problem set 6

1

a) Work done $= \int F\,dx$ = Area under F vs x curve.

$W_{0\to 2m} \approx \underline{3.2\,J}$.

b) $W_{net} = \Delta K$ $\qquad \Delta K = K_f - K_i$

$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$K_f - \frac{1}{2}(3)(1.5)^2 = 3.2\,J$

$K_f = \underline{6.58\,J}$.

c) $K_f = \frac{1}{2}mv_f^2 \qquad v_f = \sqrt{\frac{2(6.58)}{3}} = \underline{2.09}\ m/s$

d) $W_{0\to 4} = \underline{5.8}\ J$

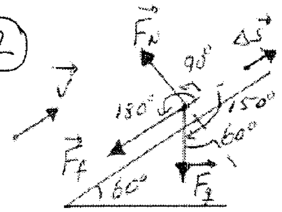
e) $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3)(1.5)^2 = 3.37\,J = 3.4\,J$

Gain in energy from $0 \to 4\,m = 5.8\,J$

Total $E = 5.8 + 3.4 = 9.2\,J \qquad = \frac{1}{2}mv_f^2$

$V_f = \underline{2.5}\ m/s$.

(2)



$m = 2\,kg$
$V_i = 3\ m/s$
$V_f = 0$
$\mu = 0.3$
$\Delta S = ?$
$W = F(\cos\theta)\Delta S$

(a) F_N = Normal force
F_g = Weight = $mg = \underline{20}$
F_f = Friction
$= (\mu mg \cos 60^\circ)$
$= (0.3)(20)\cos 60^\circ$
$= \underline{3N}$

$\boxed{F_f = 3N}$

(b) $W_N = F_N(\cos 90^\circ)\Delta S = \underline{0}$

$W_g = mg(\cos 150^\circ)\Delta S = 20(\cos 150^\circ)\Delta S = \underline{(-17.32)\Delta S}$

$W_f = F_f(\cos 180^\circ)\Delta S = 3(\cos 180^\circ)\Delta S = \underline{(-3)\Delta S}$

$\boxed{W_T = \Delta K}$

$W_g + W_N + W_f = \frac{1}{2}m(V_f^2 - V_i^2)$

$(-17.32\,\Delta S) + 0 + (-3.00\,\Delta S) = \frac{1}{2}(2)(0-9)$

$\therefore\ \Delta S = \frac{-9}{-20.32} = \boxed{0.443\,m}$ up the plane.

(a)(cont.) $W_g = (-17.32)\Delta S = -17.32\,(0.443) = -\underline{7.67J}$

$W_f = (-3)\Delta S = -3\,(0.443) = -\underline{1.33J}$

(C) Down incline:

$V_i = 0$

$\Delta S = 0.443m$ (from (b))

$\boxed{W_T = \Delta K}$

$W_g + W_N + W_f = \frac{1}{2}m(V_f^2 - V_i^2)$

where:

$W_g = mg(\cos 30^\circ)\Delta S$
$= 20(\cos 30^\circ)0.443$
$= \underline{7.67J}$

$W_f = -(\mu mg \cos 60^\circ)\Delta S$
$= -(.3)(20)\cos 60^\circ(0.443)$
$= -\underline{1.329}\ J$

$W_N = \underline{0}$

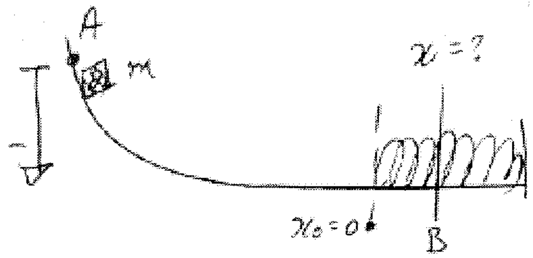
(d) $\boxed{W_T = \Delta K}$

$7.67 - 1.33 = \frac{1}{2}(2)(V_f^2 - 0)$

$V_f = \sqrt{6.34}$

$= \boxed{2.52\,m/s}$, at initial position

3



(a) $\boxed{W_T = \Delta K}$ (A → B, on block)

$W_g + W_s = \frac{1}{2}m(V_B^2 - V_A^2)$

$(-mg\Delta y - \frac{1}{2}kx^2) = 0$

$(-30)(-5) - \frac{1}{2}(400)x^2 = 0$

$x = \sqrt{\frac{150}{200}} = \boxed{0.866\,m}$

$V_A = V_B = 0$

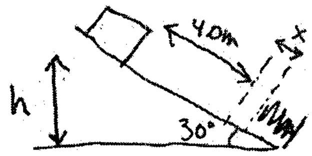
$\Delta y = -5\,m$
$m = 3\,kg$
$k = 400\,N/m$

(b) After coming to rest at B, the block will be projected to the left by the spring and it will continue along and up the ramp to its initial position, because there is no friction.

4 The orbiting astronaut and his spacecraft are both pulled toward the Earth by the force of gravity, therefore they both have weight $\left(W = F_G = G\frac{M_1M_2}{r^2}\right)$. (If they weren't pulled toward the Earth, they would not orbit – they would continue on a straight line path tangent to the orbit!) Their weight is slightly less than it is at the surface of the Earth because the separation of the centres of the masses is larger. (You should verify that an 80.0 kg astronaut on the surface of the Earth ($r = 6.38 \times 10^6$ m) has a weight of 784 N; the same astronaut orbiting 400 km above the surface ($r = 6.78 \times 10^6$ m) has a weight of 693 N.) In fact, there is no place in the Universe that is "beyond the pull of Earth's gravity", although the pull gets weaker and weaker as you travel further and further away $\left(F \propto \frac{1}{r^2}\right)$.

The astronaut 'feels' weightless because his or her apparent weight (the force which a scale would read) is zero – the astronaut is in free-fall, along with the spacecraft, and there is nothing pushing 'up' on the astronaut. A person standing in a freely falling elevator would have exactly the same sensation.

Question 5.



a) surface is frictionless

$$E_{initial} = E_{final}$$

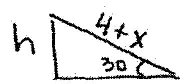
$$U_g + K + U_s = U_g + K + U_s$$

$$mgh + \tfrac{1}{2}mv^2 + 0 = 0 + 0 + \tfrac{1}{2}kx^2$$

"released" implies 0

choose 0 at bottom

since block moves a total distance of $(4+x)$ down the incline, we have.



$$h = (4+x)\sin 30$$

$$mg(4+x)\sin 30 + 0 + 0 = 0 + 0 + \tfrac{1}{2}kx^2$$

$$2(9.8)(4+x)\tfrac{1}{2} = \tfrac{1}{2}(100)x^2$$

$$39.2 + 9.8x - 50x^2 = 0$$

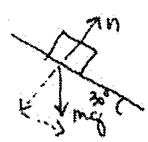
$$x = \frac{-9.8 \pm \sqrt{(9.8)^2 - 4(-50)(39.2)}}{2(-50)}$$

$$x = \frac{-9.8 \pm 89.1}{-100} = 0.989\text{m} \text{ or } -0.793\text{m}$$

b) with friction along incline

$$E_{initial} - W_{friction} = E_{final}$$

$$U_g + K + U_s - F_f d = U_g + K + U_s$$



$F_{net} = 0$

$n - mg\cos 30 = 0$

$n = mg\cos 30$

$$mg(4+x)\sin 30 + 0 + 0 - \mu(mg\cos 30)(4+x) = 0 + 0 + \tfrac{1}{2}kx^2$$

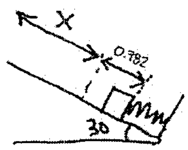
$$2(9.8)(4+x)\tfrac{1}{2} - 0.2(2)(9.8)(0.866)(4+x) = \tfrac{1}{2}(100)x^2$$

$$39.2 + 9.8x - 13.58 - 3.39x - 50x^2 = 0$$

$$25.6 + 6.4x - 50x^2 = 0$$

$$x = \frac{-6.4 \pm \sqrt{(6.4)^2 - 4(-50)(25.6)}}{2(-50)} = \frac{-6.4 \pm 71.8}{-100} = 0.782\text{m} \text{ or } -0.654\text{m}$$

c) how far up incline ie. what distance until it stops.



$$E_{initial} - W_f = E_{final}.$$

$$U_g + K + U_s - F_f d = U_g + K + U_s.$$

$$0 + 0 + \tfrac{1}{2}(100)(0.782)^2 - \mu m g \cos 30 (x + 0.782) = mg(x + 0.782)\sin 30 + 0 + 0$$

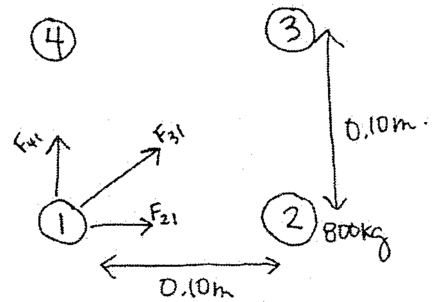
$$30.6 - 0.2(2)(9.8)(0.866)(x + 0.782) = 2(9.8)(x + 0.782)\tfrac{1}{2}$$

$$30.6 - 3.39x - 2.65 = 9.8x + 7.66$$

$$20.29 = 13.19x$$

$$\boxed{x = 1.54\,m}$$

Question 6.



consider the mass labelled ①. It will experience a force of attraction from each of the spheres 2, 3 and 4. Arrows on the diagram above show the direction of each force (along the lines connecting their centers)

first lets find the magnitude of each force.

$$F_{21} = \frac{G m_2 m_1}{r_{21}^2} = \frac{6.67\times10^{-11}(800)(800)}{(0.1)^2} = 4.27\times10^{-3}\,N$$

this force is directed in the x-direction.

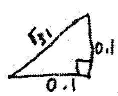
$$F_{41} = \frac{G m_4 m_1}{r_{41}^2} = \frac{6.67\times10^{-11}(800)(800)}{(0.1)^2} = 4.27\times10^{-3}\,N$$

this force is directed in the y-direction

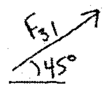
$$F_{31} = \frac{G m_3 m_1}{r_{31}^2} = \frac{6.67\times10^{-11}(800)(800)}{(0.1)^2+(0.1)^2} = 2.13\times10^{-3}\,N$$

from pythagorus

this force has x- and y- components



$r_{31}^2 = (0.1)^2 + (0.1)^2$



$$F_{31}^x = 2.13\times10^{-3}\cos 45 = 1.51\times10^{-3}\,N$$

$$F_{31}^y = 2.13\times10^{-3}\sin 45 = 1.51\times10^{-3}\,N$$

The net force is the vector sum of all three forces.

$$F_{net}^x = 4.27\times10^{-3} + 0 + 1.51\times10^{-3} = 5.78\times10^{-3}\,N$$

$$F_{net}^y = 0 + 4.27\times10^{-3} + 1.51\times10^{-3} = 5.78\times10^{-3}\,N$$

$$|F| = \sqrt{F_x^2 + F_y^2} = 8.17\times10^{-3}\,N$$

$$\tan\theta = \frac{F_y}{F_x} = 45^\circ \qquad \boxed{F = 8.17\times10^{-3}\,N @ 45^\circ}$$

NOTE: The specific angle the net force makes depends on the mass you choose.

For any of the masses here, the net force is directed along the diagonal towards the opposite mass.

Question 7.

$m_{Earth} = 5.97\times10^{24}\,kg.$

$r_{Earth} = 6370\,Km.$

$r_{moon} = 1738\,Km$

$g_{moon} = 1.62\,m/s^2$

$\frac{\rho_{moon}}{\rho_{Earth}}$ ← what we are seeking.

recall $\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3}$ ↑ sphere.

$\rho_{moon} = \frac{m_{moon}}{\frac{4}{3}\pi r_{moon}^3}$ $\quad \rho_{Earth} = \frac{m_{Earth}}{\frac{4}{3}\pi r_{Earth}^3}$

$$\frac{\rho_{moon}}{\rho_{Earth}} = \frac{m_{moon}\, r_{Earth}^3}{r_{moon}^3\, m_{Earth}} = \frac{m_{moon}}{m_{Earth}}\left(\frac{r_{Earth}}{r_{moon}}\right)^3$$

$$= \frac{m_{moon}}{5.97\times10^{24}}\left(\frac{6370}{1738}\right)^3$$

how to get m_{moon}?

We know

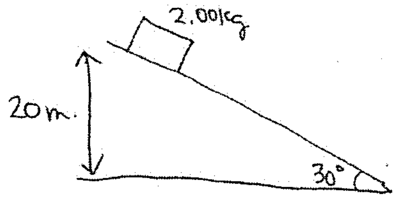
$$g_{moon} = \frac{G m_{moon}}{r_{moon}^2}$$

$$1.62 = \frac{6.67\times10^{-11}\, m_{moon}}{(1738\times10^3)^2}$$

$$m_{moon} = 7.336\times10^{22}\,kg.$$

$$\frac{\rho_{moon}}{\rho_{Earth}} = \frac{7.336\times10^{22}}{5.97\times10^{24}}\left(\frac{6370}{1738}\right)^3 \approx \boxed{0.605}$$

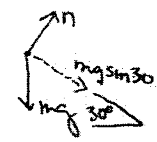
Question 8.



a): $U_g = mgh$

$= 2(9.8)(20) = 392\,J$

b) $F_{net} = ma$



$mg\sin 30 = ma$

$a = 9.8\sin 30$

$a = 4.9\,m/s^2$ down incline

$V = V_0 + at$

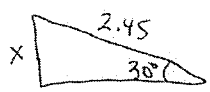
$V = 0 + 4.9(1) = 4.9\,m/s.$

↑ starts from rest

c) at 1s, the box has slid a distance down the incline

$\Delta x = v_0 t + \frac{1}{2}at^2$

$\Delta x = 0 + \frac{1}{2}(4.9)(1)^2 = 2.45\,m$



this corresponds to a height

$x = 2.45\sin 30 = 1.225\,m$

(down from the 20m it was initially above the ground)

$U_g = mg(20 - 1.225) = 2(9.8)(18.775) = 368\,J$

$K = \frac{1}{2}mv^2 = \frac{1}{2}(2)(4.9)^2 = 24\,J$ (total is still 392J)

d) at the bottom

$E_{initial} = E_{final}$

$U_g + K = U_g + K$

$mgh + 0 = 0 + \frac{1}{2}mv^2$

$2(9.8)(20) = \frac{1}{2}(2)v^2$

$v = 19.8\,m/s.$

the kinetic energy should be equal to the potential energy at the top, since all of it is transferred (no loss due to friction etc...)

Question 9.

* I and II slide with no friction so the sphere I does not roll. In both cases: $U_i + \cancel{K_i} + \cancel{W_{nc}} = K_f + \cancel{U_f}$

$$mgh = \frac{1}{2}mv^2$$

which means that they have the same final kinetic energy.

* III has friction on it so if we assume that the sphere slides without slipping, we can write

$$U_i + \cancel{K_i} + \cancel{W_{nc}} = K_{f\,lin} + K_{f\,rot} + \cancel{U_f}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

for a sphere $I = \frac{2}{5}mr^2$

for rolling without slipping $v = \omega r$

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{5}mv^2$$

Plugging back into the conservation of energy statement we find:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$mgh = \frac{7}{10}mv^2$$

→ So the sphere's speed at the bottom of the incline is <u>smaller</u> than in cases I and II

(<u>Not convinced</u>? in case III we have $v = \sqrt{\frac{10}{7}gh}$, in cases I and II we have $v = \sqrt{2gh}$)

You should also take a minute to think about the fact that the potential energy goes into translational kinetic energy <u>and</u> rotational kinetic energy so it makes sense that v in case III should be smaller. The total amount of final kinetic energy in cases I, II and III is the same, however (equal to mgh)

* Now if the block in $\underline{IV}$ slides with friction, the conservation of energy statement becomes

$$U_i + \cancel{K_i} + W_{nc} = \cancel{U_f} + K_f$$

$$mgh - F_F \cdot s = \frac{1}{2} m v^2$$

The work done by friction transforms mechanical energy into thermal energy so the final kinetic energy is smaller than in cases I, II, III

And after all this we determine that the answer is $\underline{\underline{d}}$!

#10

$$K_{total} = K_{translational} + K_{rotational}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{4}m\cancel{r^2}\left(\frac{v^2}{\cancel{r^2}}\right)$$

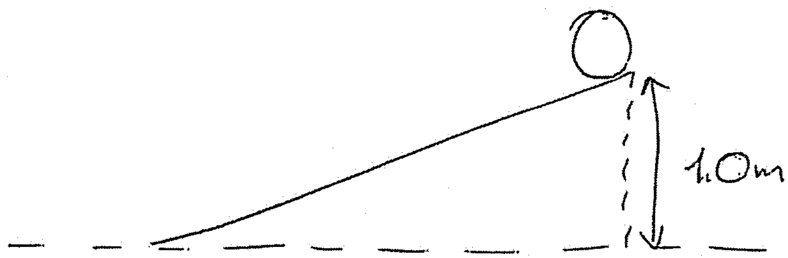
$$= \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$= \frac{3}{4}mv^2$$

$$= \frac{3}{4}(10)(5)^2$$

$$= 187.5\ \text{J}$$

(11)



a) frictionless ramp.

$$U_i + \cancel{K_i}^{\,0} + \cancel{W_{nc}}^{\,0} = \cancel{U_f}^{\,0} + K_f$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \cdot 9.81 \cdot 1} = \underline{\underline{4.4}}\ \frac{m}{s}$$

b) ramp with enough friction for rolling without slipping.

$$U_i + \cancel{K_i} + \cancel{W_{nc}} = \cancel{U_f} + K_f$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$mgh = \frac{3}{4}mv^2$$

$$v = \sqrt{\frac{4}{3} \cdot g \cdot h} = \sqrt{\frac{4}{3} \cdot 9.81} = \underline{\underline{3.62}}\ \frac{m}{s}$$