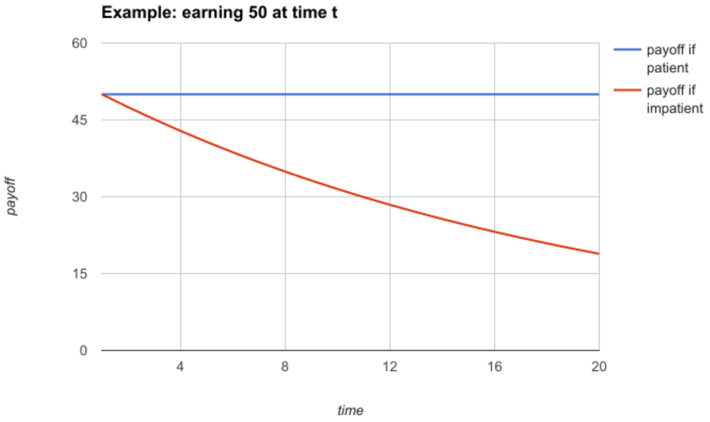
  Impatient player earning x after t rounds of negotiation is worth $x \cdot (0.95)^t$

  - 95% is the *discount factor*

  Patient player no cost of waiting

- There is a 20% probability that a player is patient.
- Each player knows his type but does not know the opponent's type.

In class game: bargaining with patient / impatient types



In class game: instructions

- Turn on your phone/tablet/laptop (exceptionally!)
- Open the email you received from me. Look up your number and whether you are patient or impatient. **Do not tell anybody about it!**
- Open a browser and connect to **http://bit.ly/2plsM05** QR code
- Find your game (i.e. look for your number in column A).
- Suppose you are number x and you play against number y in the game “x & y”
  - Number x makes the first offer. **An offer is the fraction of surplus that accrue to player x.**
  - Player y accept or rejects. If he rejects, then he makes an offer. **An offer is the fraction of surplus that accrues to player x.**
  - And so on
- Let me know when you are ready...
- ... you have 10 min to complete the game.

In class game: discussion

- What was your strategy?
- Would you have played differently if the discount factor for the impatient type was 99%?
- Who got the highest score?

Stats for today's game

War of attrition

- Two firms, each with a valuation V randomly drawn between \$100 million and \$800 million,
- Choice: how long to stay in the game. Each year in the game “costs” 100 million,
- The last firm in the market earns its valuation,
- Question: given your assigned valuation, how do you play?

Discussion points for the war of attrition

1. How would you articulate in words the trade-off that you face in waiting one extra period?
2. How does this trade off change with your valuation?
   - Therefore, how does waiting time depend on valuation?
3. How does this trade off change over time?
4. Does it make sense to fight for a short amount of time?

Your strategies